A Faster Algorithm for Constrained Correlation Clustering

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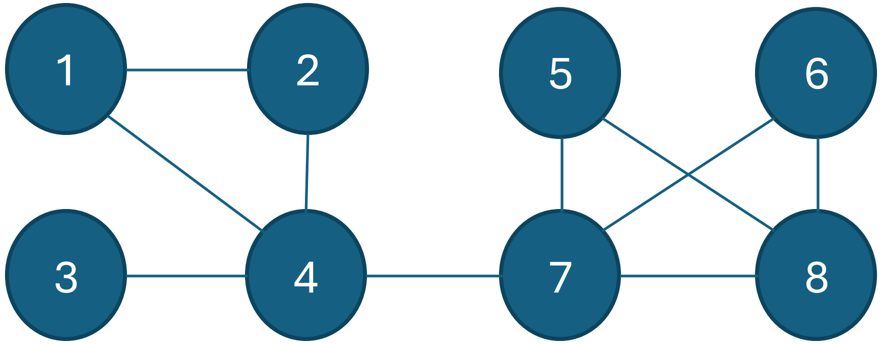
(Unconstrained) Correlation Clustering

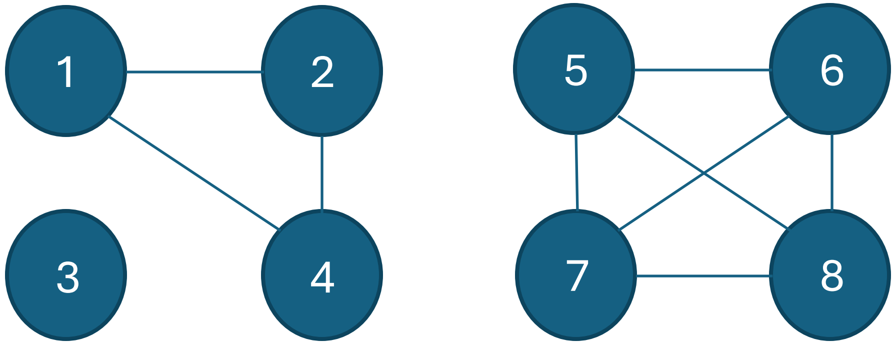
- **Input:** A complete set of *preferences*
- **Output:** Partition of vertices violating as few preferences as possible

(Unconstrained) Correlation Clustering

Input



Output (cost 3)



(Unconstrained) Correlation Clustering

- **Input:** A complete set of *preferences*

- **Output:** Partition of vertices violating as few preferences as possible

- Introduced by Bansal, Blum, and Chawla [Machine Learning '04]
- Randomized 3-apx by Ailon, Charikar, and Newman [STOC '05]
- 1.437-apx by Cao *et al.* [STOC '24]

Constrained Correlation Clustering

- **Input:** A complete set of *preferences,*
  a subset of which are *hard constraints*
- **Output:** Partition of vertices violating as few preferences as possible, and satisfying all hard constraints

- Introduced by van Zuylen, Hegde, Jain, and Williamson [SODA '07]
- 3-approximation in $O(n^{3\omega})$ time

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- **New:** 16-approximation in $\tilde{O}(n^3)$ time

