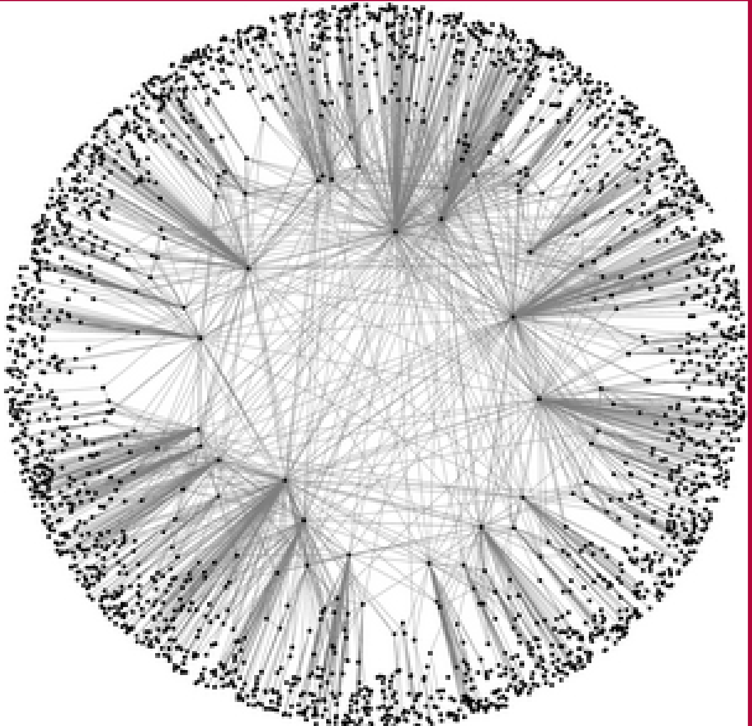
Hasso Plattner Institut
Digital Engineering · Universität Potsdam



Hyperbolic Random Graphs

Clique Number and Degeneracy

Sam Baguley, Yannic Maus, Janosch Ruff, George Skretas

STACS'25

Motivation

Theory

Practice

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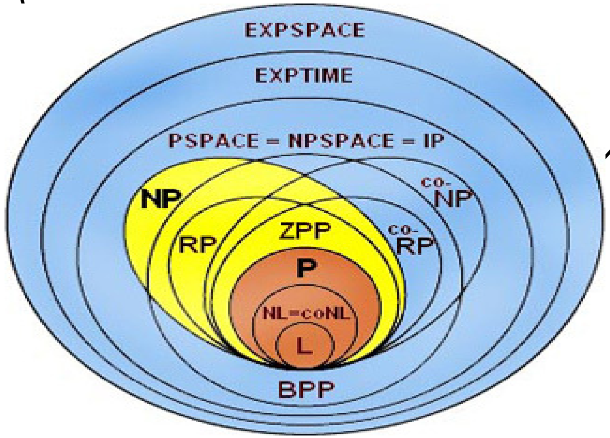

Practice

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NP-hard problems are hard.

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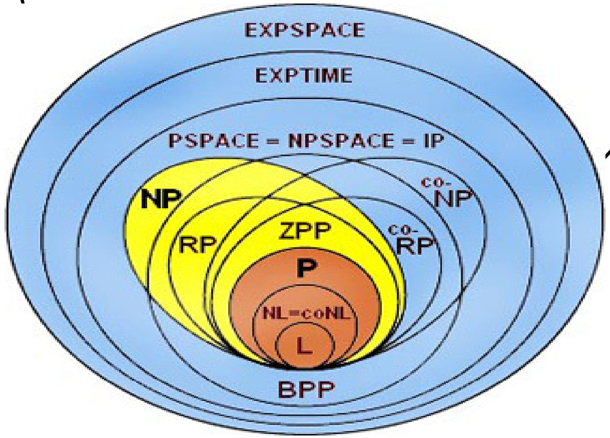

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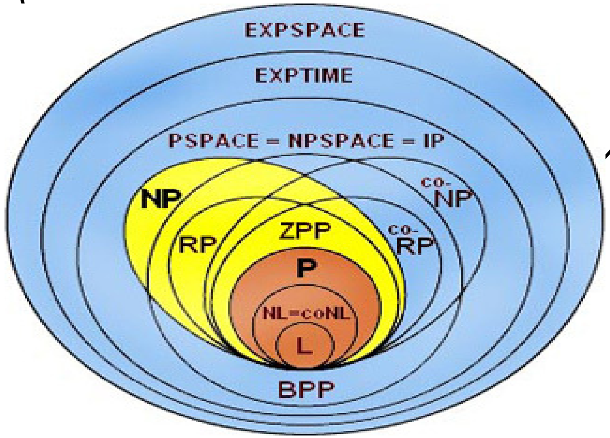

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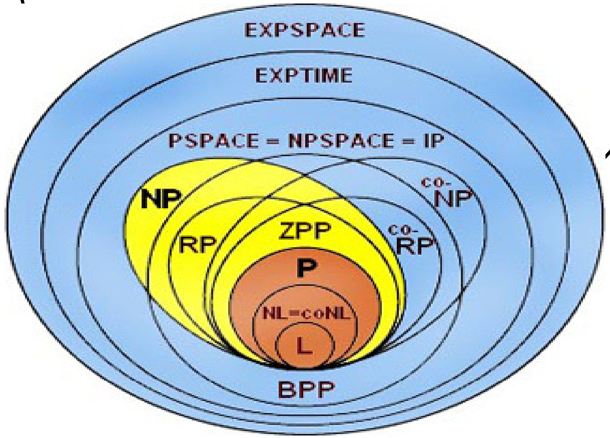

Heuristics give efficient solutions.

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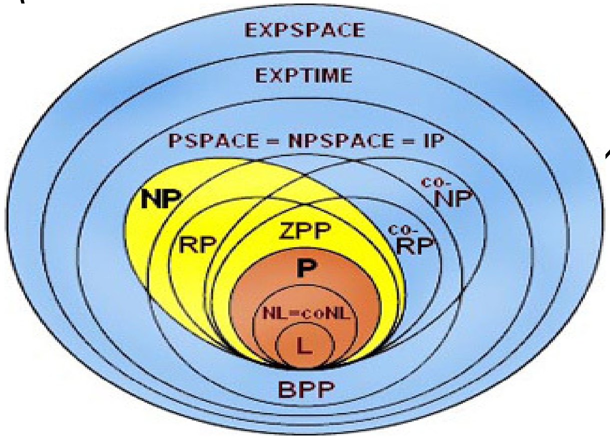

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Theory-practice-gap

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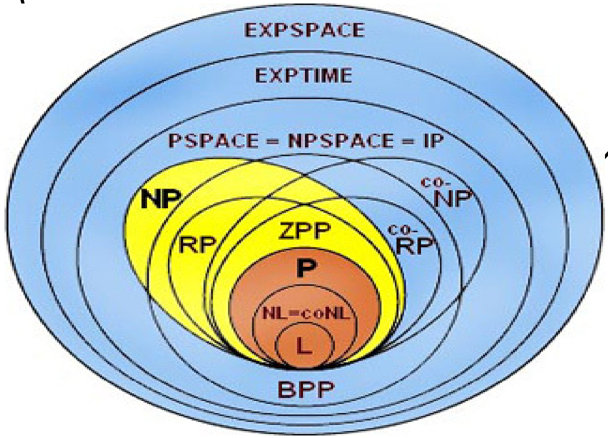

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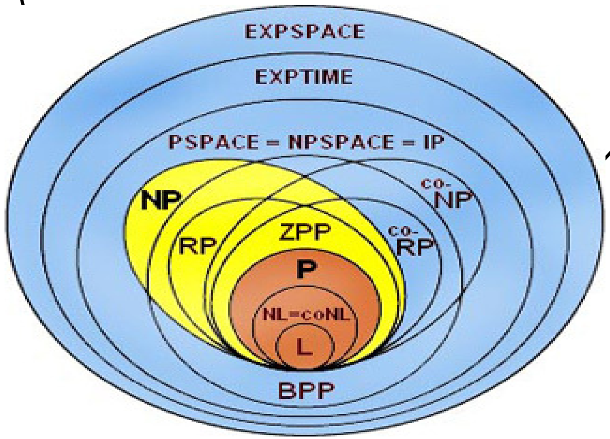

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run-time on *realistic* distributions

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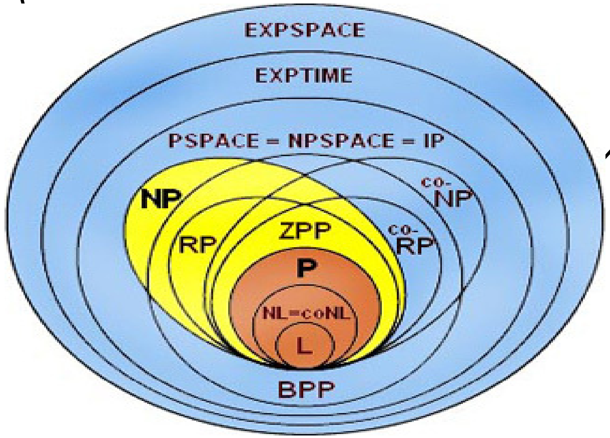

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What are *realistic* distributions?

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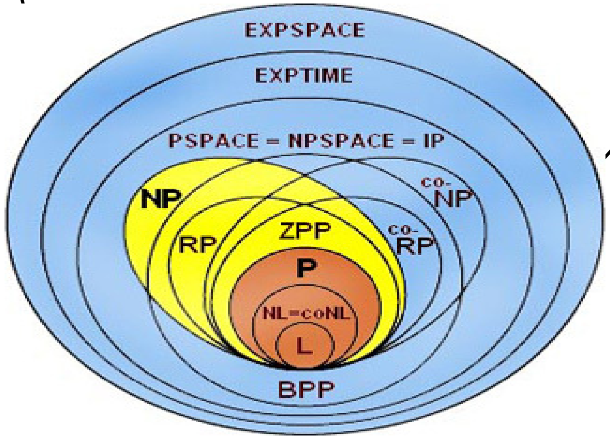

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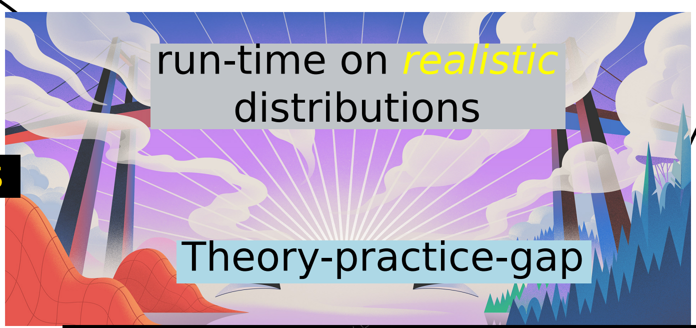

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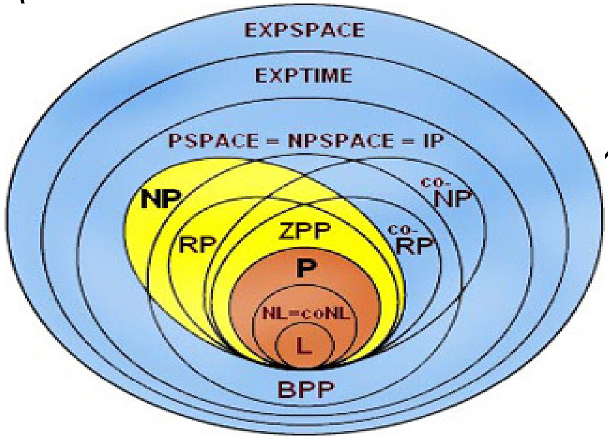

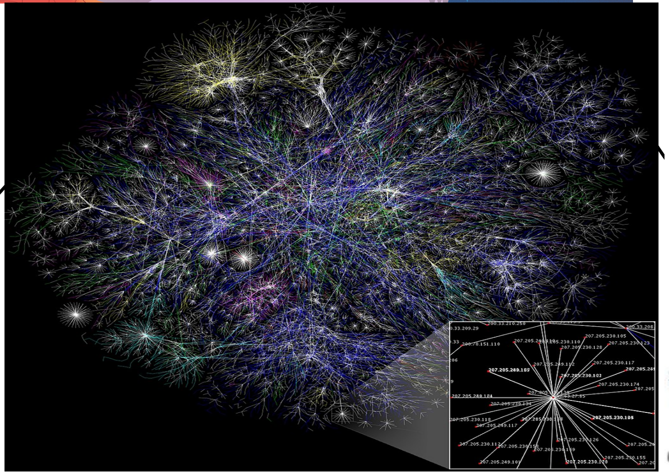

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What are their (algorithmic) properties?

