Improved Approximation Algorithms for (1,2)-TSP and Max-TSP Using Path Covers in the Semi-Streaming Model

Ermiya Farokhnejad, University of Warwick

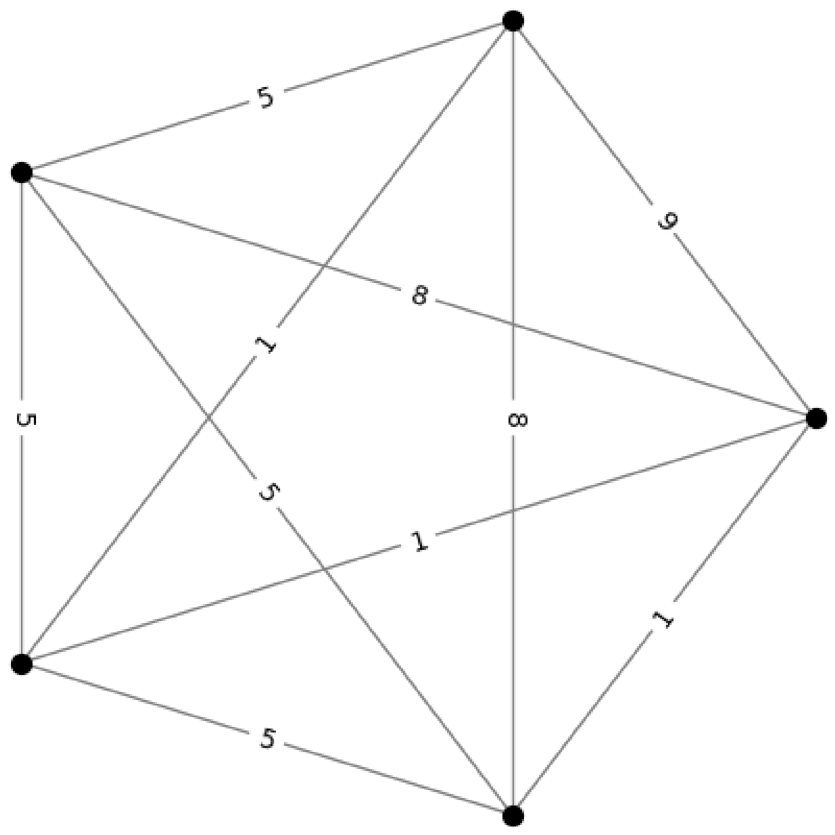
Joint work with:

Sharareh Alipour, Tehran Institute for Advanced Studies (TeIAS)

Tobias Mömke, University of Augsburg

TSP:

Given a complete graph:



Find a tour with minimum cost

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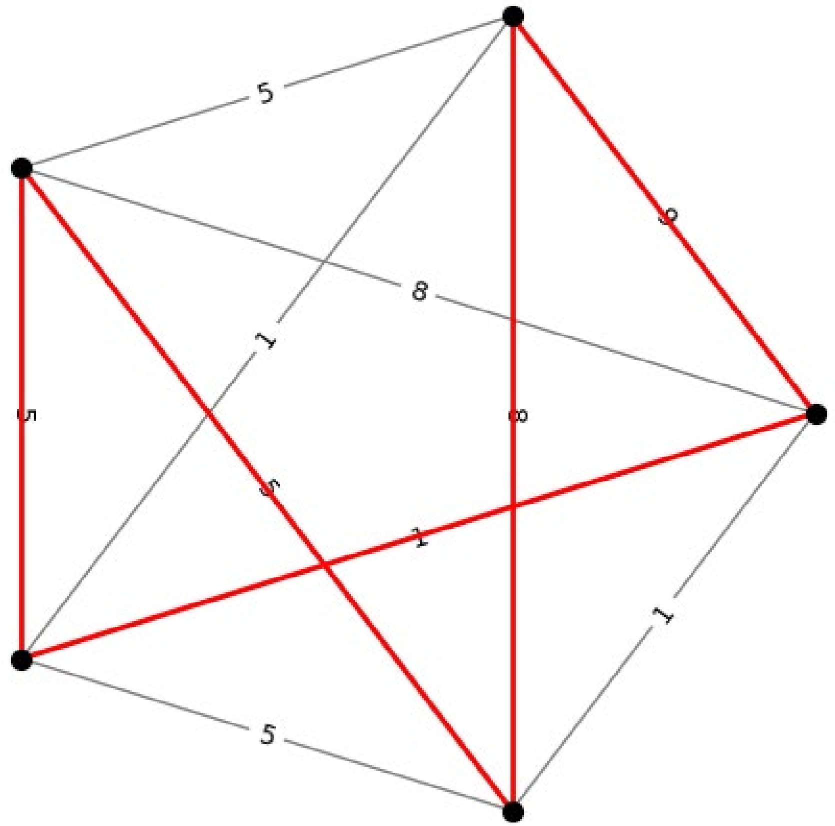

Cost: 28

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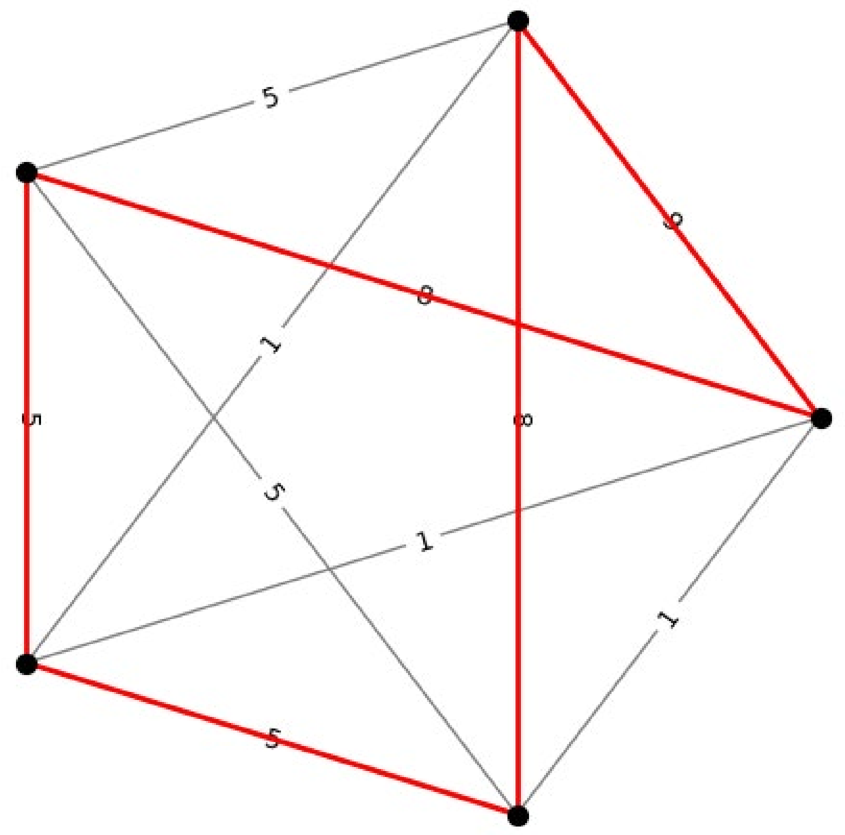

Cost: 35

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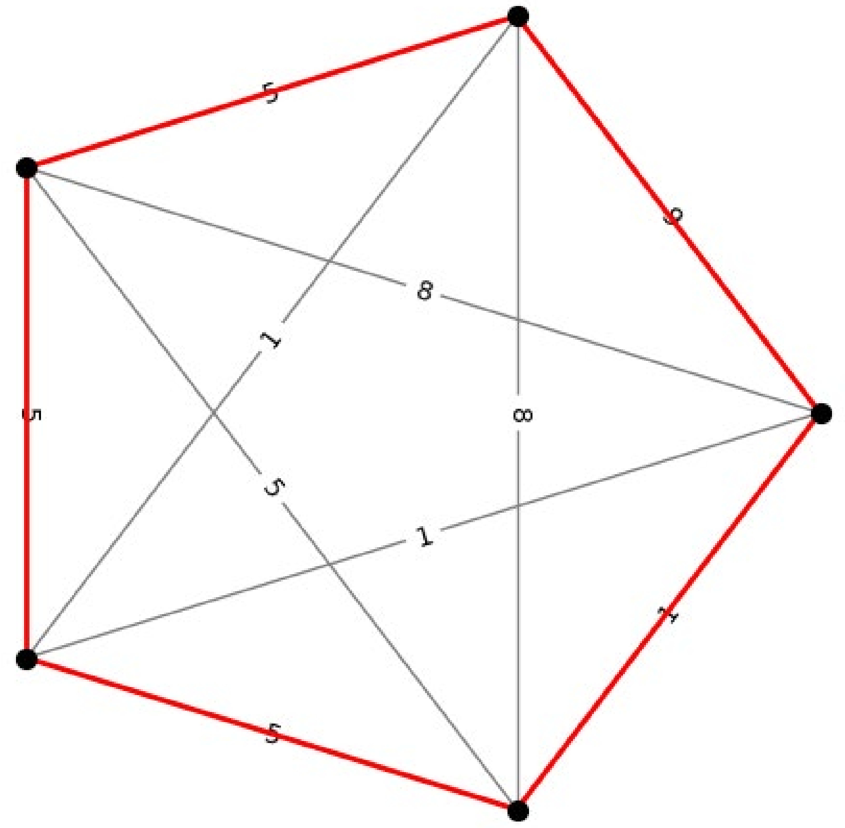