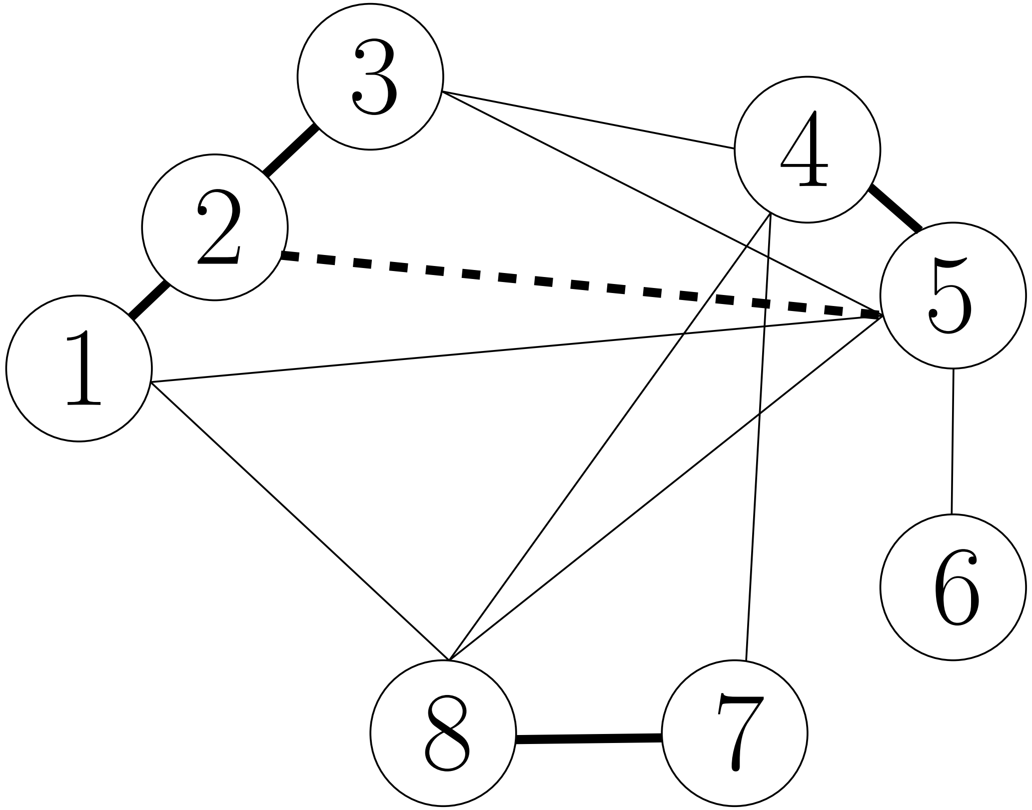
A Fast Approximation Algorithm (**new**)

1. Modify instance to one having *nice* neighborhoods
2. Forget hard constraints
3. Run *Pivot* algorithm on unconstrained instance

Nice Neighborhoods

- If (u, v) is a *friendly* pair they now have the same neighborhood
- If (u, v) is a *hostile* pair they now have no common neighbor

Transforming to Unconstrained Clustering



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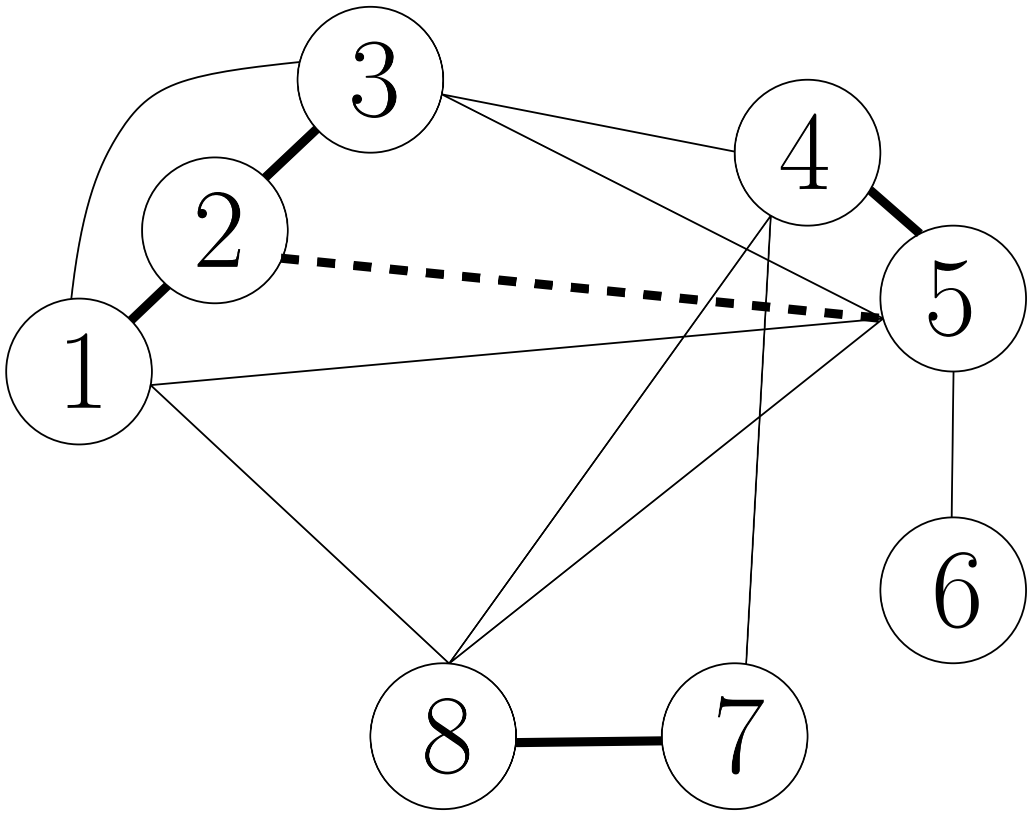