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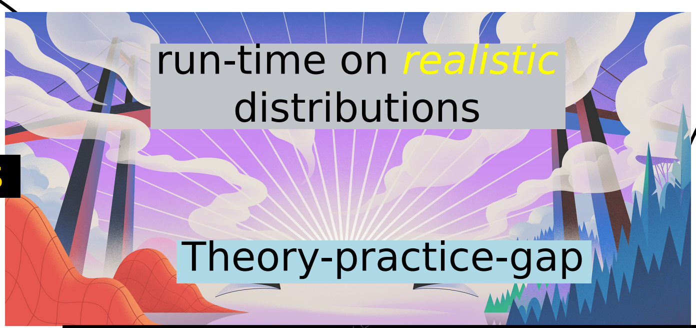

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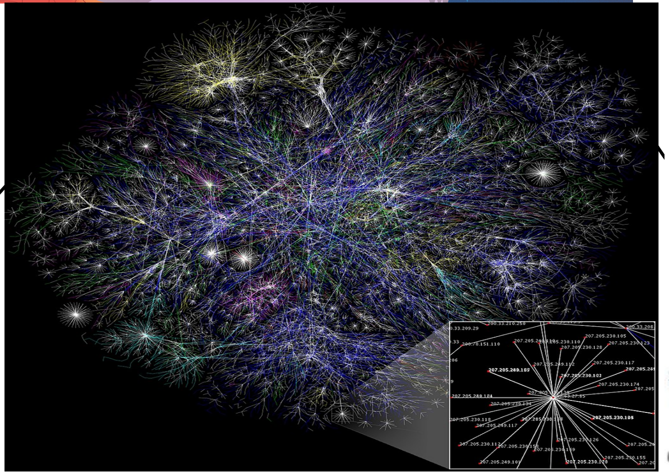

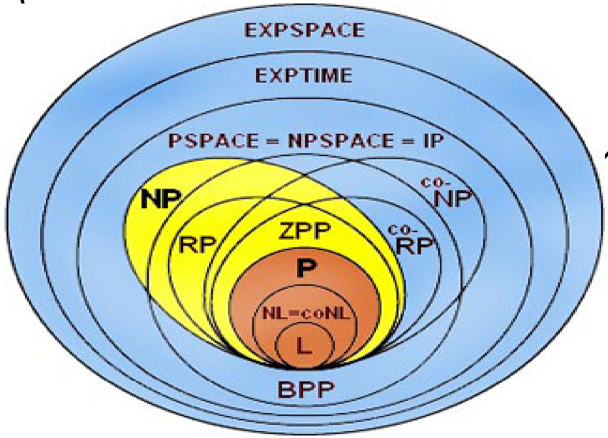

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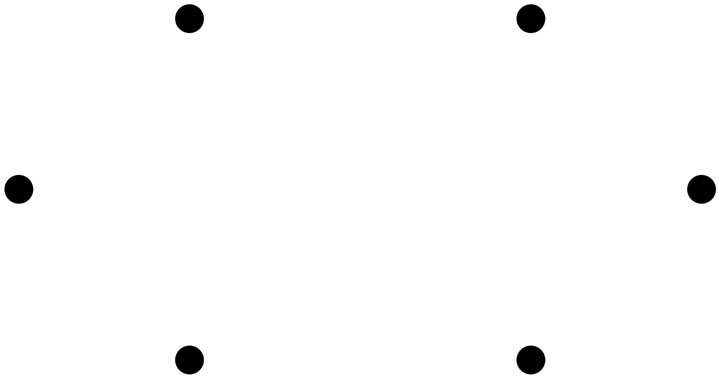

Toy example: $n = 6$

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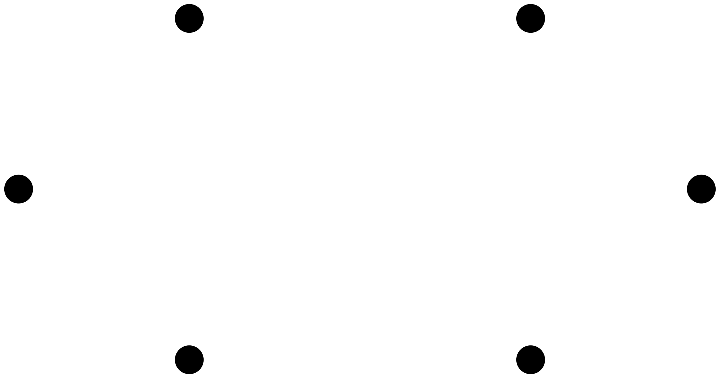

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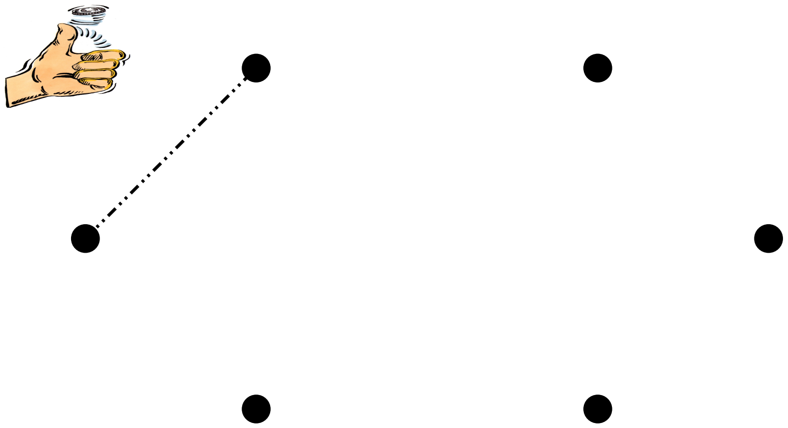

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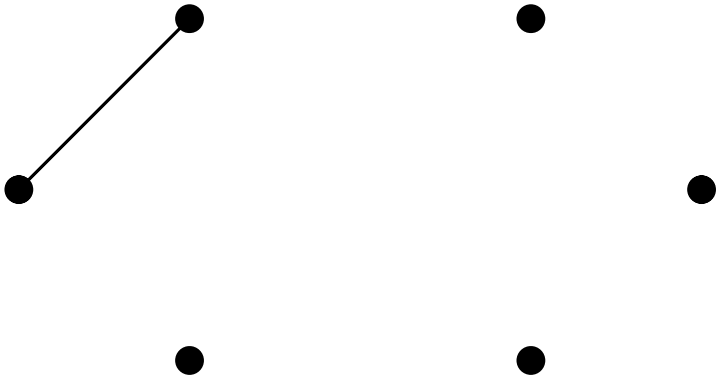

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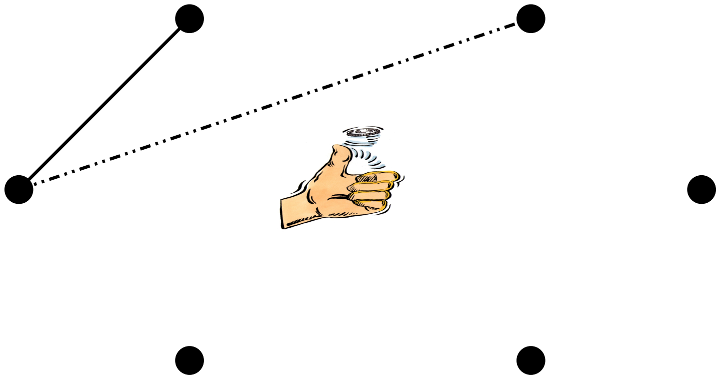

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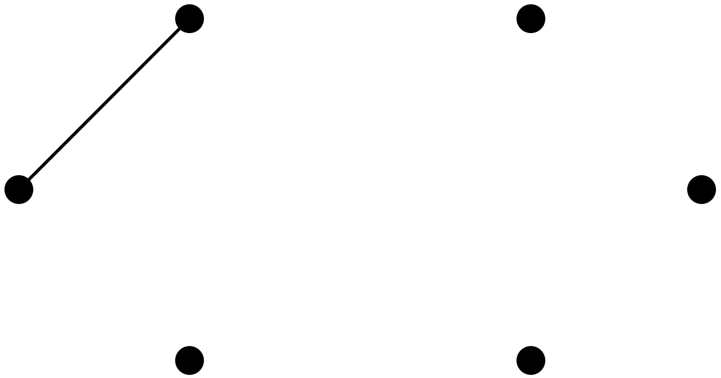

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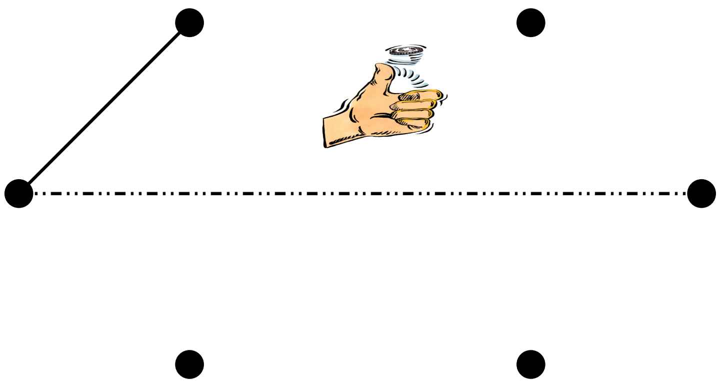

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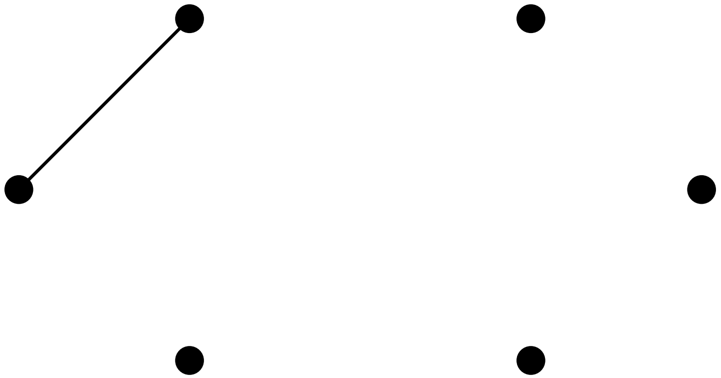

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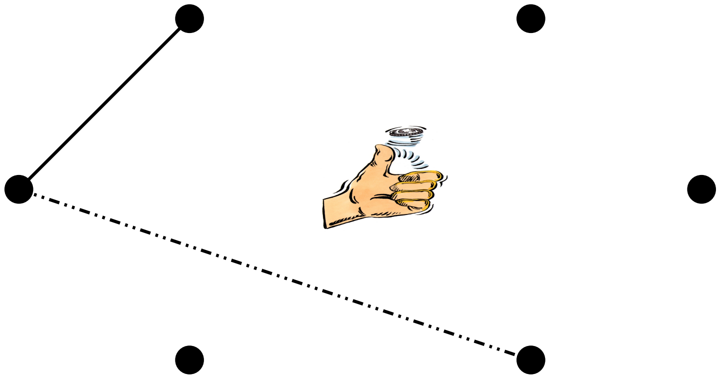

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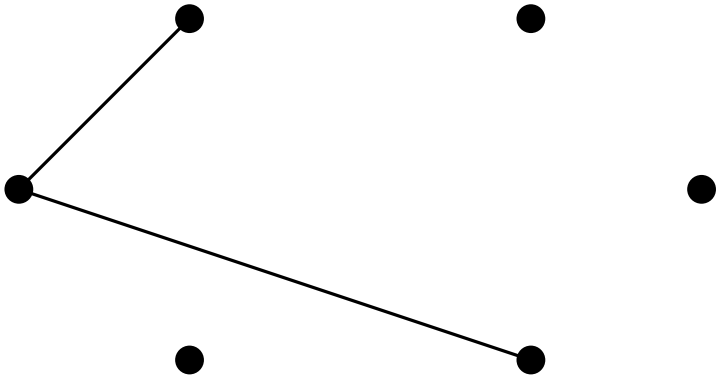

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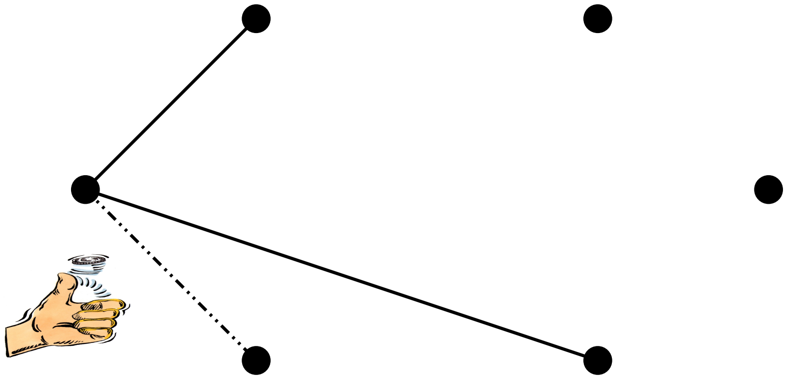

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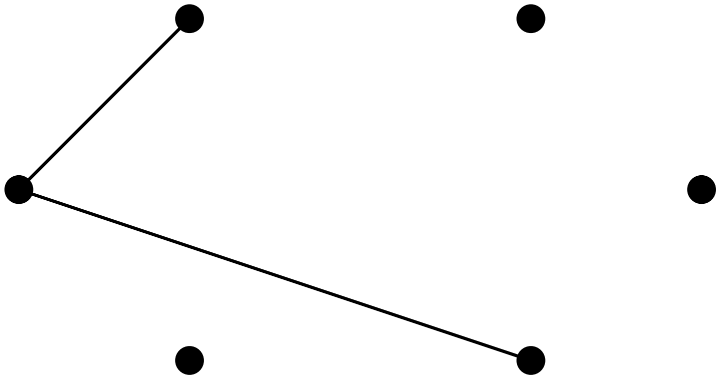

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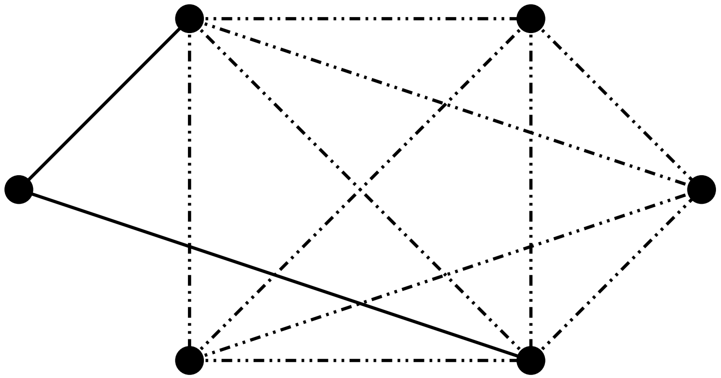