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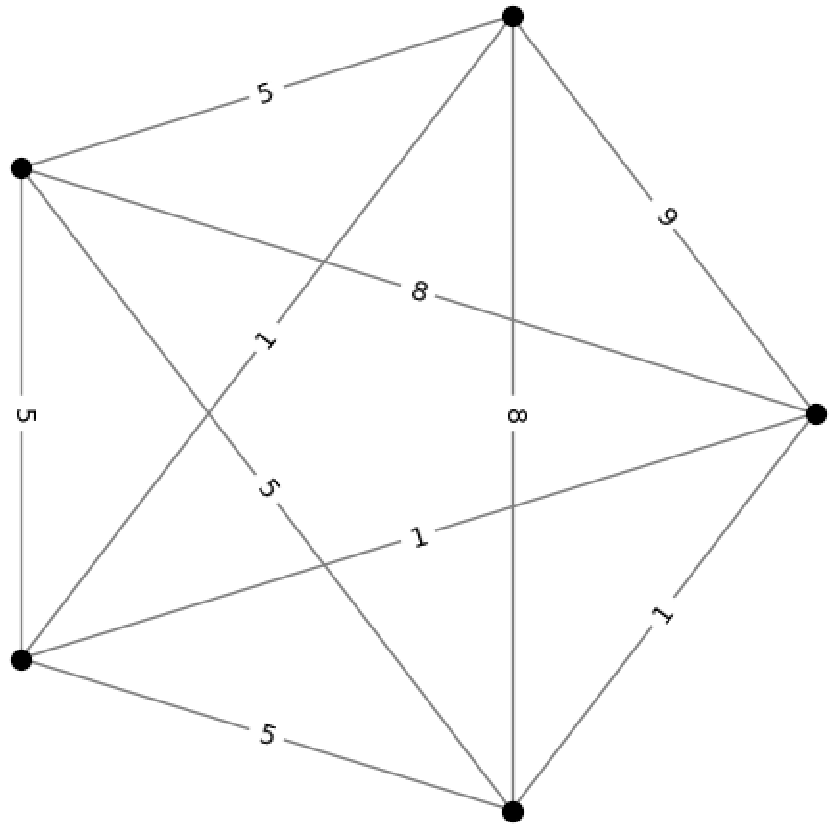

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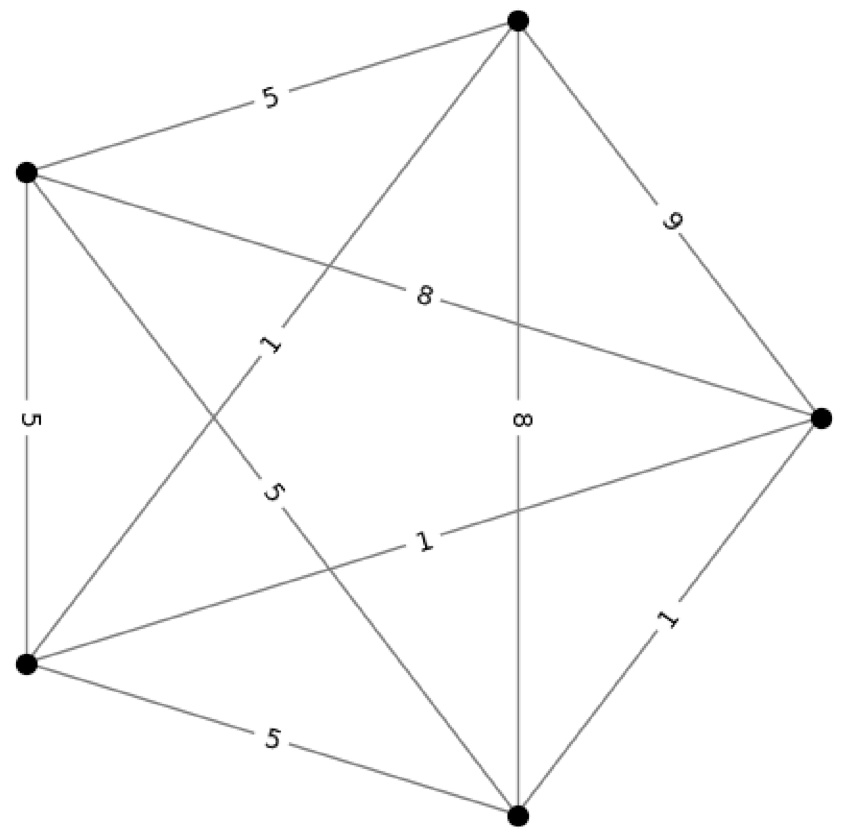
Variants:



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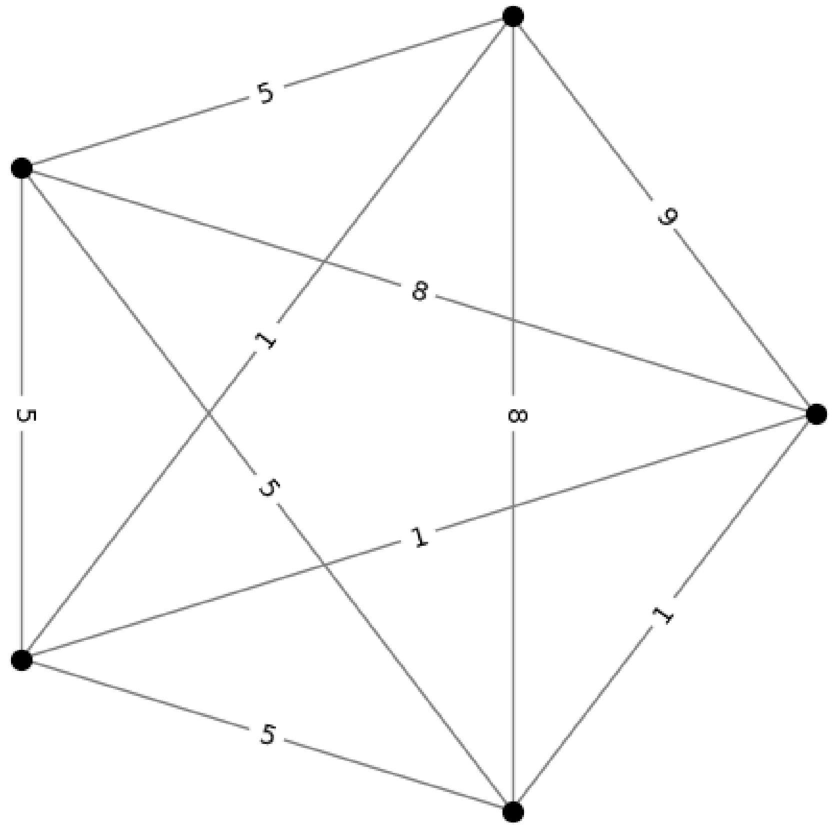

Variants:

Metric TSP

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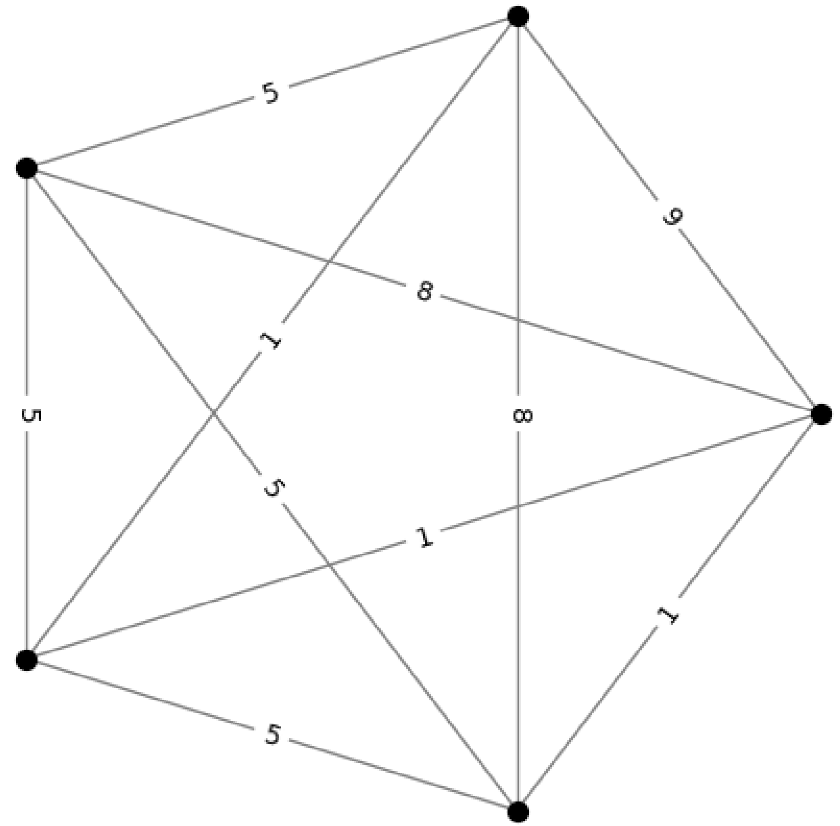