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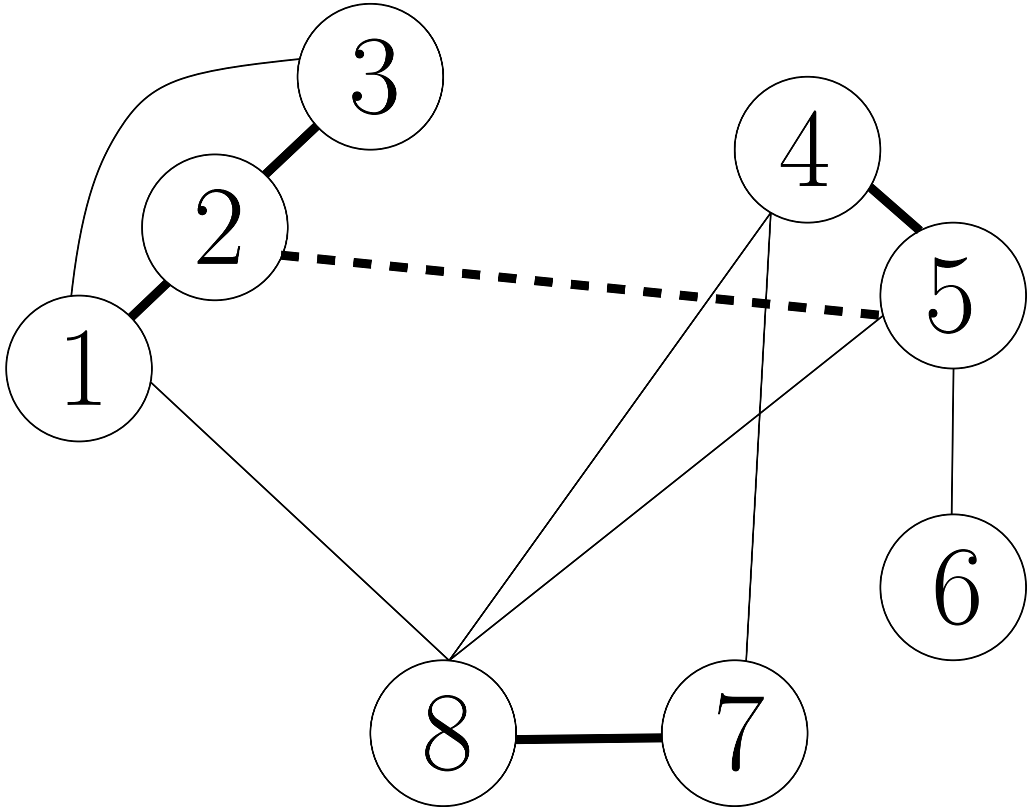