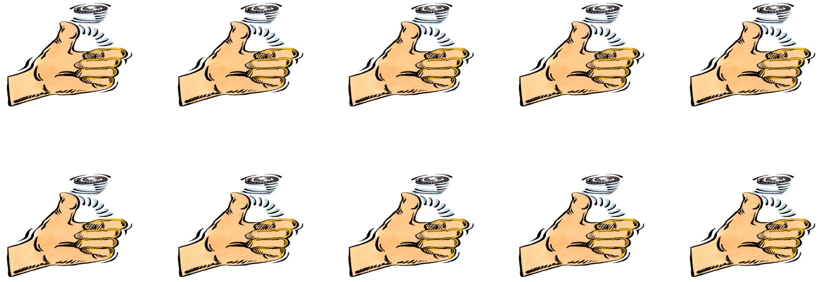

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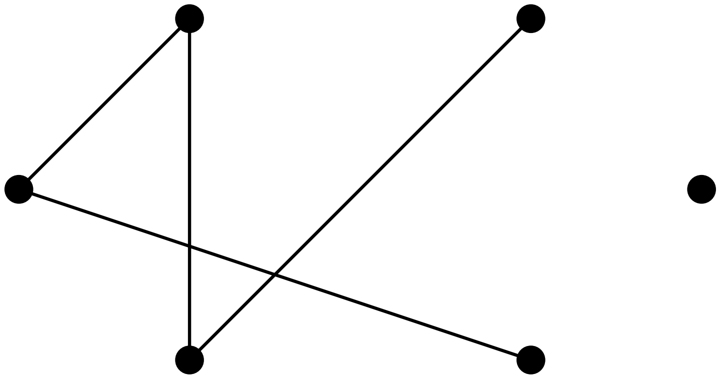

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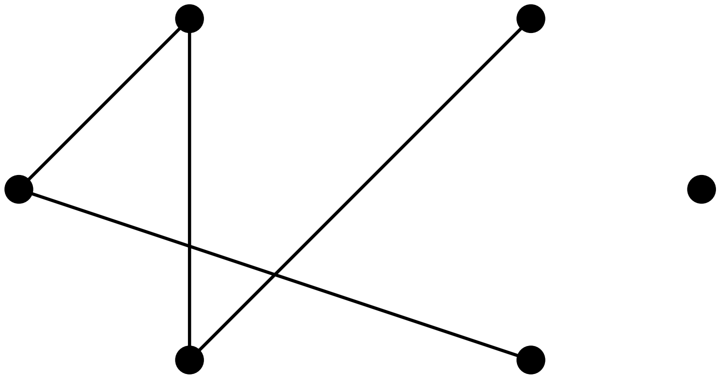

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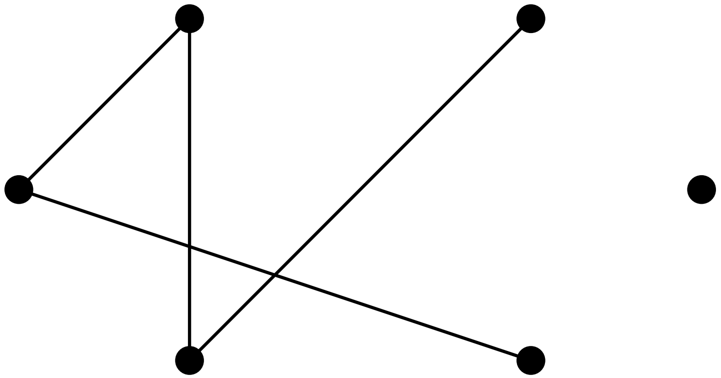

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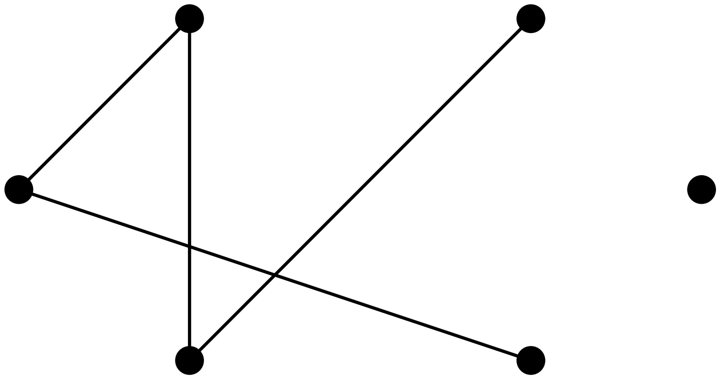

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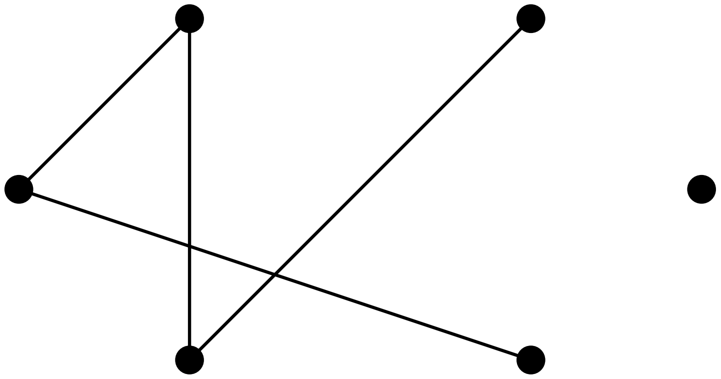

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Probability that the world continues to exist tomorrow is smaller..

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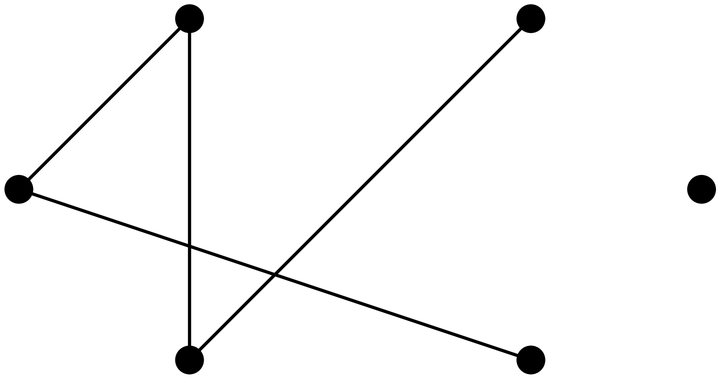

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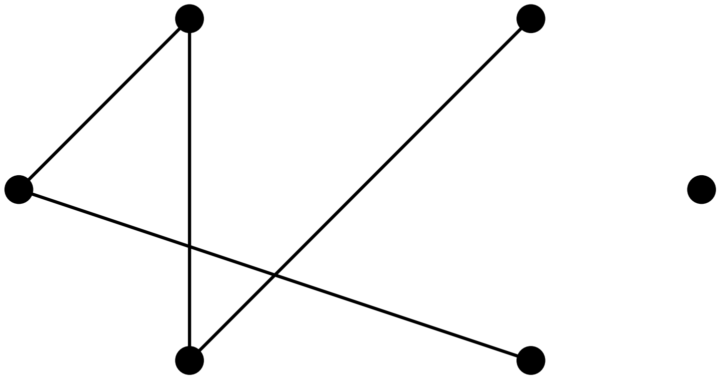

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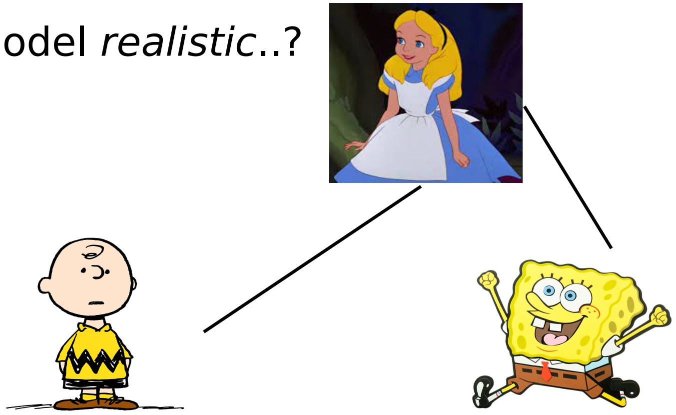

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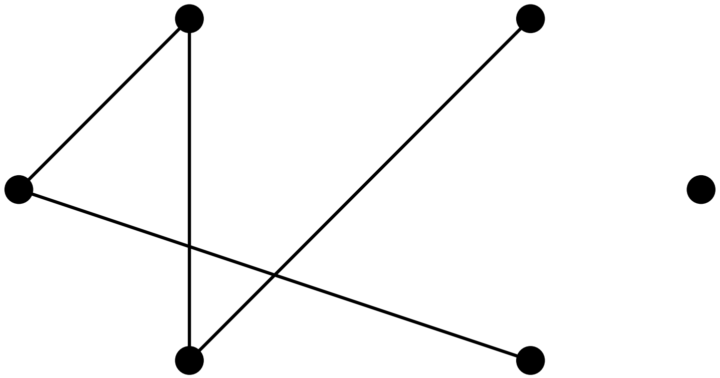

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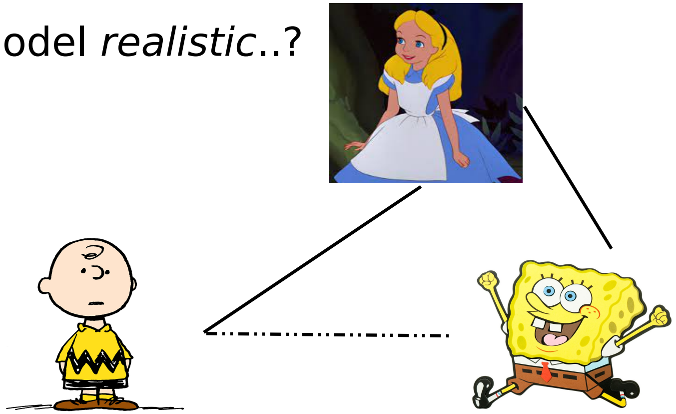

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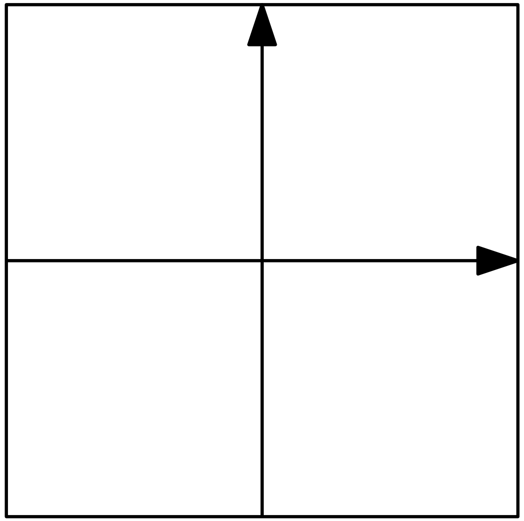

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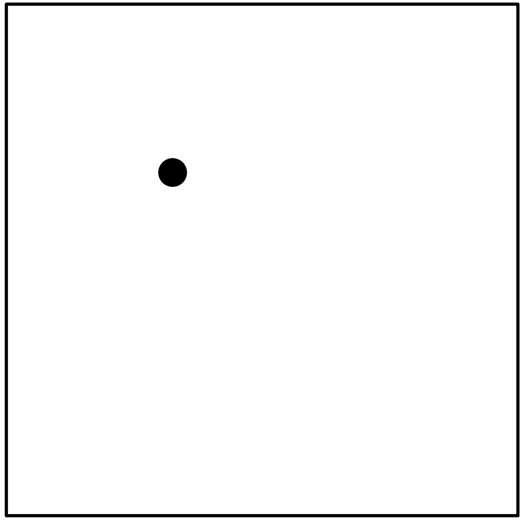

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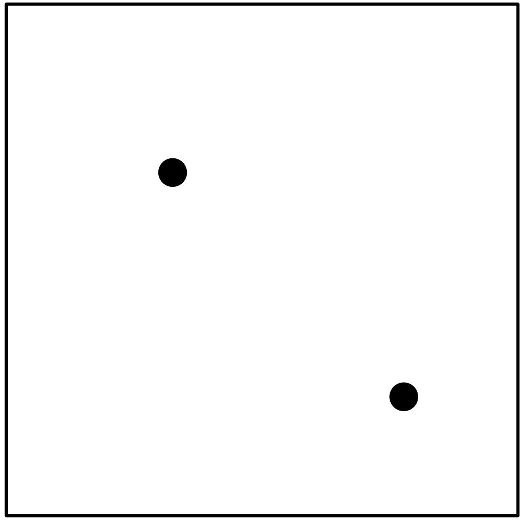

Gamble coordinates
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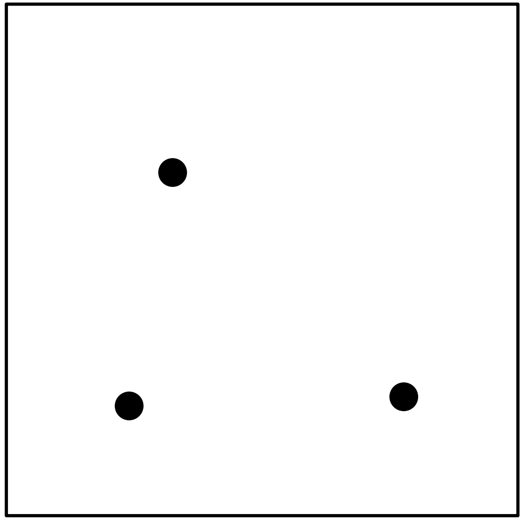

