A Strongly Polynomial Algorithm for Linear Programs with at most Two Non-zero Entries per Row or Column

Daniel Dadush
Centrum Wiskunde & Informatica, Amsterdam, The Netherlands

Zhuan Khye Koh
Centrum Wiskunde & Informatica, Amsterdam, The Netherlands

Bento Natura
Columbia University, New York, USA

Neil Olver
London School of Economics and Political, UK

László A. Végh
University of Bonn, Germany

Abstract

We give a strongly polynomial algorithm for minimum cost generalized flow, and hence for optimizing any linear program with at most two non-zero entries per row, or at most two non-zero entries per column. Primal and dual feasibility were shown by Végh (MOR '17) and Megiddo (SICOMP '83) respectively. Our result can be viewed as progress towards understanding whether all linear programs can be solved in strongly polynomial time, also referred to as Smale's 9th problem.

Our approach is based on the recent primal-dual interior point method (IPM) due to Allamigeon, Dadush, Loho, Natura and Végh (FOCS '22). The number of iterations needed by the IPM is bounded, up to a polynomial factor in the number of inequalities, by the *straight line complexity* of the central path. Roughly speaking, this is the minimum number of pieces of any piecewise linear curve that multiplicatively approximates the central path.

As our main contribution, we show that the straight line complexity of any minimum cost generalized flow instance is polynomial in the number of arcs and vertices. By applying a reduction of Hochbaum (ORL '04), the same bound applies to any linear program with at most two non-zeros per column or per row.

To be able to run the IPM, one requires a suitable initial point. For this purpose, we develop a novel multistage approach, where each stage can be solved in strongly polynomial time given the result of the previous stage. Beyond this, substantial work is needed to ensure that the bit complexity of each iterate remains bounded during the execution of the algorithm. For this purpose, we show that one can maintain a representation of the iterates as a low complexity convex combination of vertices and extreme rays. Our approach is black-box and can be applied to any log-barrier path following method.

2012 ACM Subject Classification Theory of computation → Linear programming

Keywords and phrases Linear Programming, Strongly Polynomial Algorithms, Interior Point Methods

Digital Object Identifier 10.4230/LIPIcs.STACS.2025.74

Category Invited Talk