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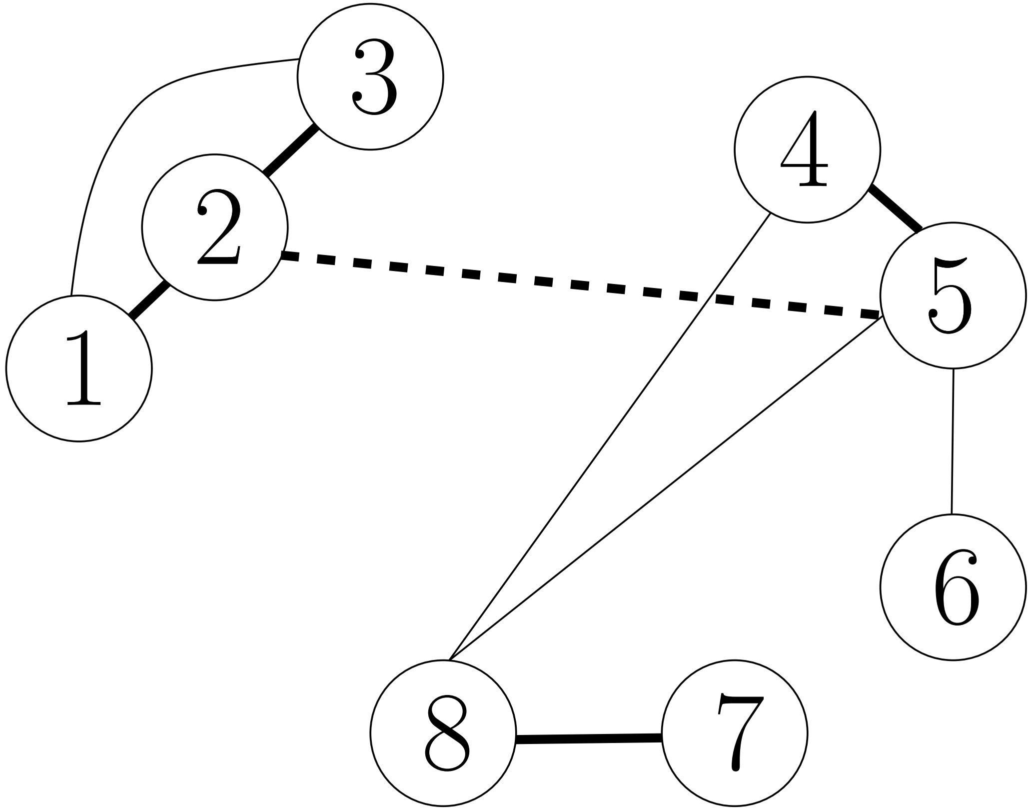

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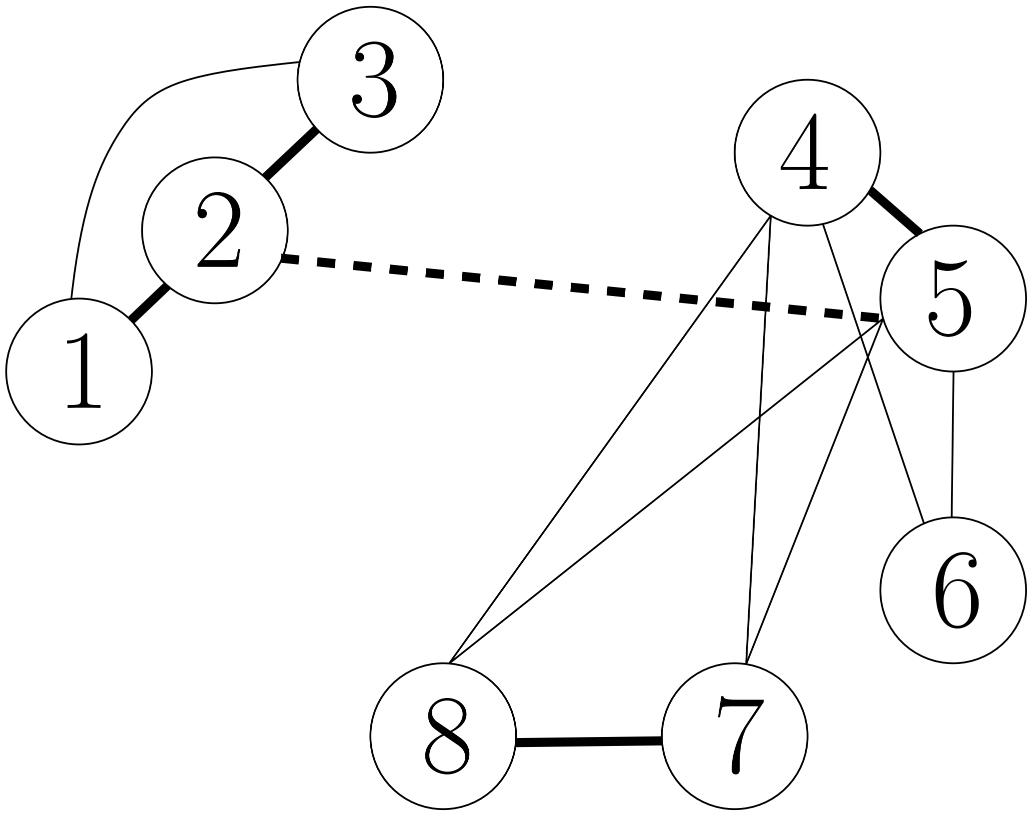