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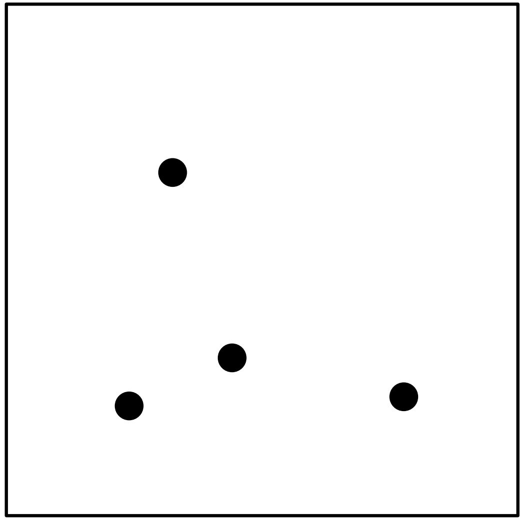

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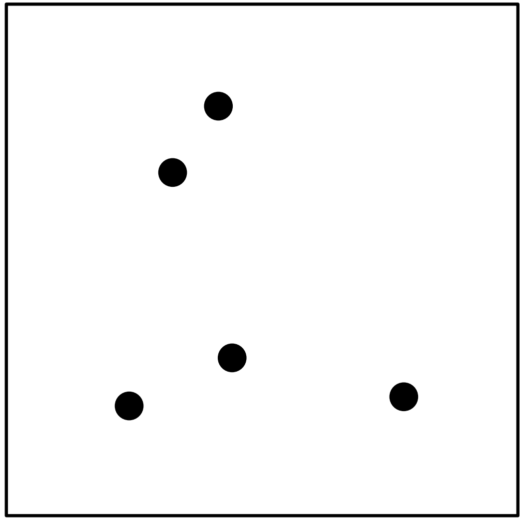

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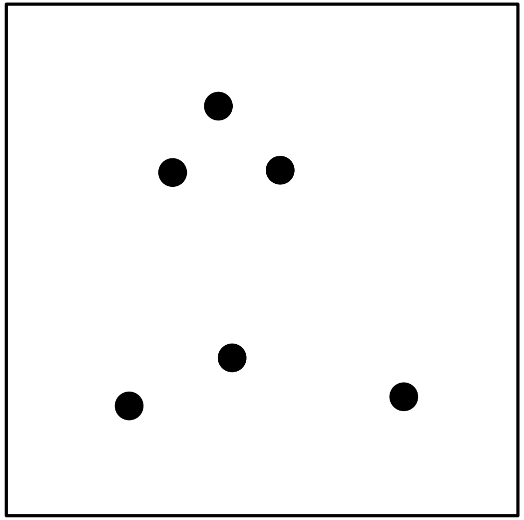

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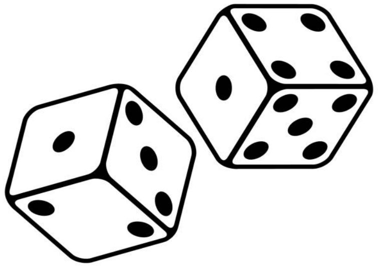

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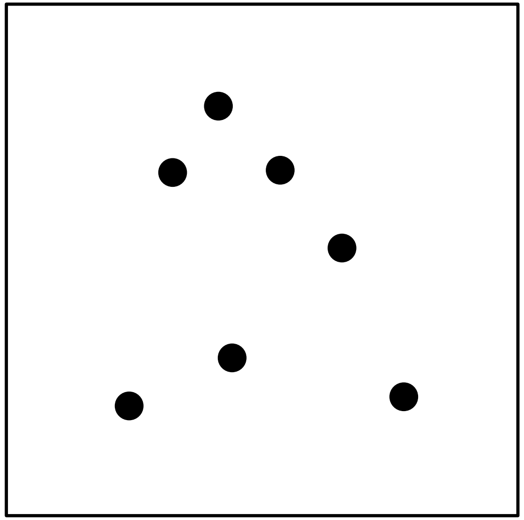

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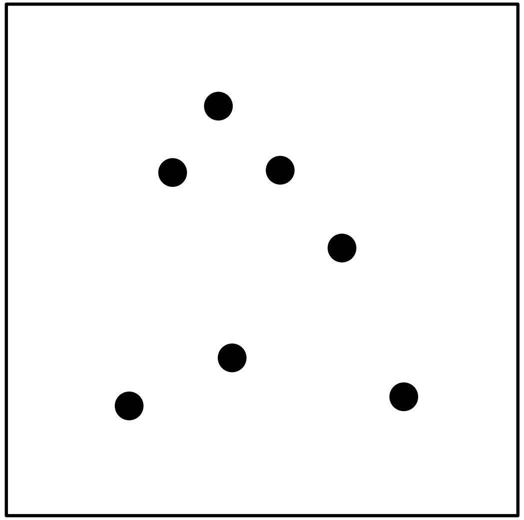

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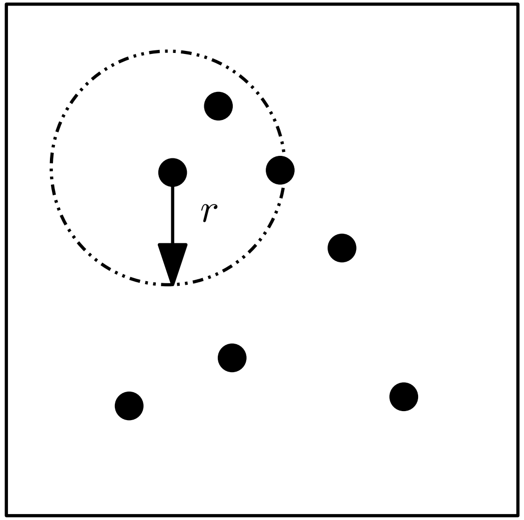

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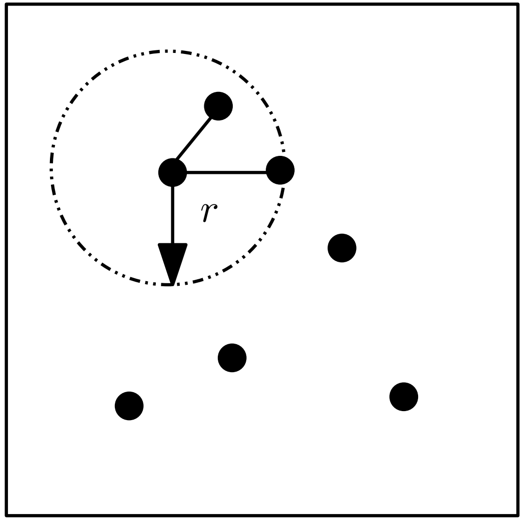

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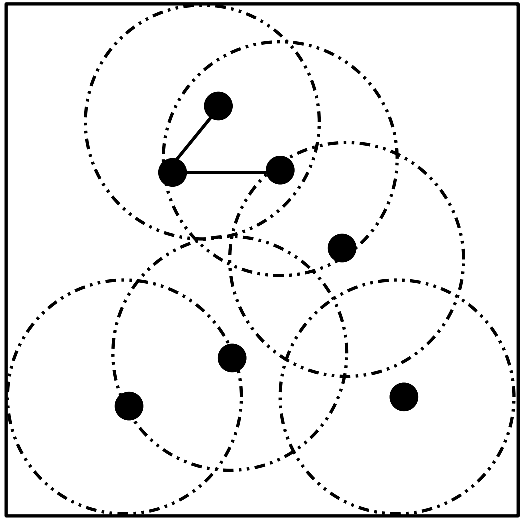

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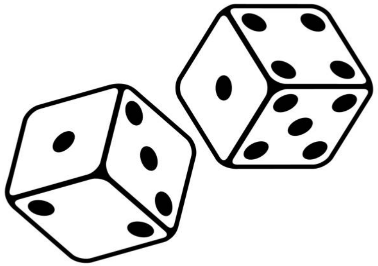

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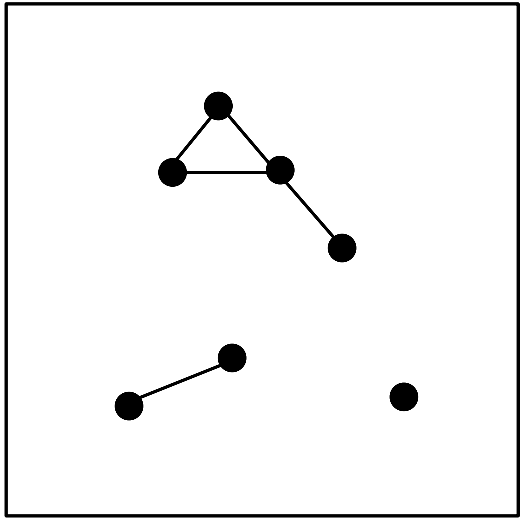

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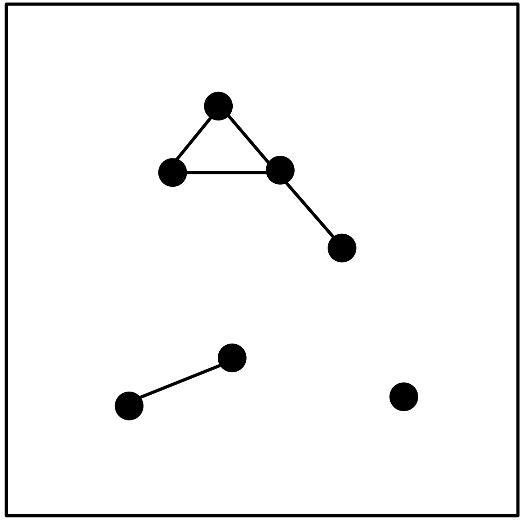

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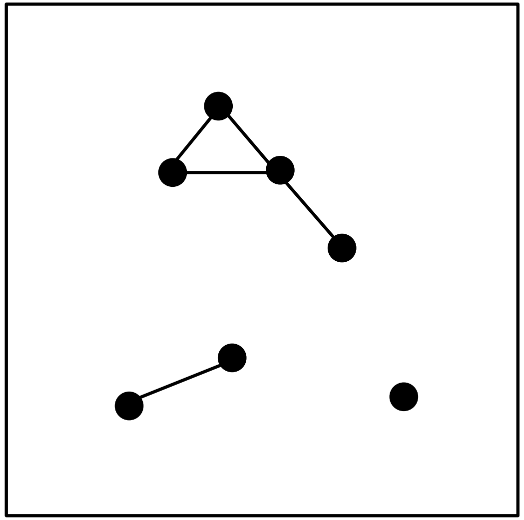

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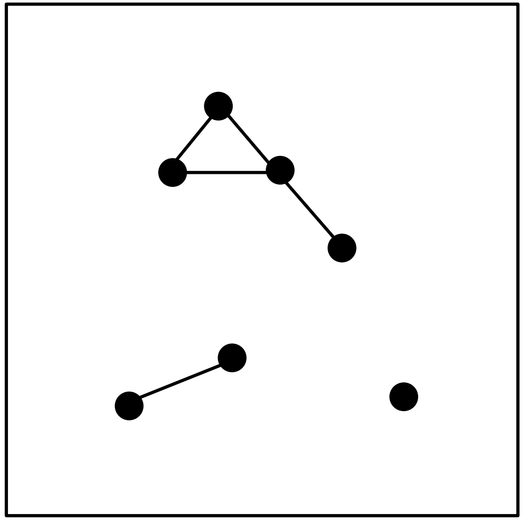

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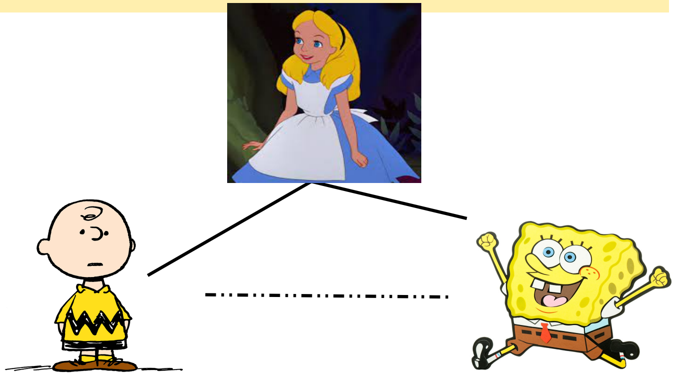

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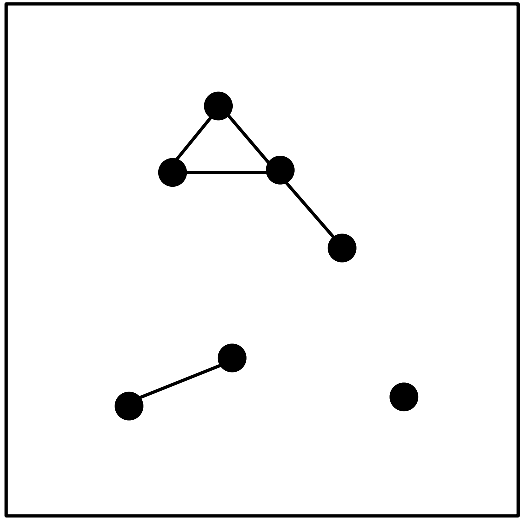