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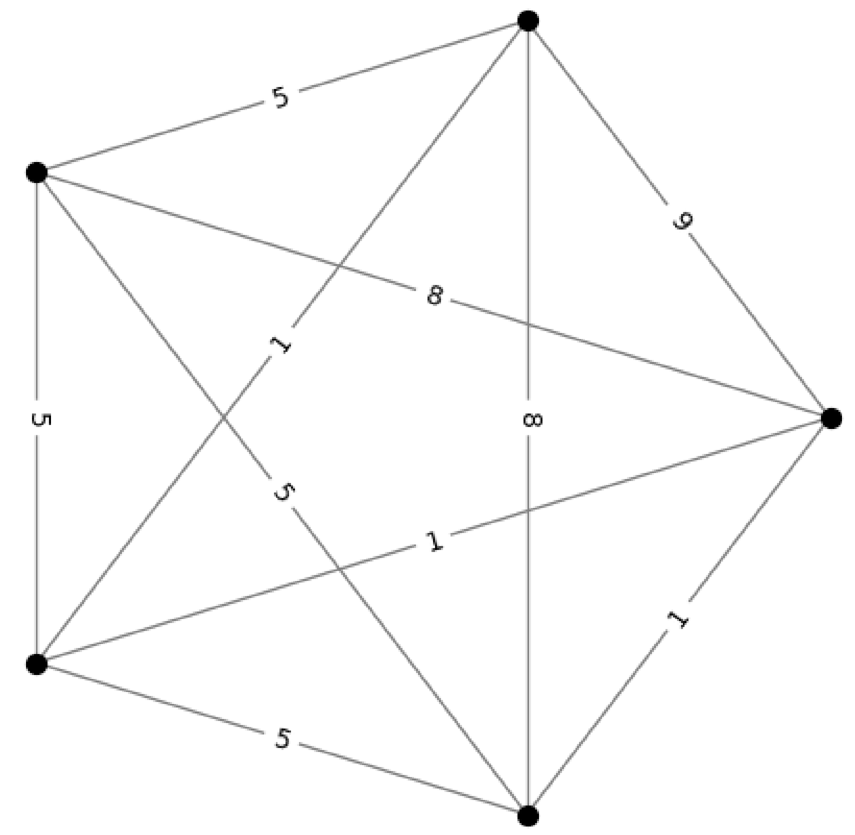

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(1,2)-TSP

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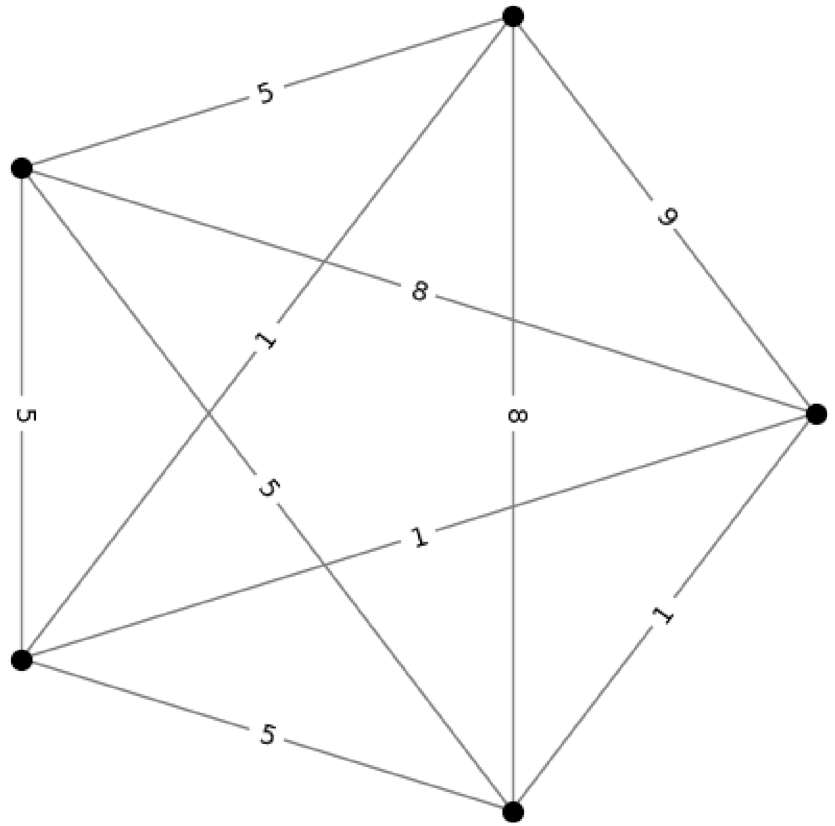

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Max-TSP

Previous Work:

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Behnezhad, Roghani, Rubinstein, Saberi [ICALP 2024]

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Question:

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$(1,2)$-TSP: $(3/2+\epsilon)$-approx 1 pass

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it is very hard to break $3/2$-approx in 1 pass.

Question:

Better approximation by more passes?

Our results:

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Theorem

Given an instance of (1, 2)-TSP , there is a $(4/3+\epsilon)$-approximation algorithm that runs in poly$(1/\epsilon)$ passes in the semi-streaming model.

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Theorem

Given an arbitrary weighted graph, there is a $(7/12-\epsilon)$-approximation algorithm for MAX-TSP that runs in poly$(1/\epsilon)$ passes in the semi-streaming model.

Main Idea:

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Reduction to Maximum Matching

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[Huang, Su] [PODC23]:

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Maximum Weighted Matching in Semi-Streaming Model

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$(1-\epsilon)$-approximation

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$(1-\epsilon)$-approximation   poly$(1/\epsilon)$ passes

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Reduction to Maximum Matching

rest of the talk

[Huang, Su] [PODC23]:

Maximum Weighted Matching in Semi-Streaming Model

$(1-\epsilon)$-approximation

poly$(1/\epsilon)$ passes

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union of vertex-disjoint paths

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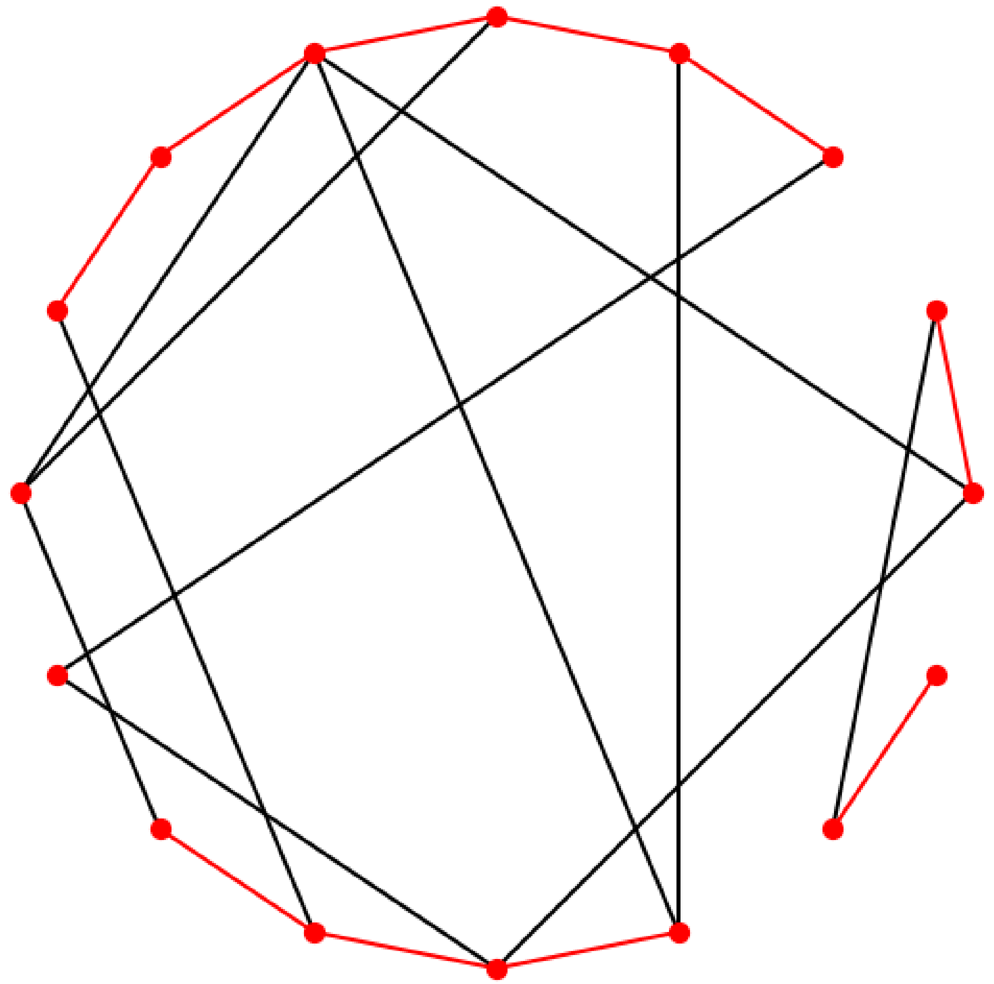