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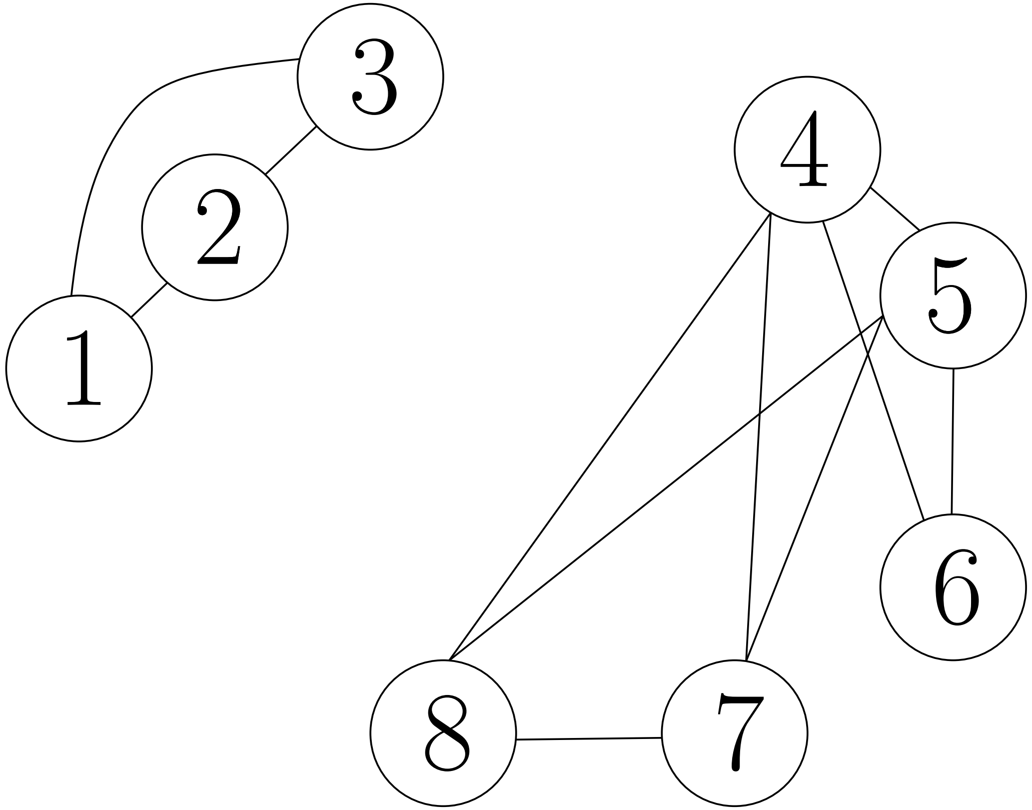

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Pivot for Unconstrained Correlation Clustering

1. $C = \emptyset$
2. while $V \neq \emptyset$ do
   3. Choose a pivot node u
   4. Add a cluster containing u and all its neighbors to C
   5. Remove u and its neighborhood from G
6. return C

Pivot for Unconstrained Correlation Clustering

1. $C = \emptyset$
2. while $V \neq \emptyset$ do
    3. Choose a pivot node u **(but how?)**
    4. Add a cluster containing u and all its neighbors to C
    5. Remove u and its neighborhood from G
6. return C