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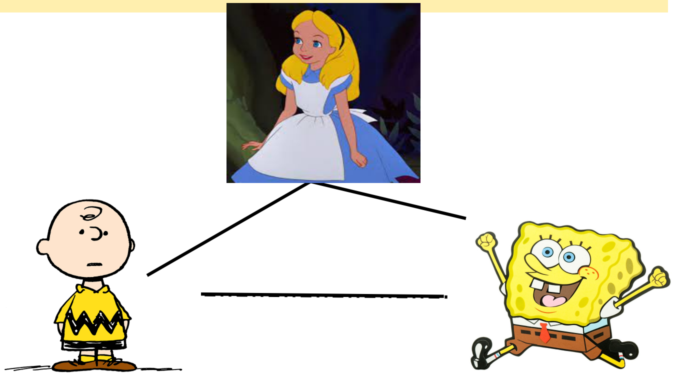
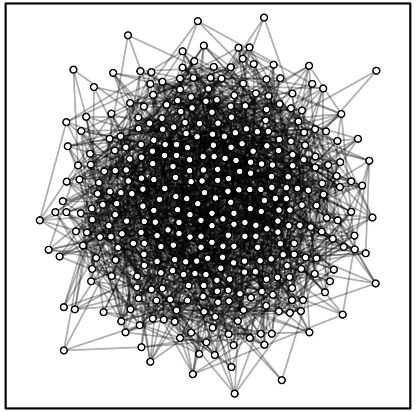


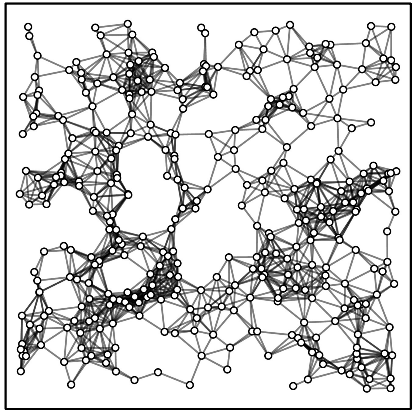
Network Science



Erdős–Réyni

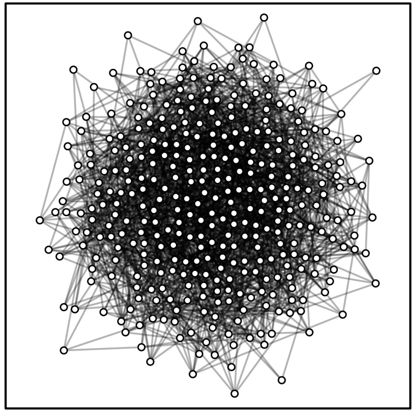
Gilbert

Network Science



geometric

locality



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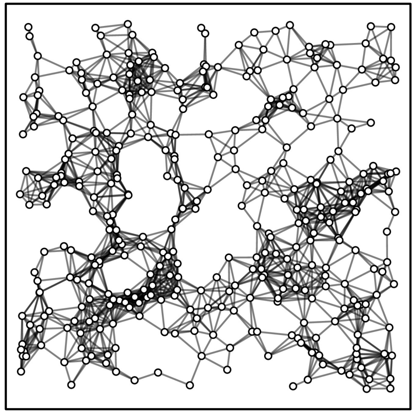

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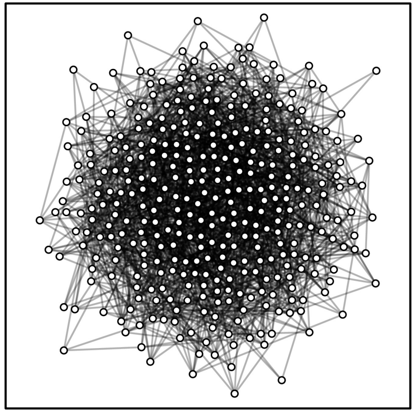
Clustering Coefficient:
Many Triangles

locality



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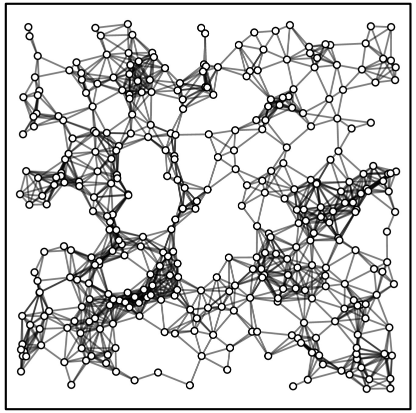

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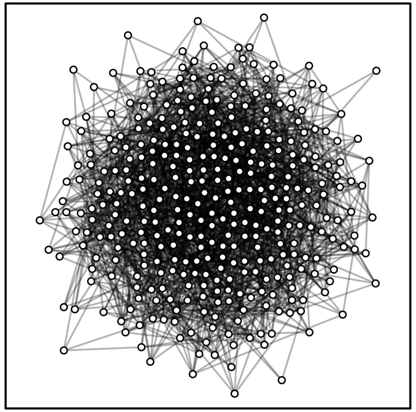

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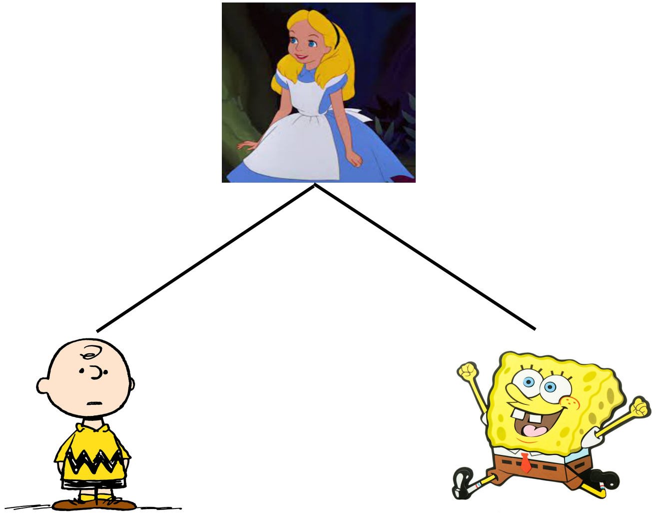

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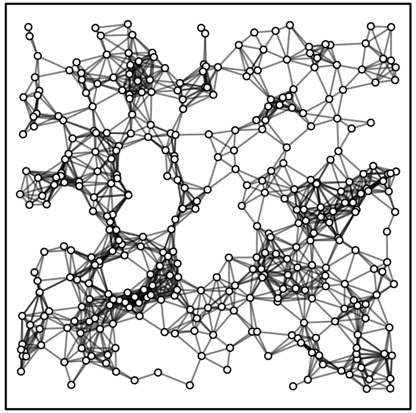

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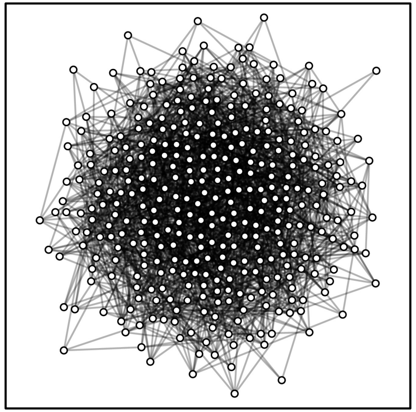

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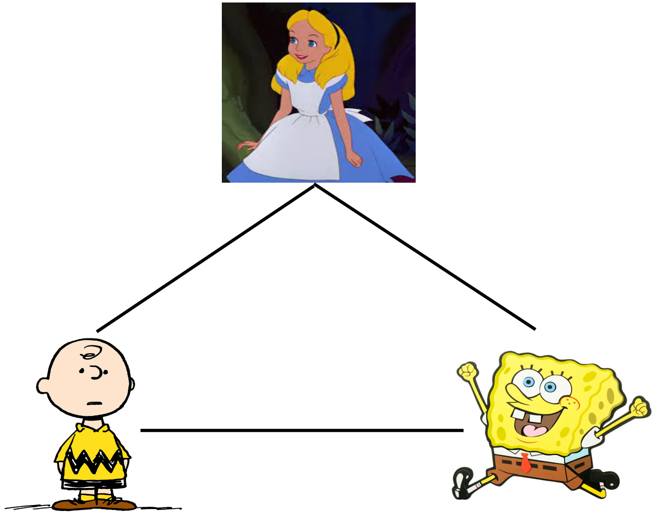

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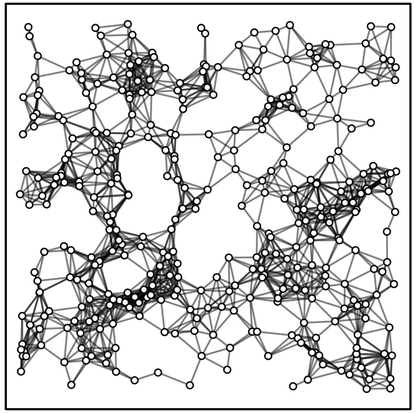

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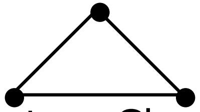
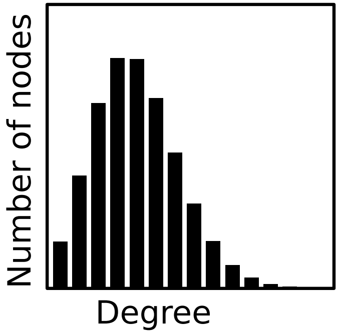
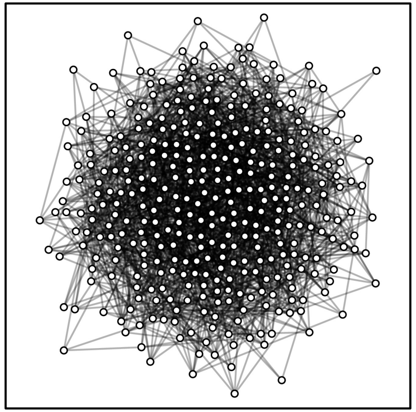


homogeneous degree distribution

locality





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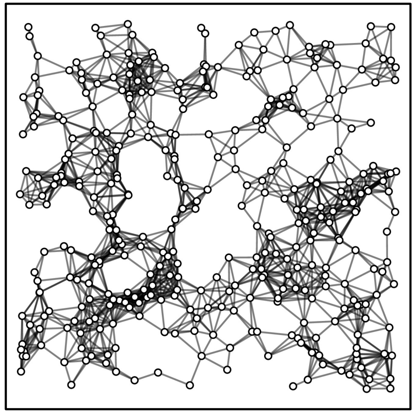

heterogeneity →

geometric

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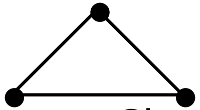

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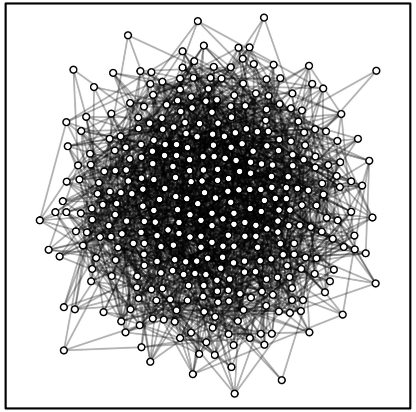

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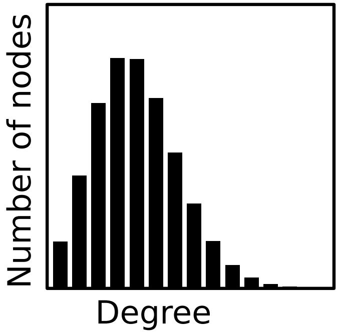


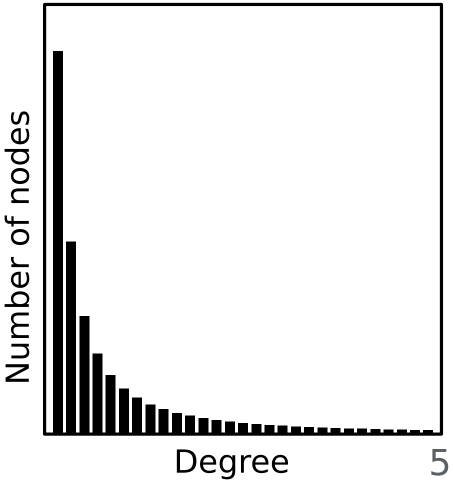
Erdős–Réyni

Gilbert

Chung–Lu

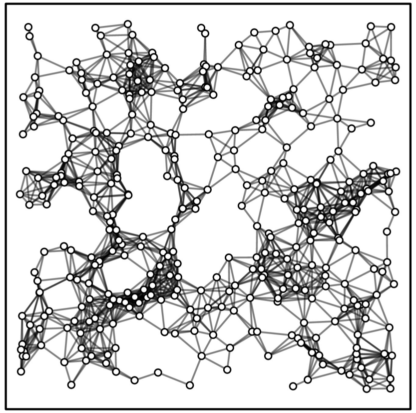
(soft) configuration

power law degree distribution



Network Science

heterogeneity →



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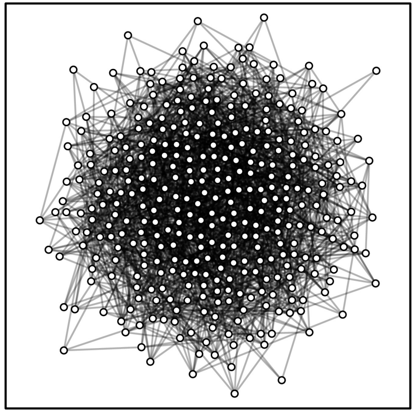

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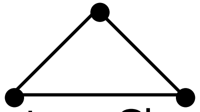