Related problem: Maximum Path Cover (MPC)

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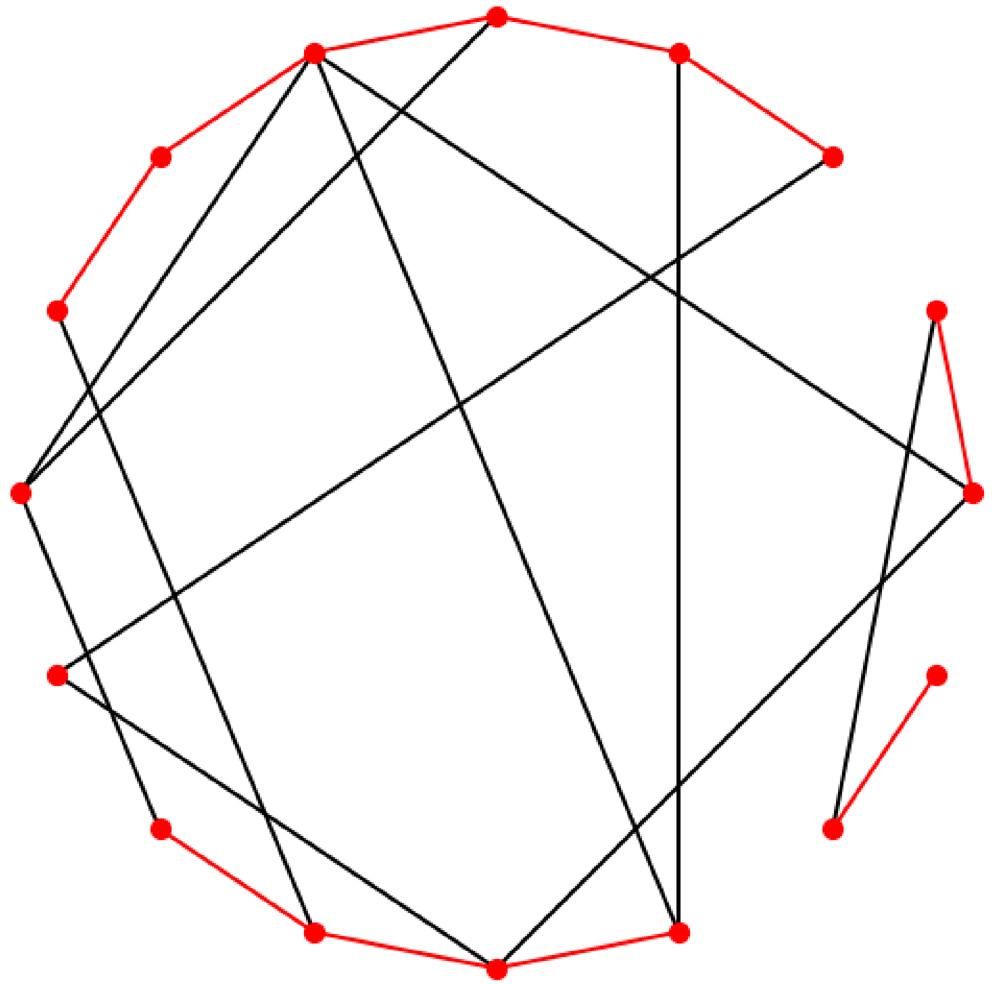
Find a path cover
with maximum number of edges



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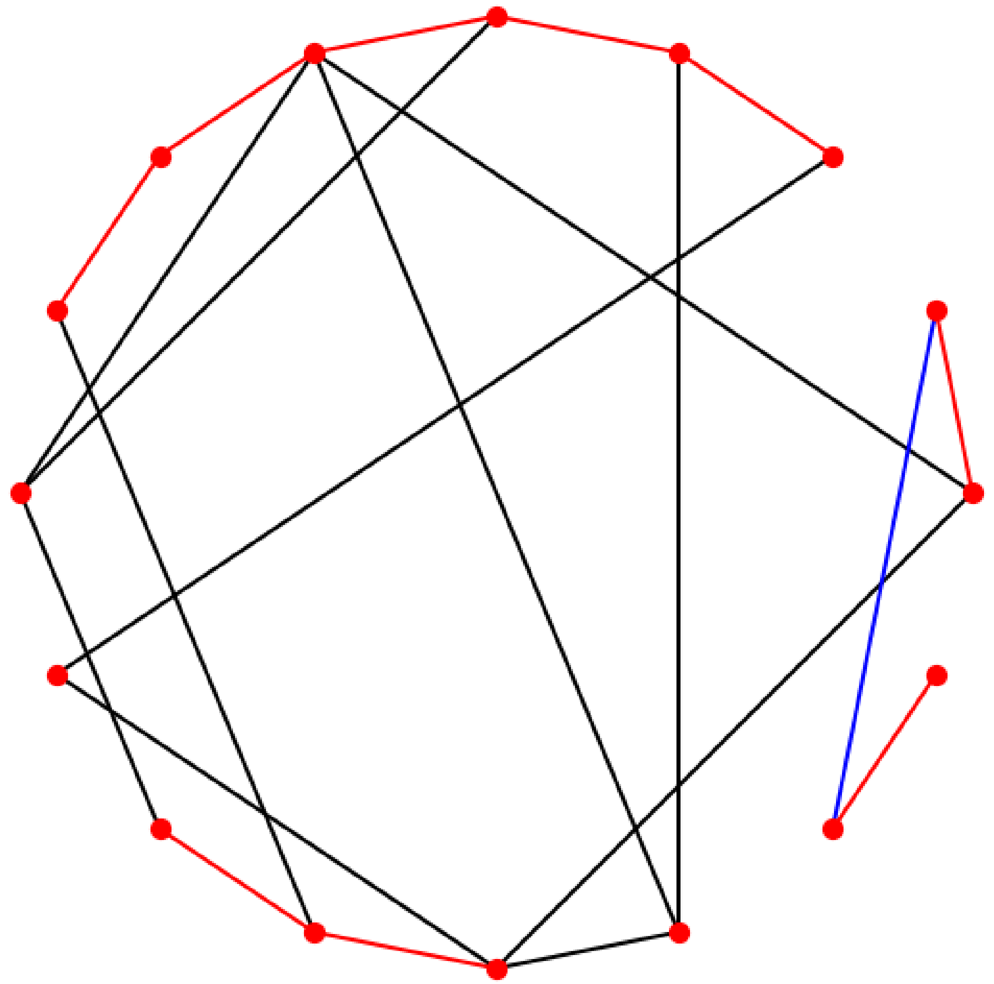