Properties

- APX: $(1+\sqrt{5}) + (2+\sqrt{5}) \cdot$ Apx(Pivot)
- Time: $O(n\, m)$ + Pivot

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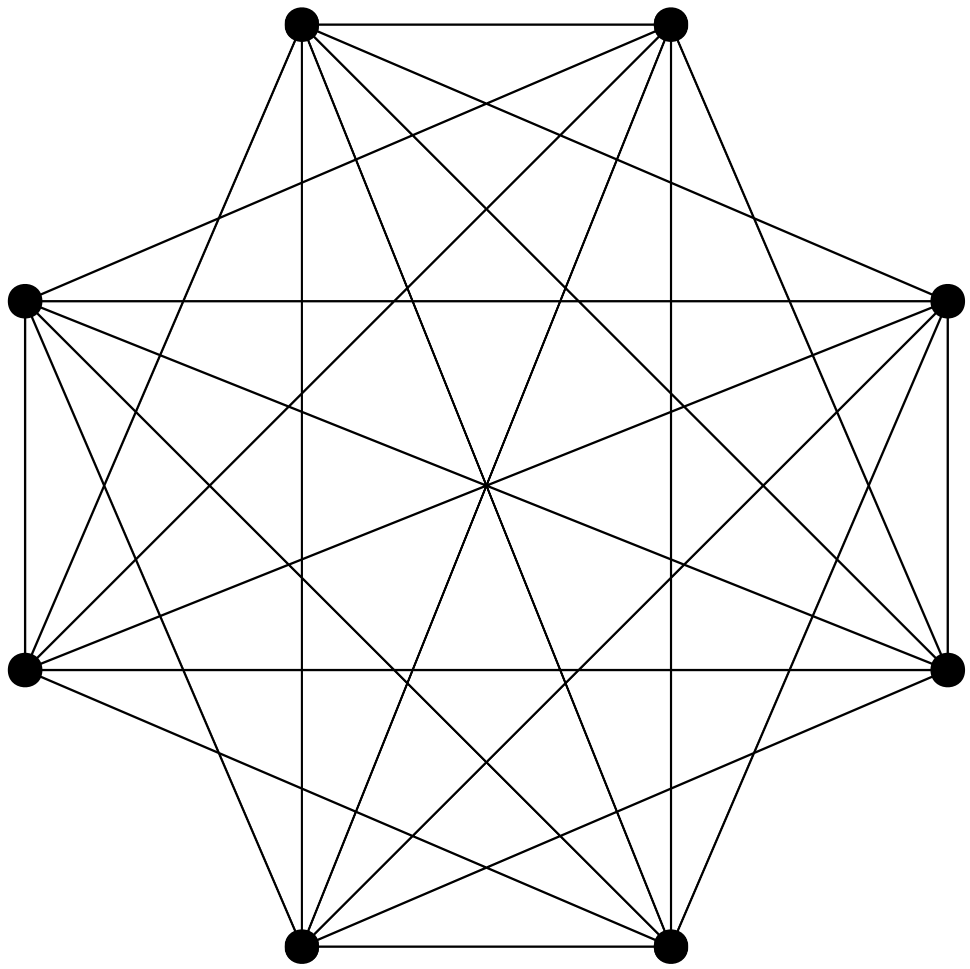
  - Solves a *covering* LP
- **Q:** Better approximations in $\tilde{O}(n^3)$ time?