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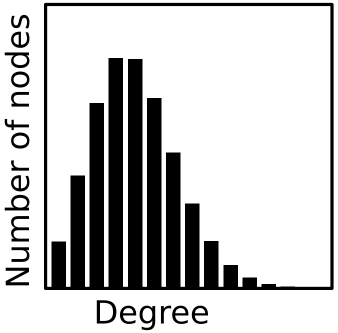

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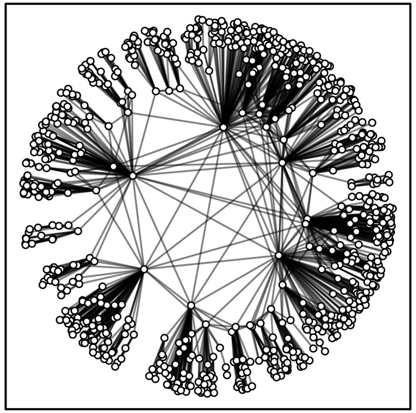

power law
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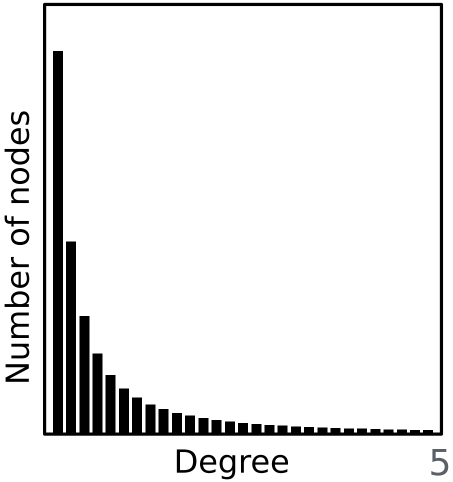
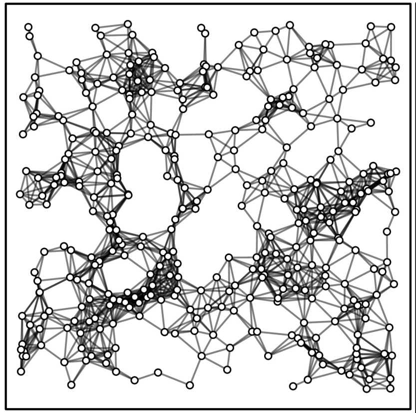


Network Science



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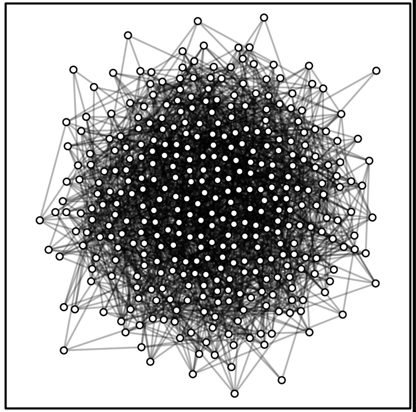

heterogeneity

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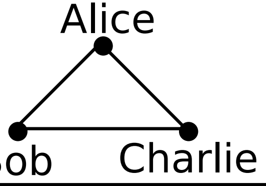

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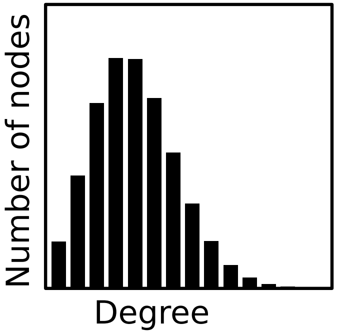

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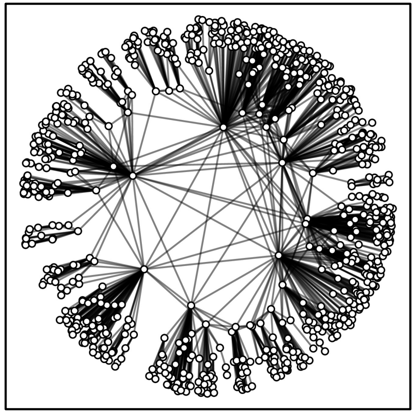

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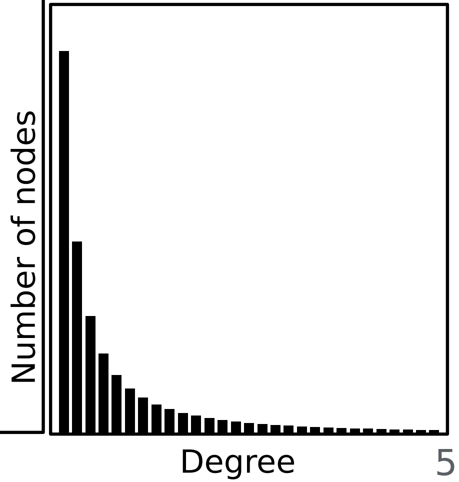

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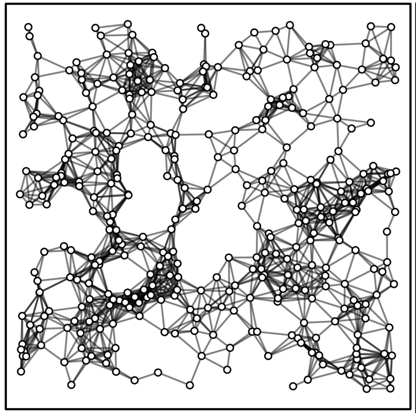

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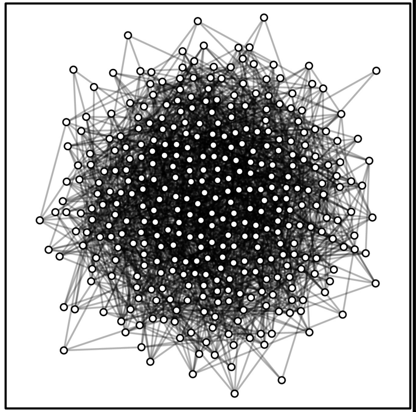

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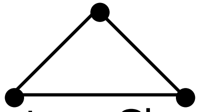

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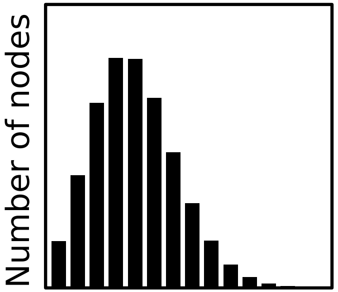

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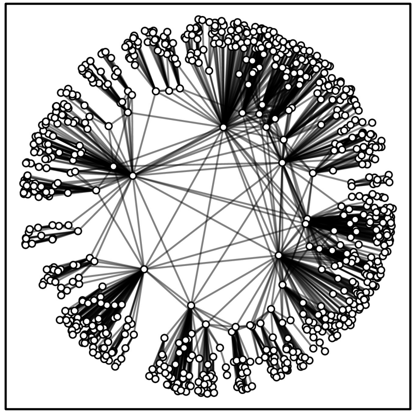

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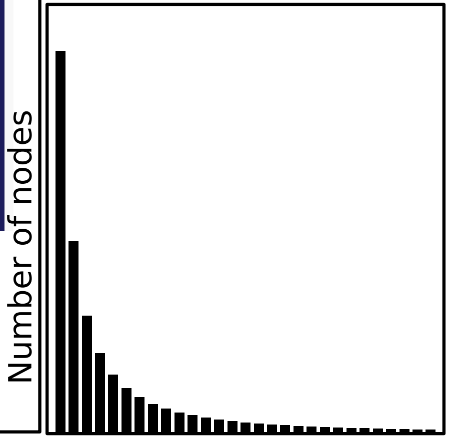

Hyperbolic Random Graphs

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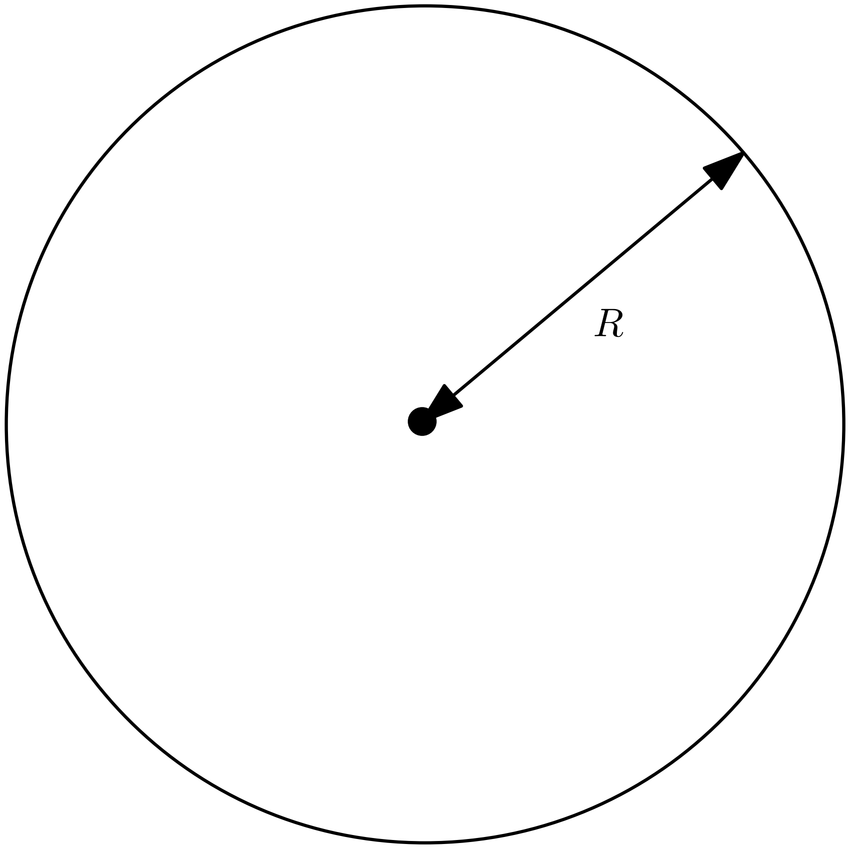
Disk with radius $R = 2\log(n) + C$

Hyperbolic Disk $\mathcal{D}_R$



Hyperbolic Random Graphs

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Hyperbolic geometry: HRG $G \sim \mathcal{G}(n, \alpha, C)$.

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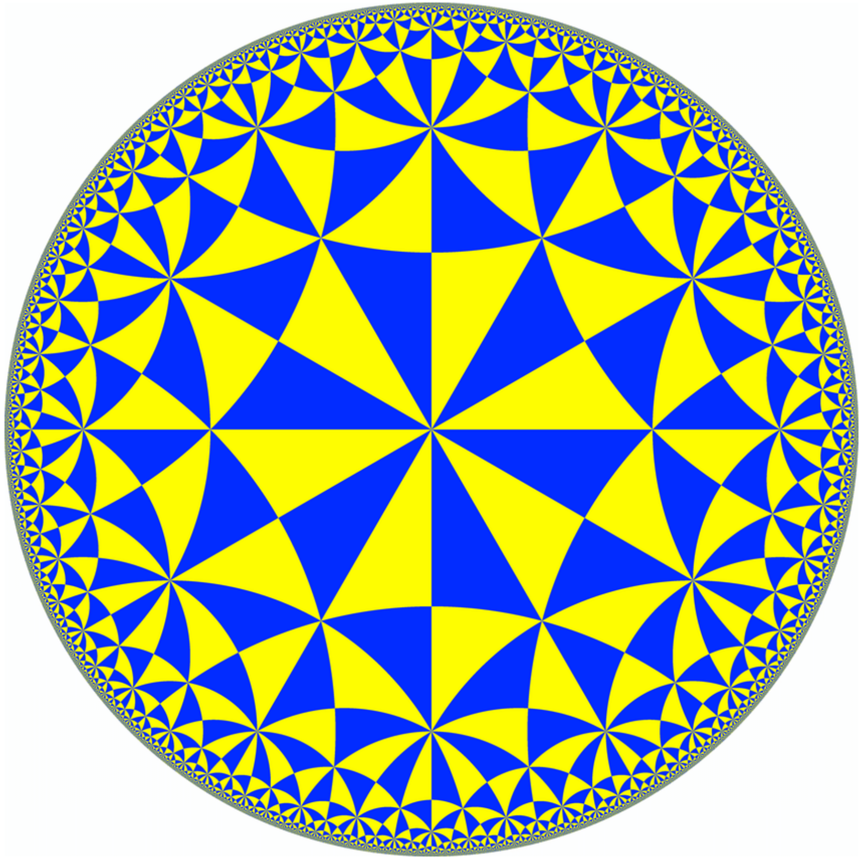