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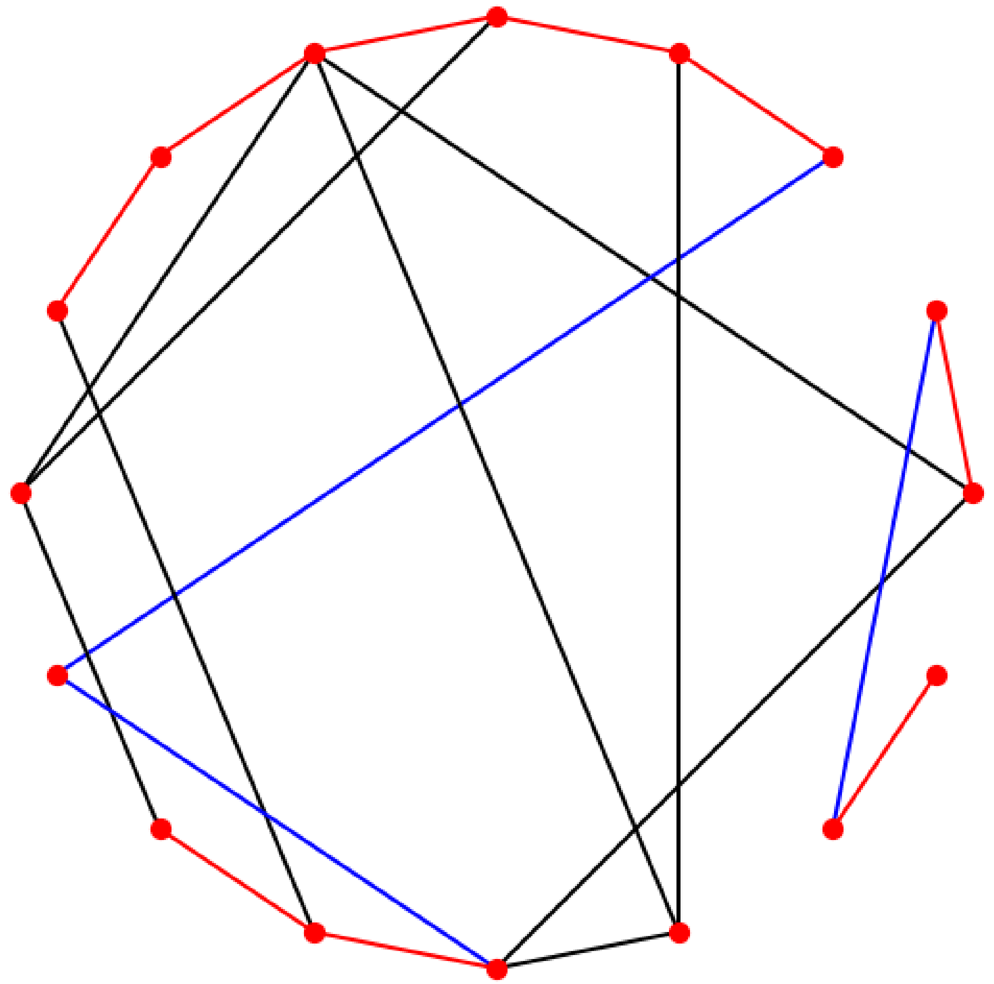
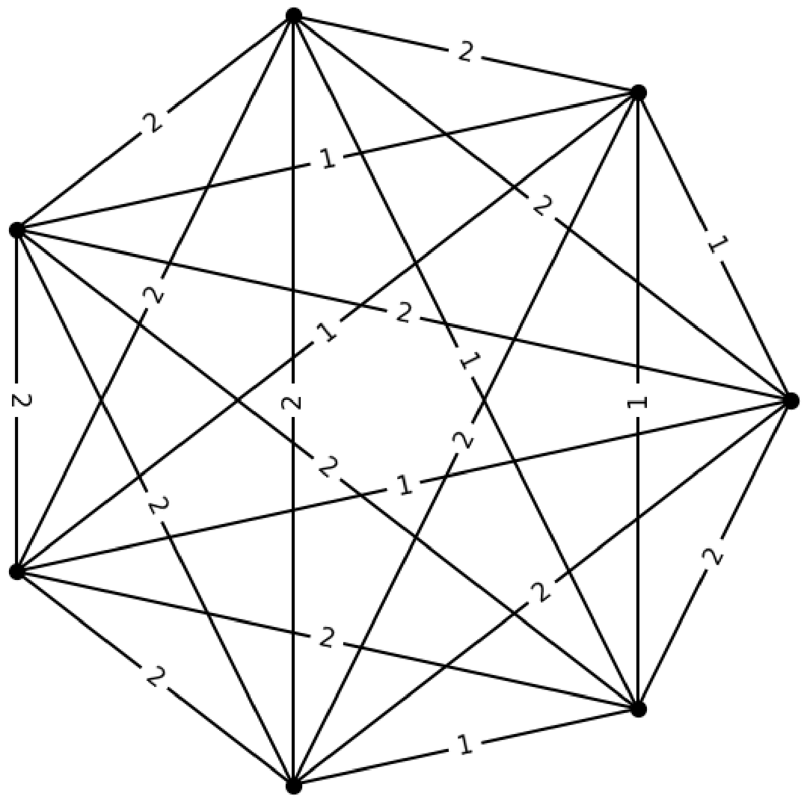
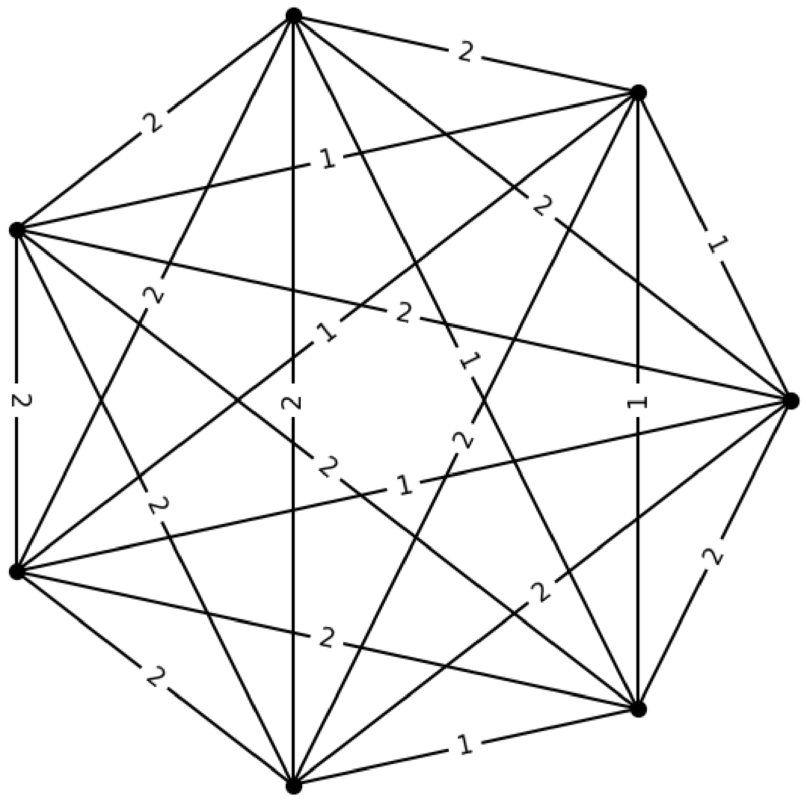


Rlation between MPC and (1, 2)-TSP:



Rlation between MPC and $(1, 2)$-TSP:

step 1: focus on edges of weight 1



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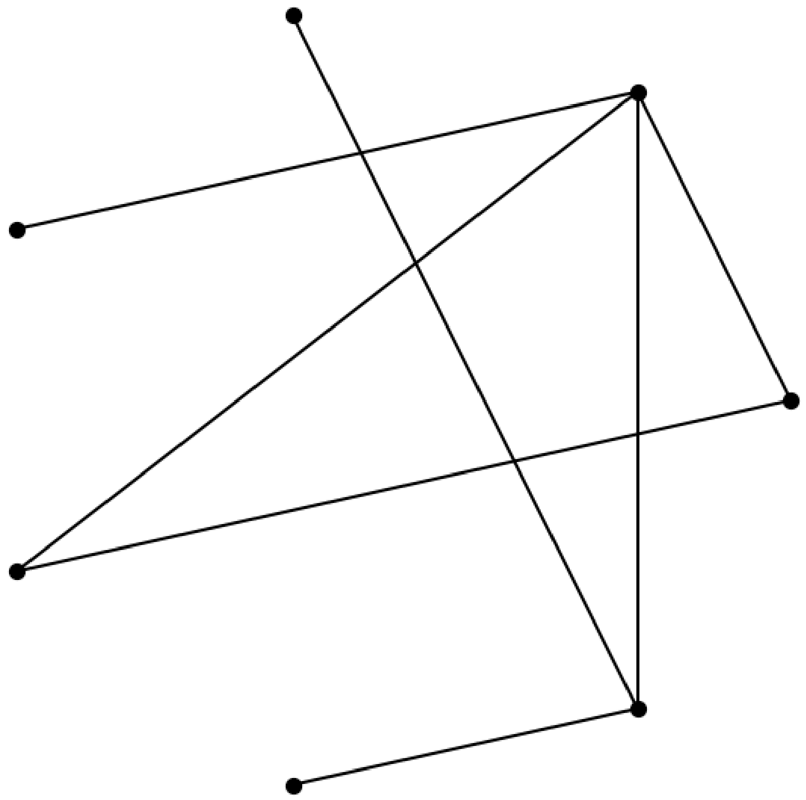