The Limit of Pivoting



The Limit of Pivoting

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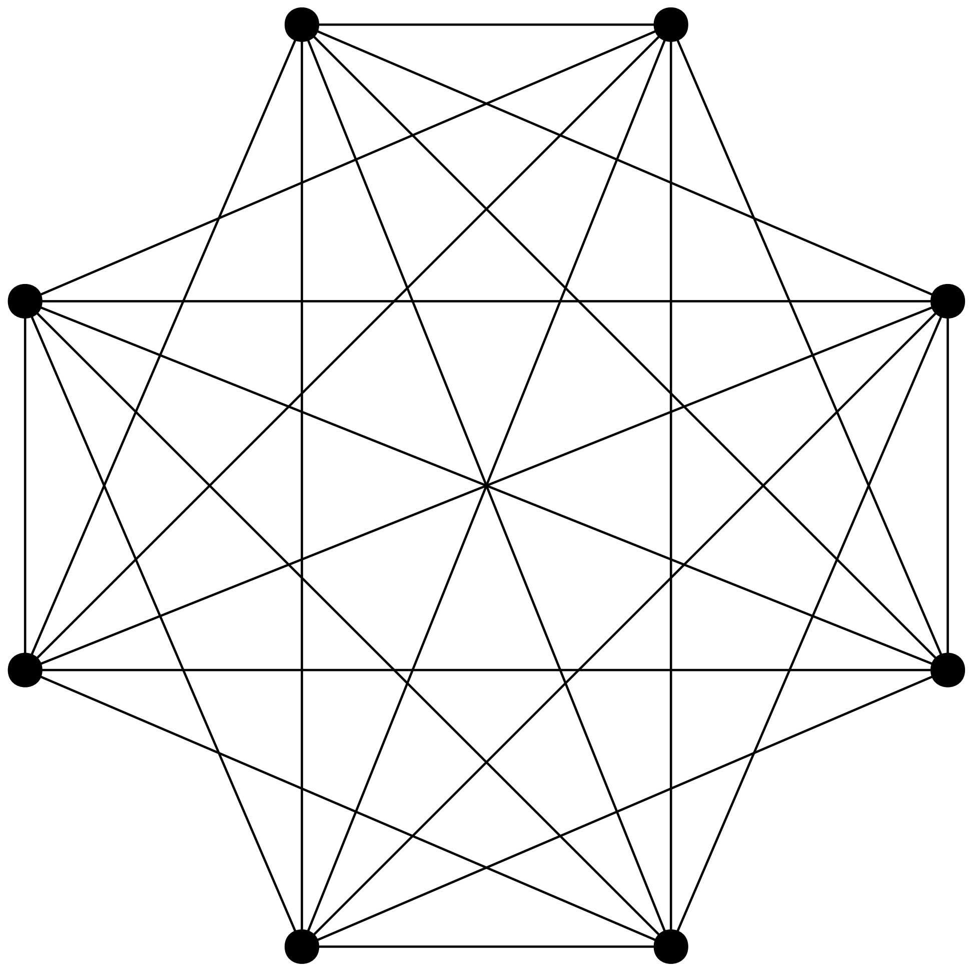
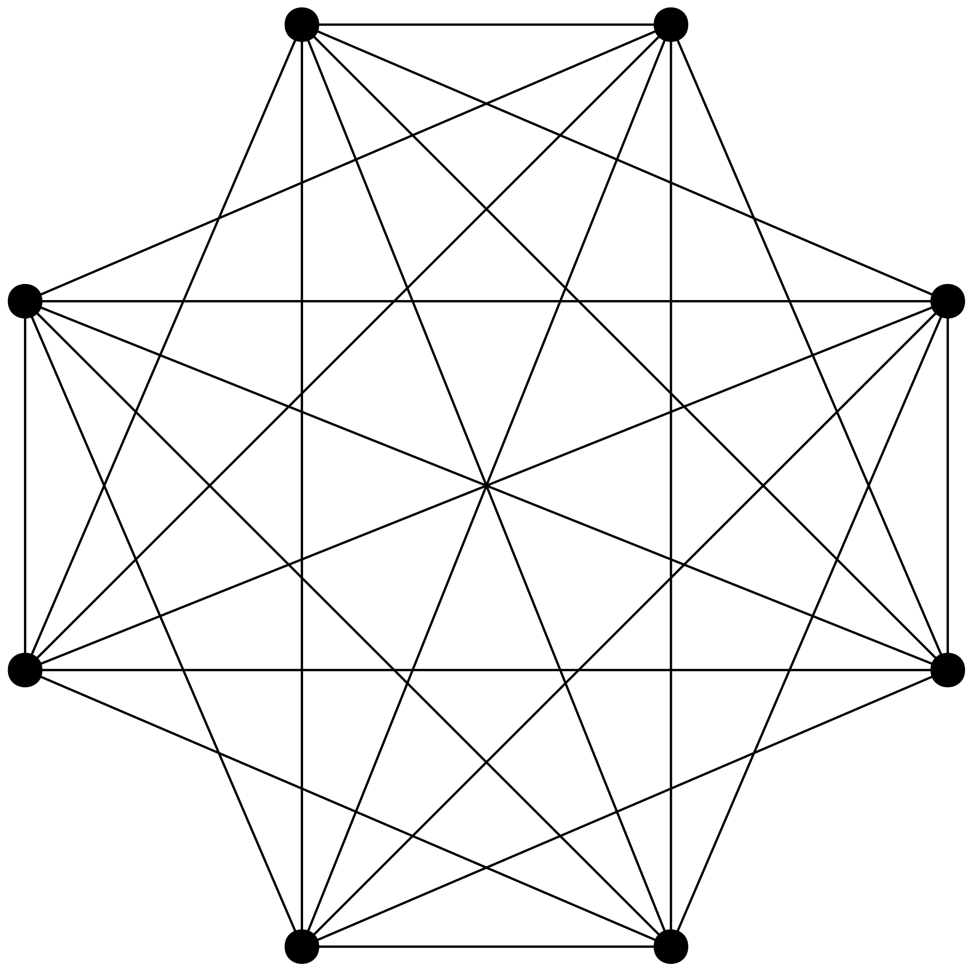


The Limit of Pivoting

- OPT: A single cluster,
  cost = $n/2$

- PIVOT: (n-1) vertices + singleton,
  cost ≈ 1/2 n + n



Thank You

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