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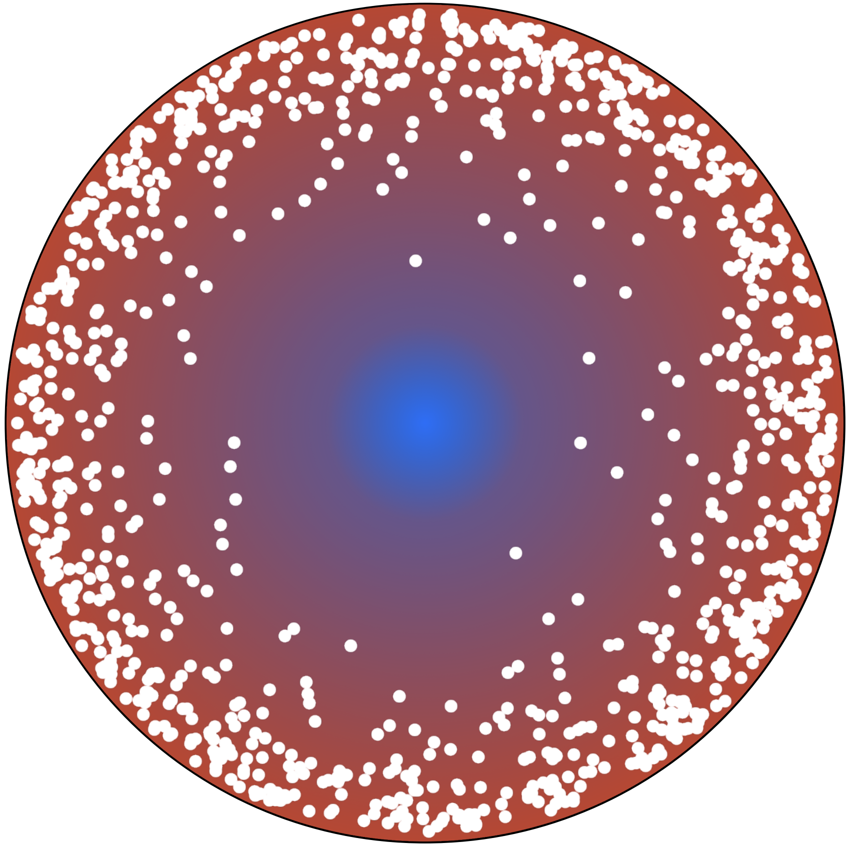

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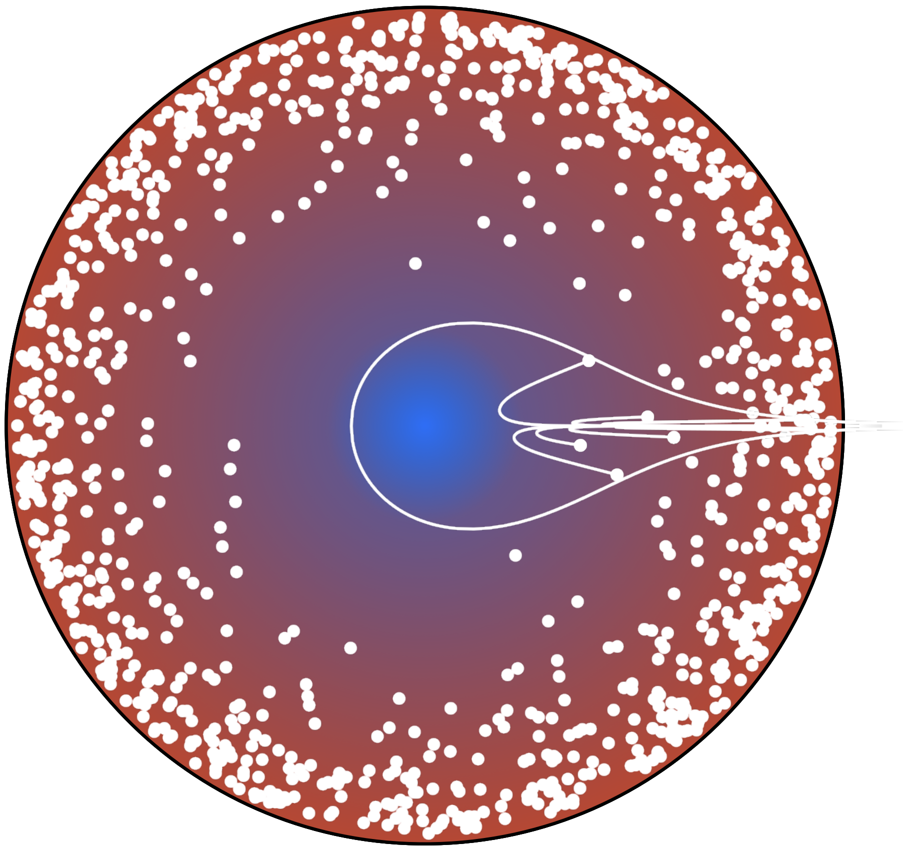

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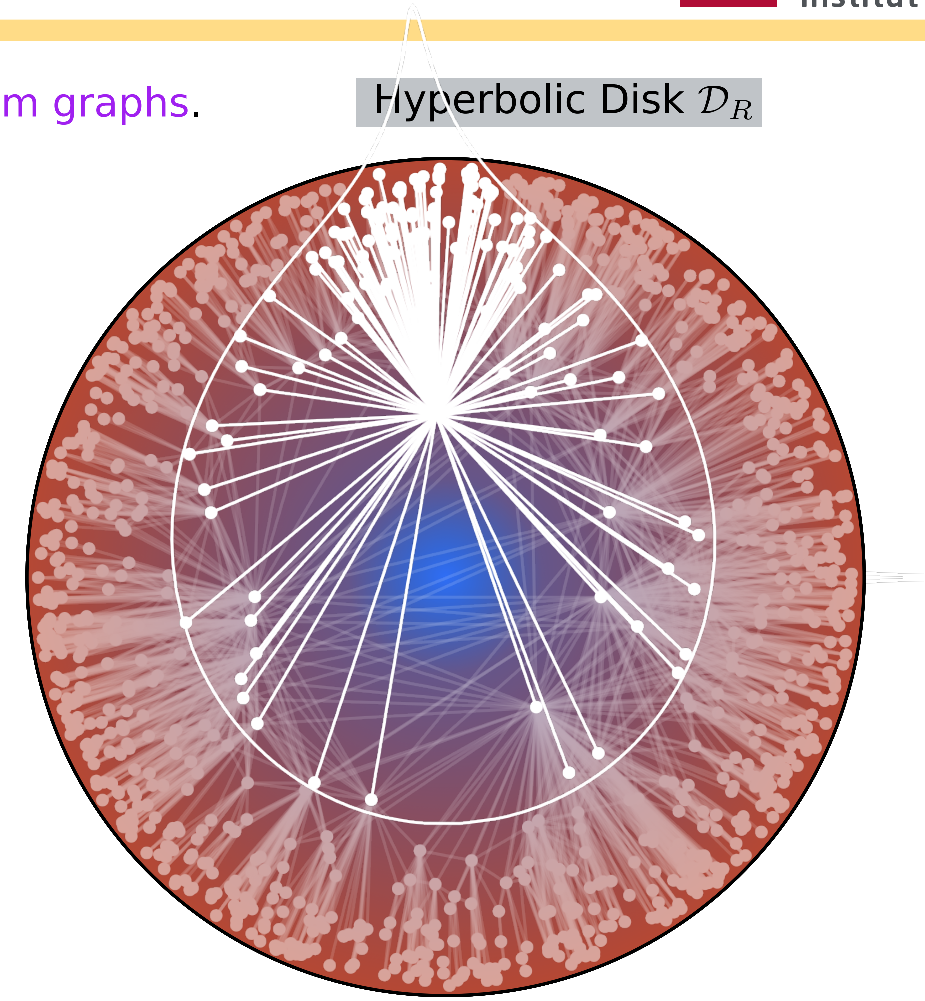

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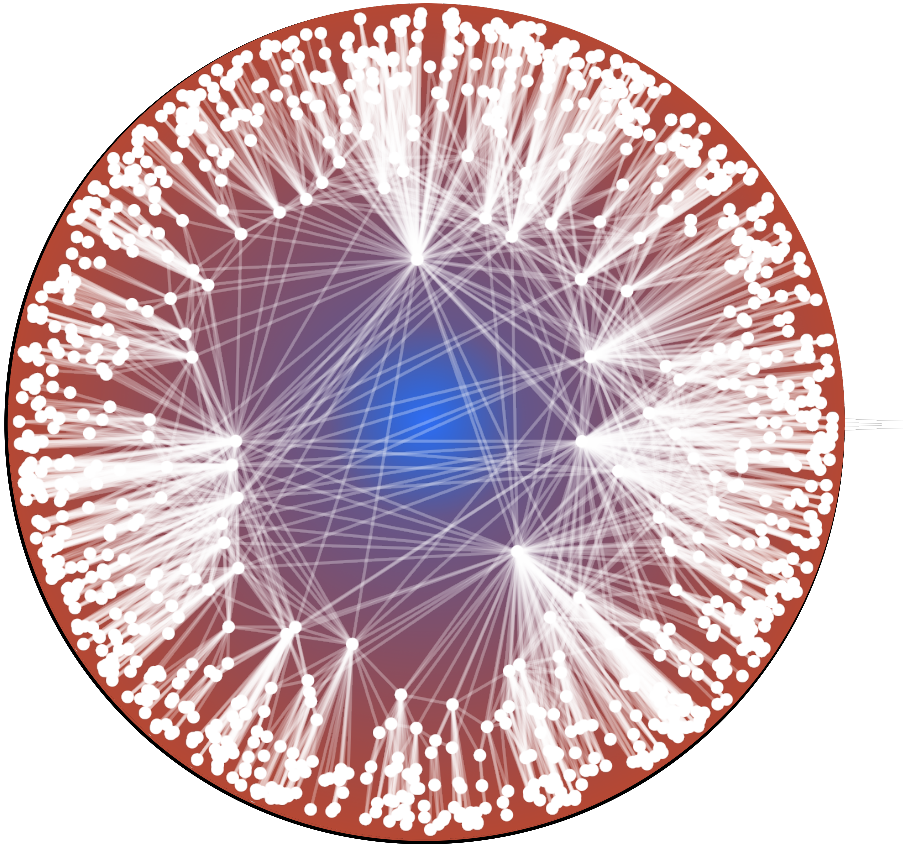

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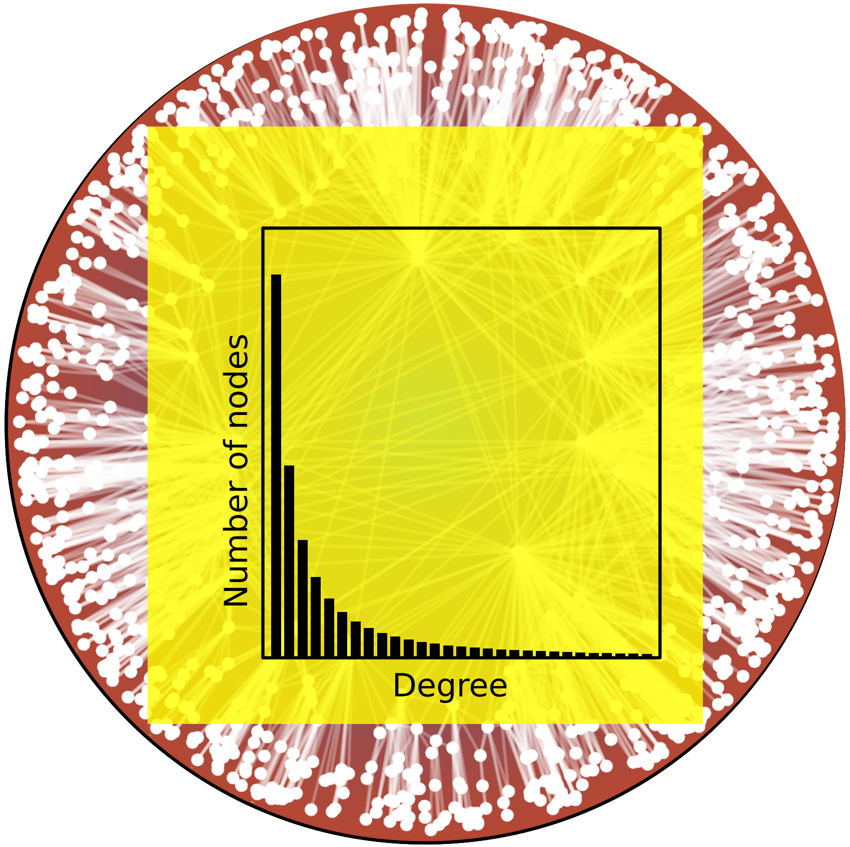

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Probability that vertex u has degree k is $\sim k^{-(2\alpha+1)}$

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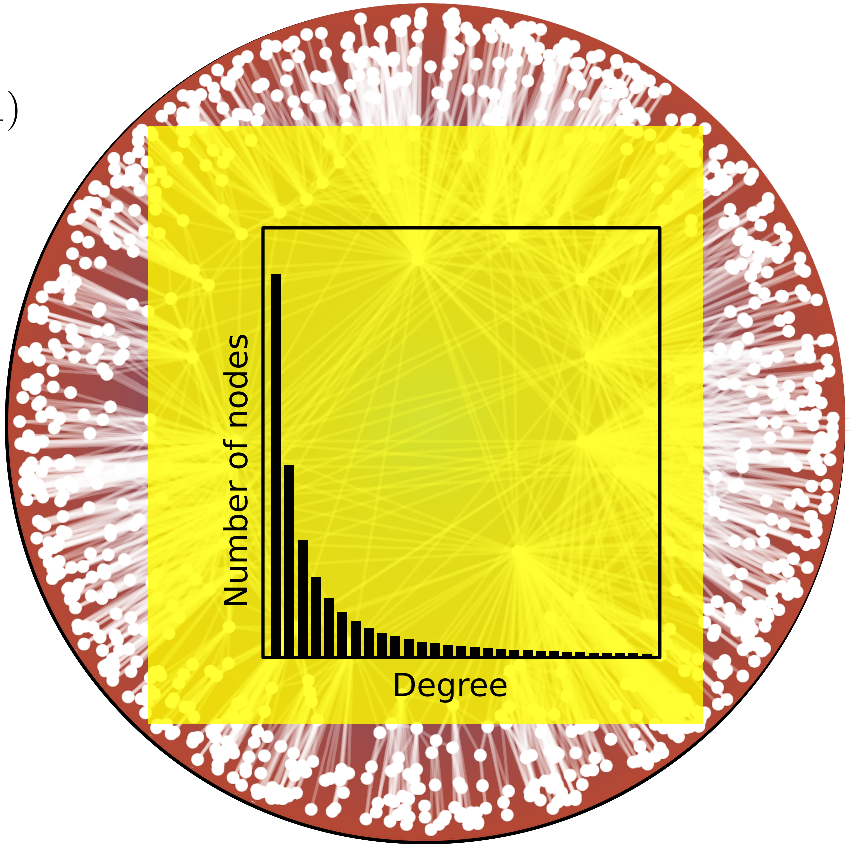

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- Diameter is $\Theta(\log(n))$ with high probability.