Rlation between MPC and (1, 2)-TSP:

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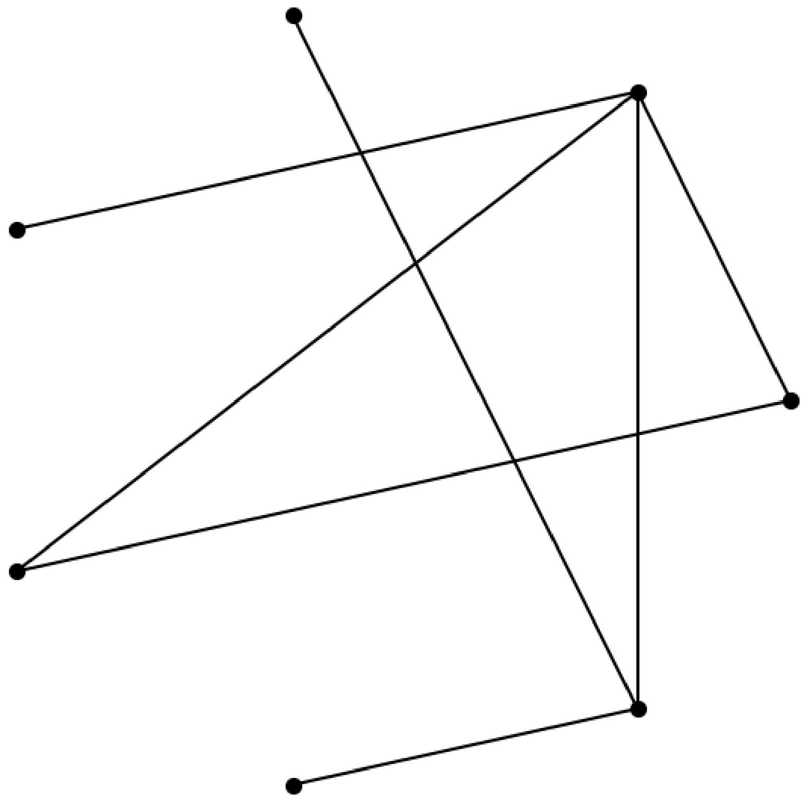

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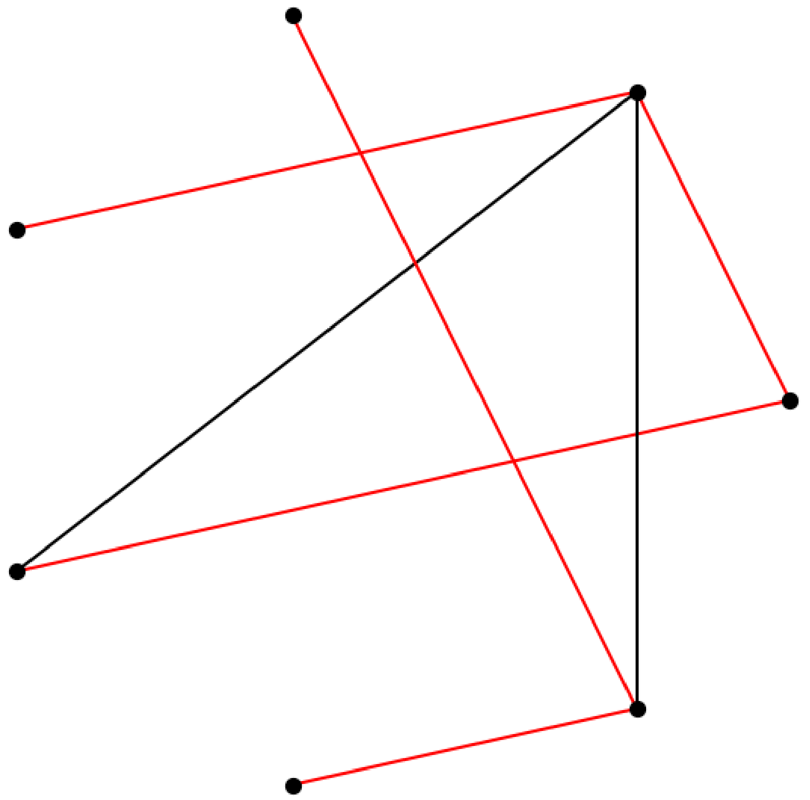

Rlation between MPC and $(1, 2)$-TSP:

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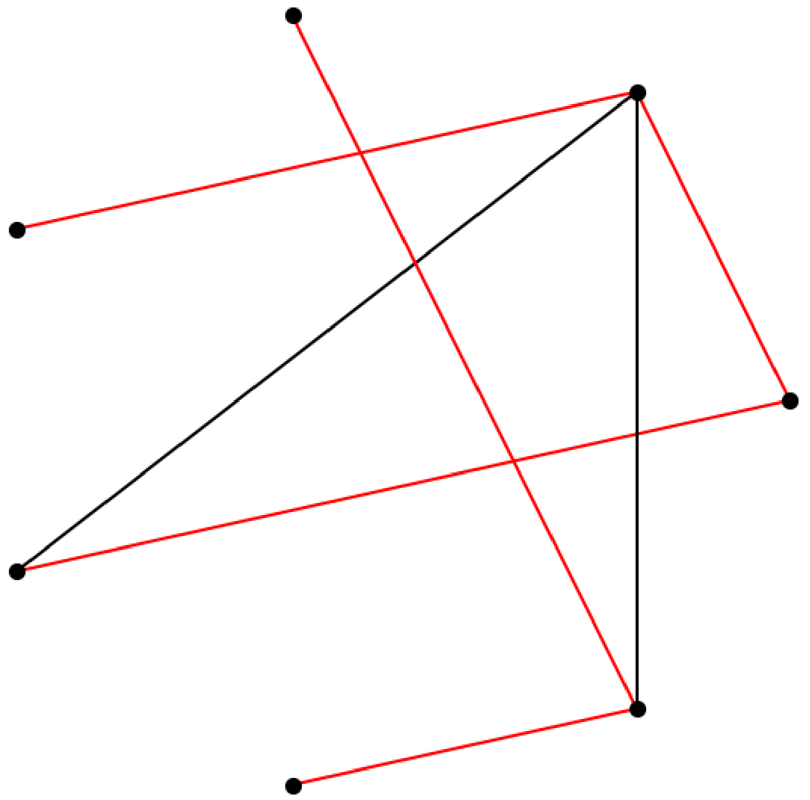
step 3: complete it to a tour



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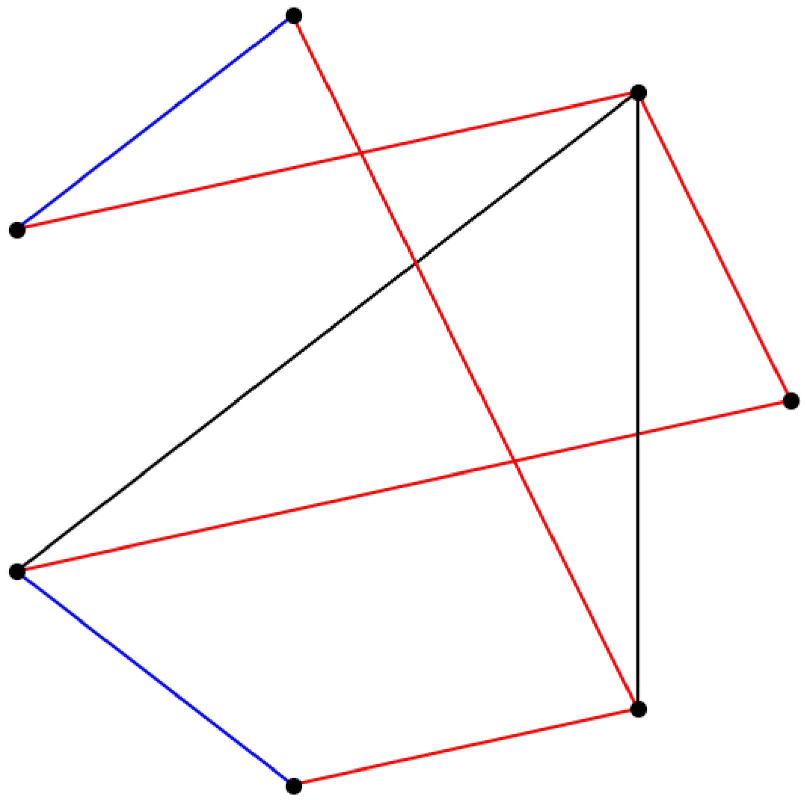
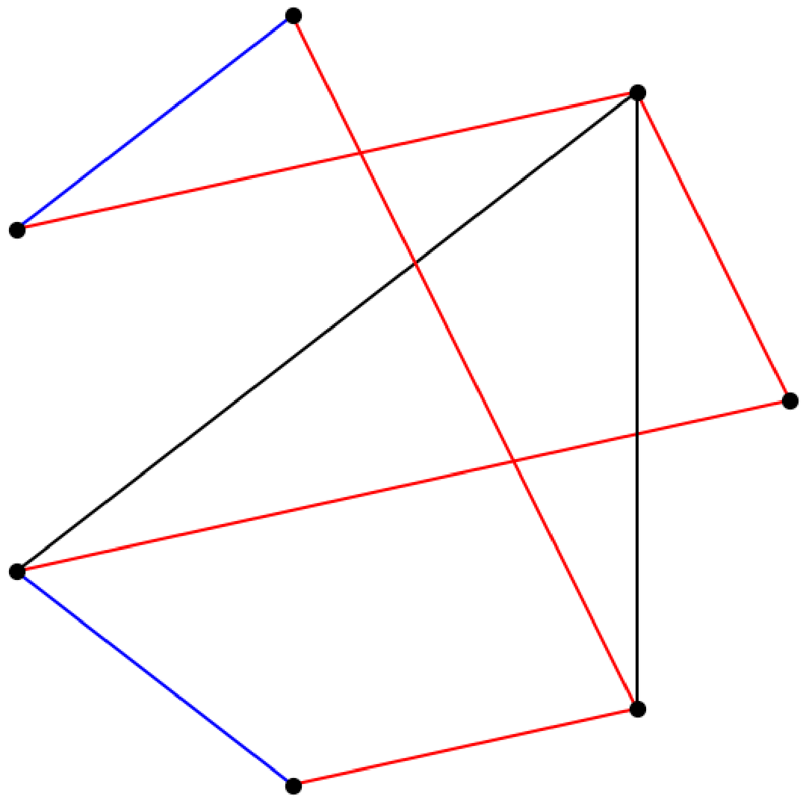


Rlation between MPC and $(1, 2)$-TSP:

ρ: length of the path cover, T: TSP cost

$$T \approx 2n - \rho$$



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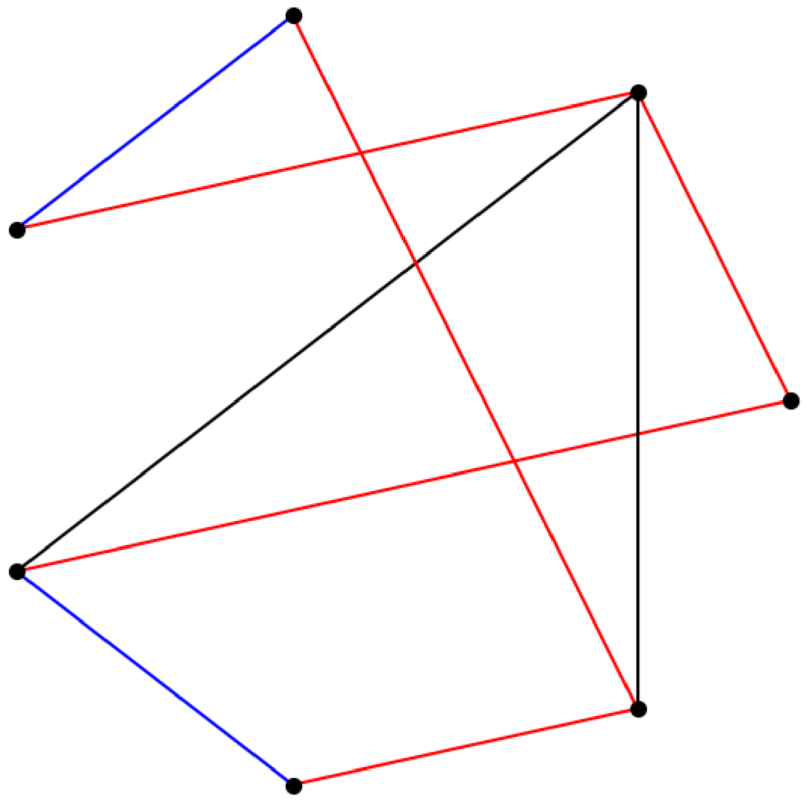
α-approximation of MPC

$$\Longleftrightarrow$$

$\approx (2-\alpha)$-approximation of $(1,2)$-TSP



Algorithm for MPC:

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(1/2)-approximation of MPC

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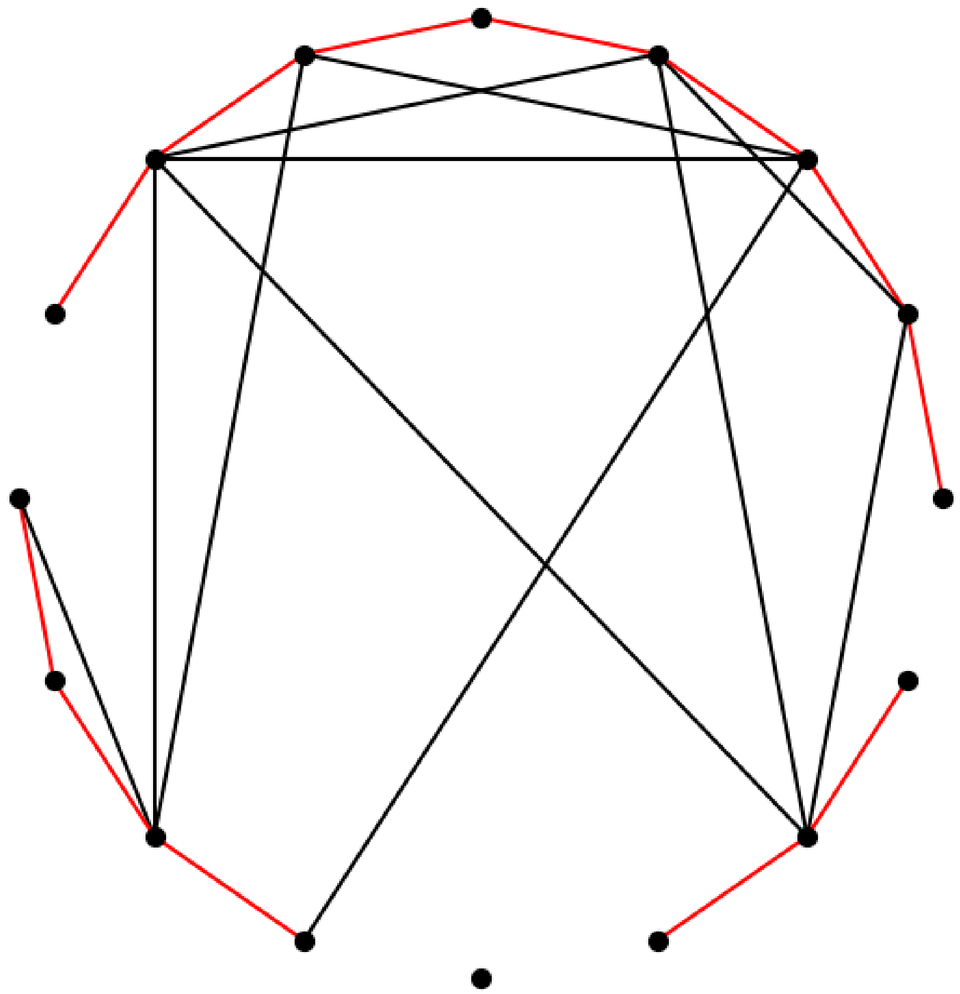