(Kiwi, Mitsche ANALCO'15; Friedrich, Krohmer ICALP'15; Müller,Staps Advances in Applied Probability'19)

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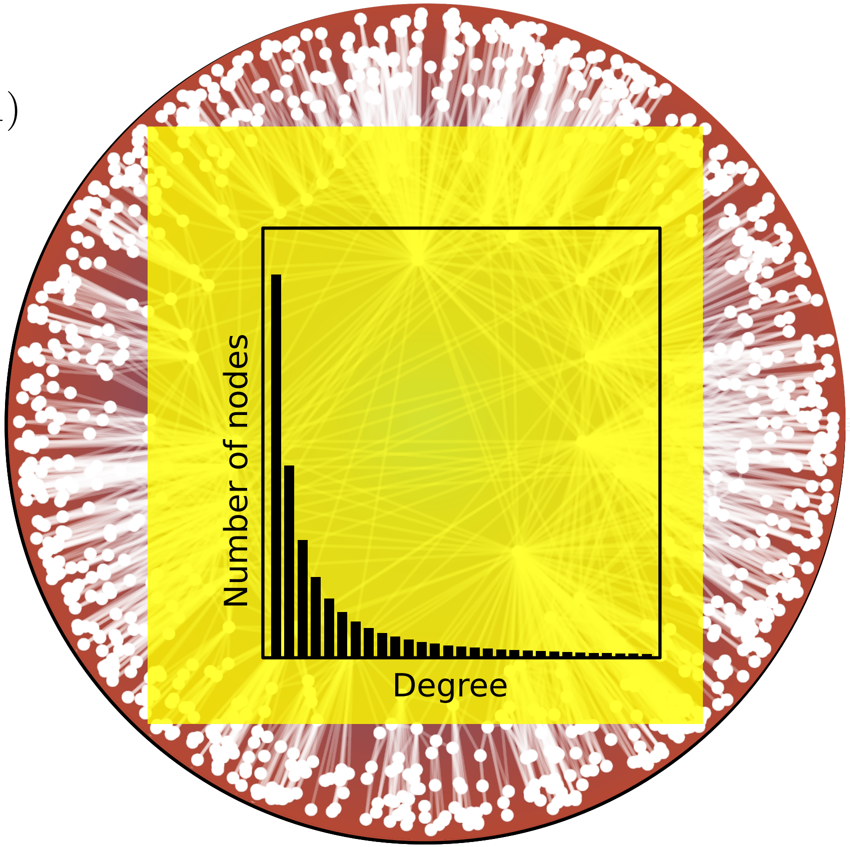

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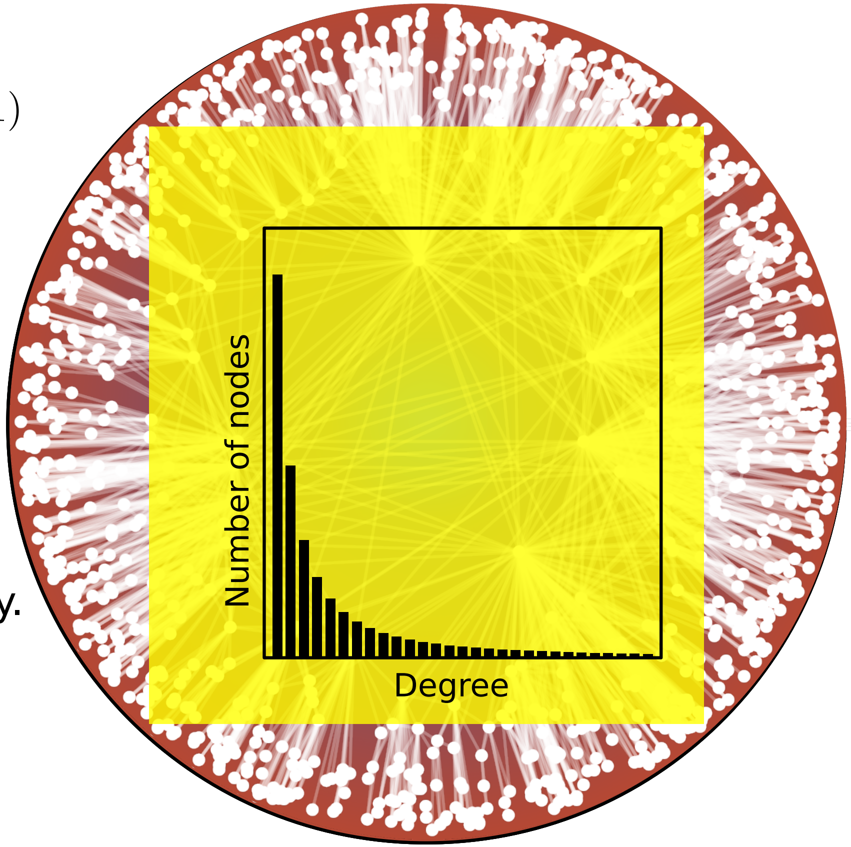
- Giant component with extremely high probability.

(Bode, Fountoulakis, Müller Electron. J. Comb.’15; Fountoulakis, Müller Ann. Appl. Probab’18; Bläsius, Friedrich, Katzmann, R., Zeif ESA’23)



Algorithmic Results

- The treewidth is of size $\Theta\left(n^{1-\alpha}\right)$ with high probability.

  (Bläsius, Friedrich, Krohmer ESA'16)

$tw \in n^{\varepsilon} \cap o(\sqrt{n})$

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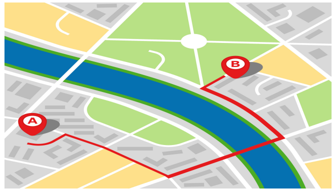

Worst-case $\Omega(n)$

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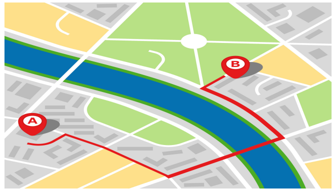

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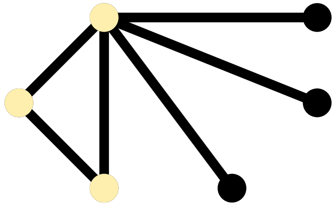

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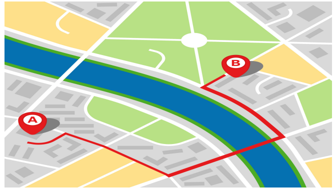

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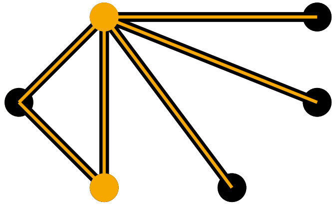

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(Baguley, Maus, R., Skretas STACS'25)

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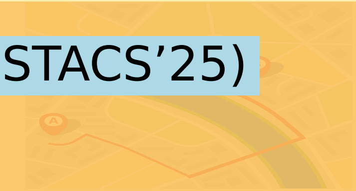

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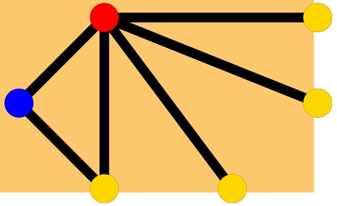

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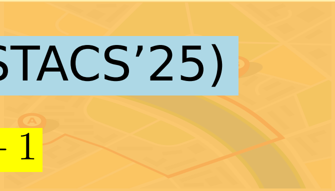

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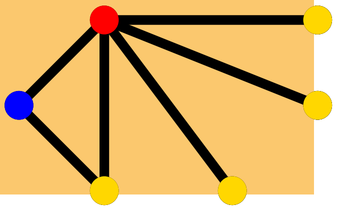

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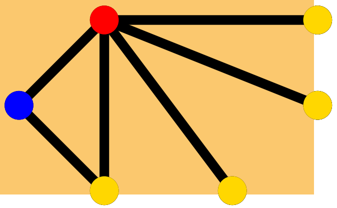

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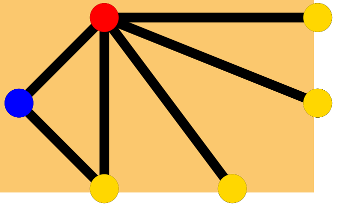

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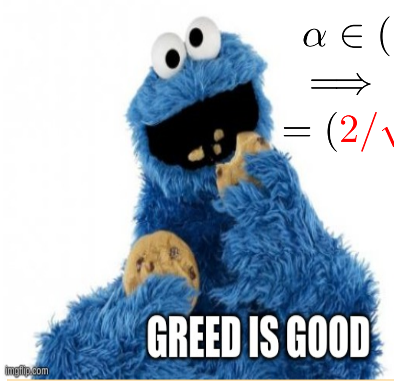

$$\alpha \in (1/2, 1) \implies (4/3)^{\alpha} = (2/\sqrt{3}, 4/3)$$

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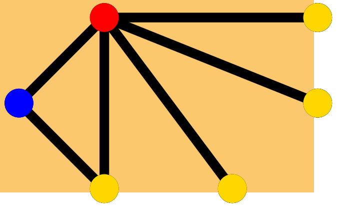
- The gap is at most $(4/3)^{\alpha}$ giving a linear-time greedy algorithm to approximate colouring with this gap.



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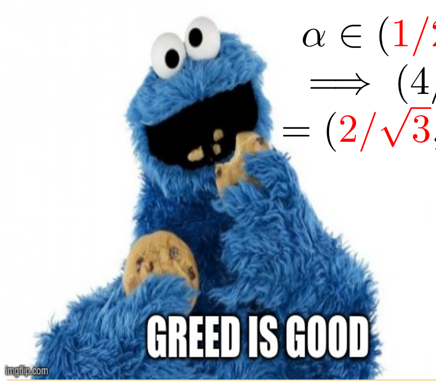

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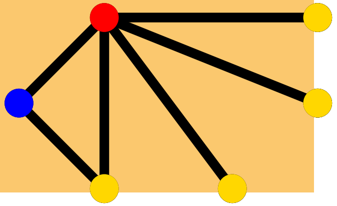
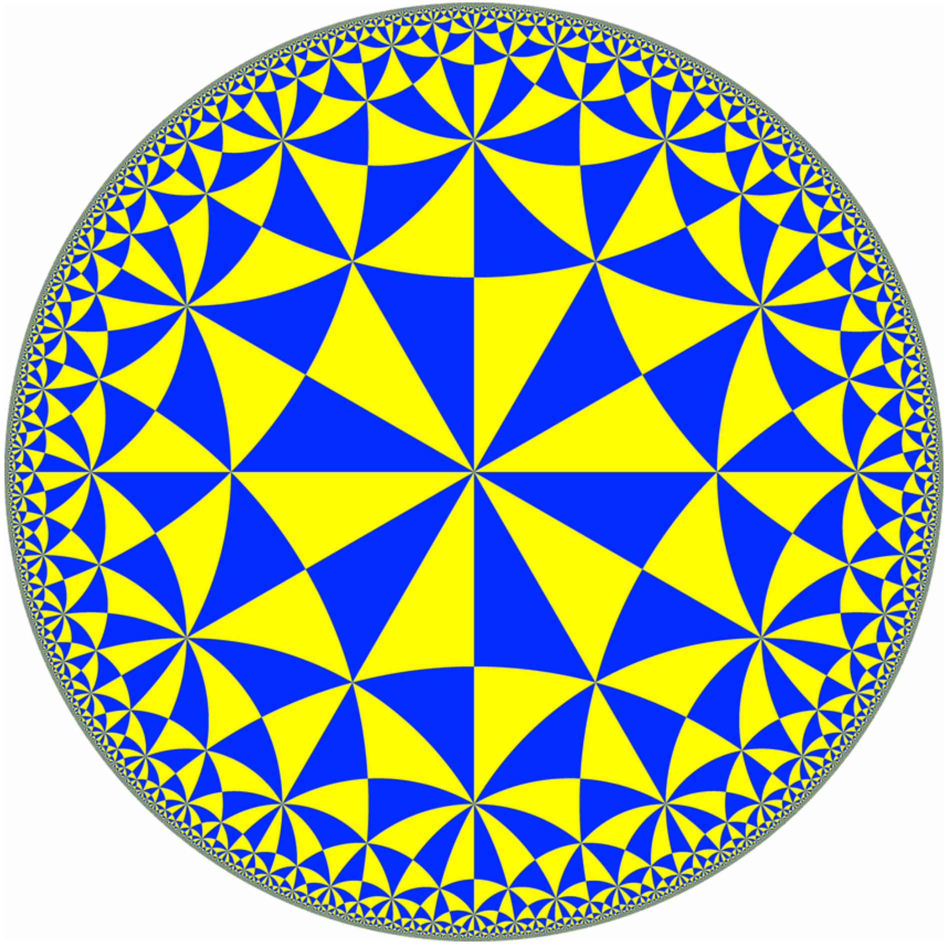
What is the Chromatic Number of a Hyperbolic Random Graph?



Hyperbolic Disk

Thank



YOU!