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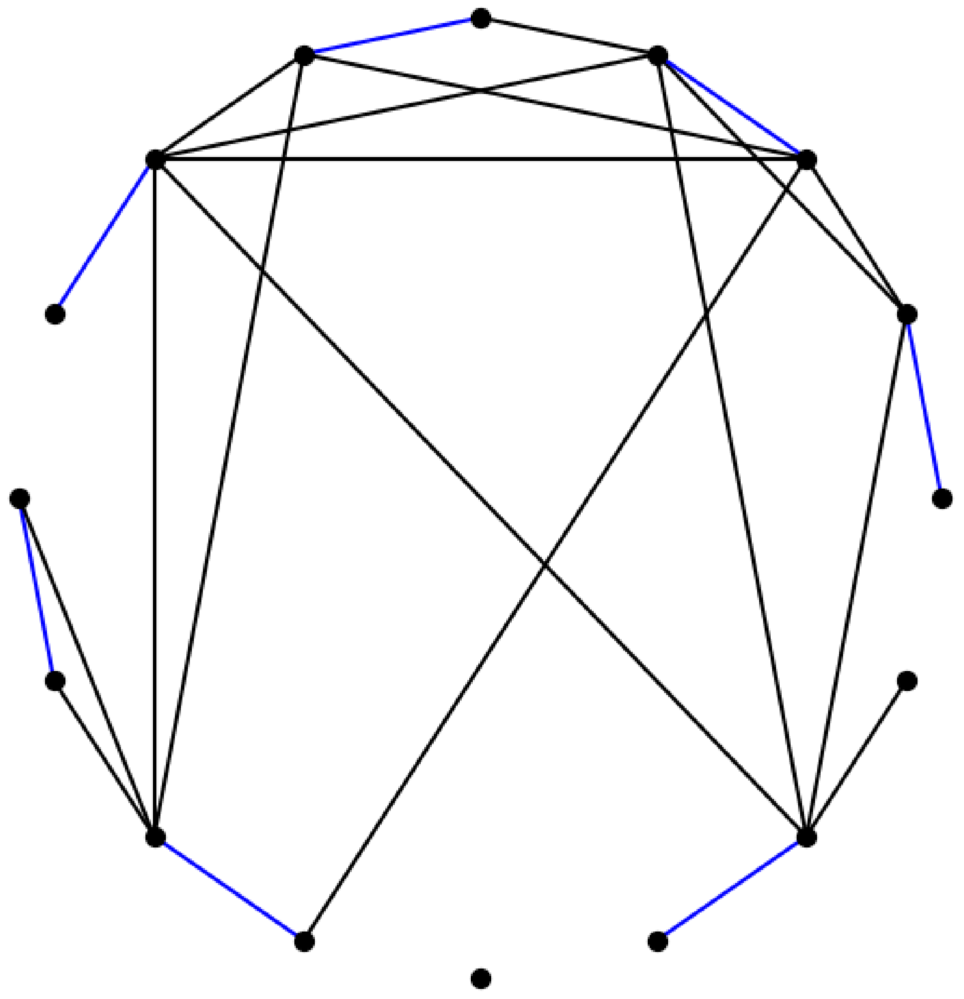

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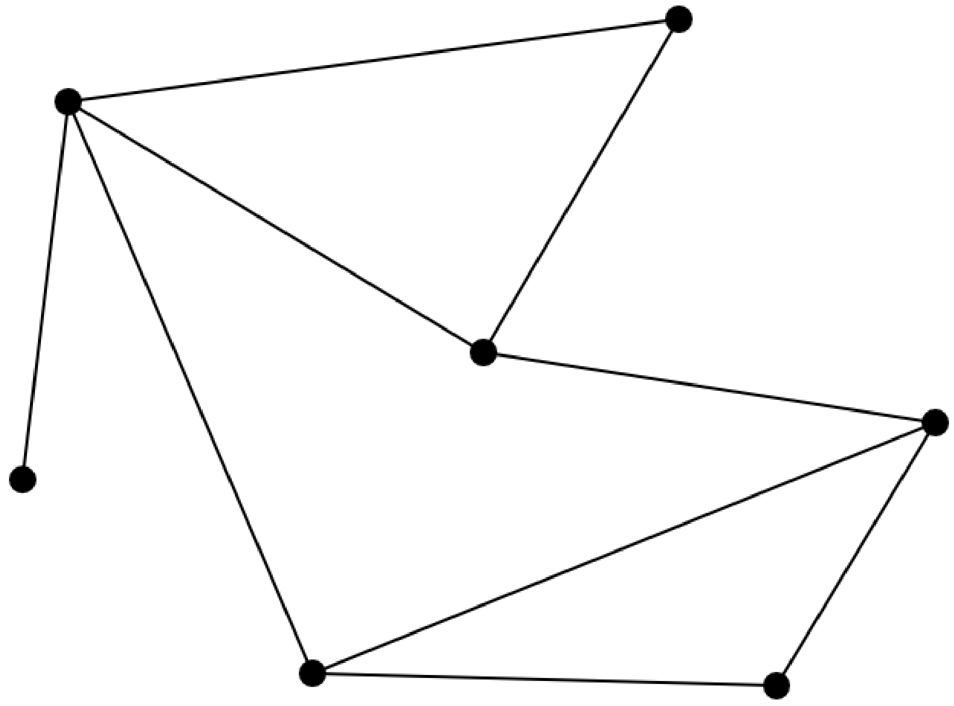

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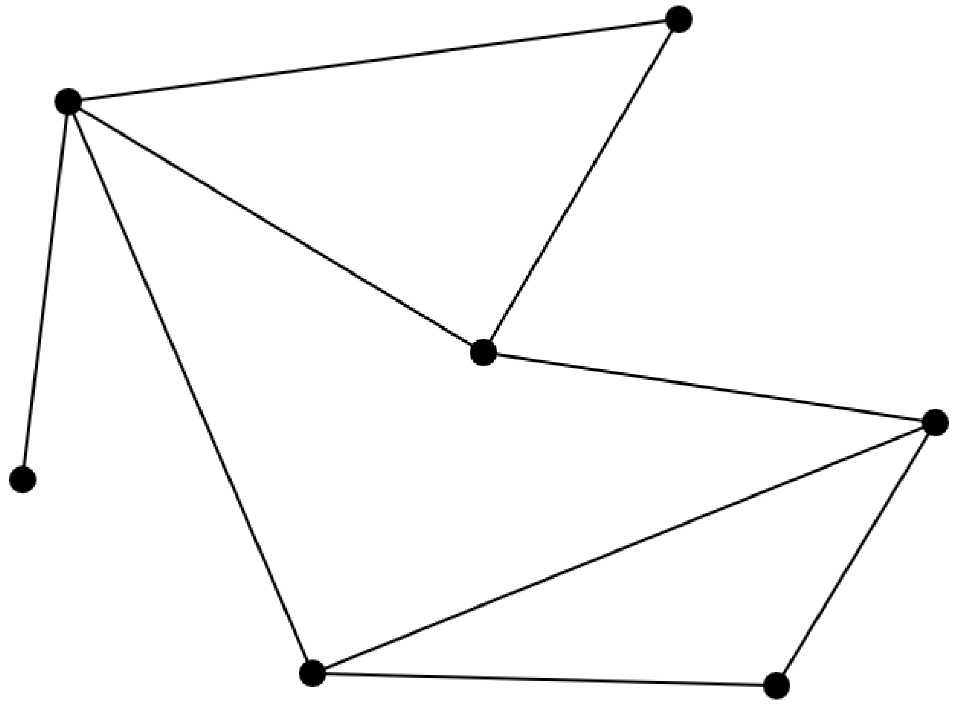

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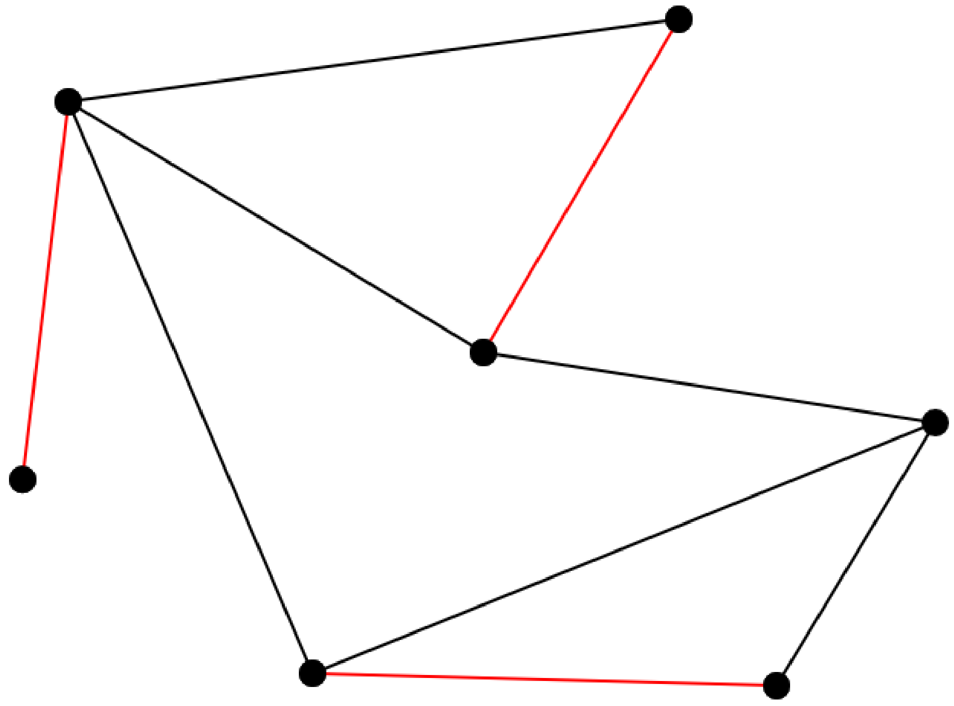

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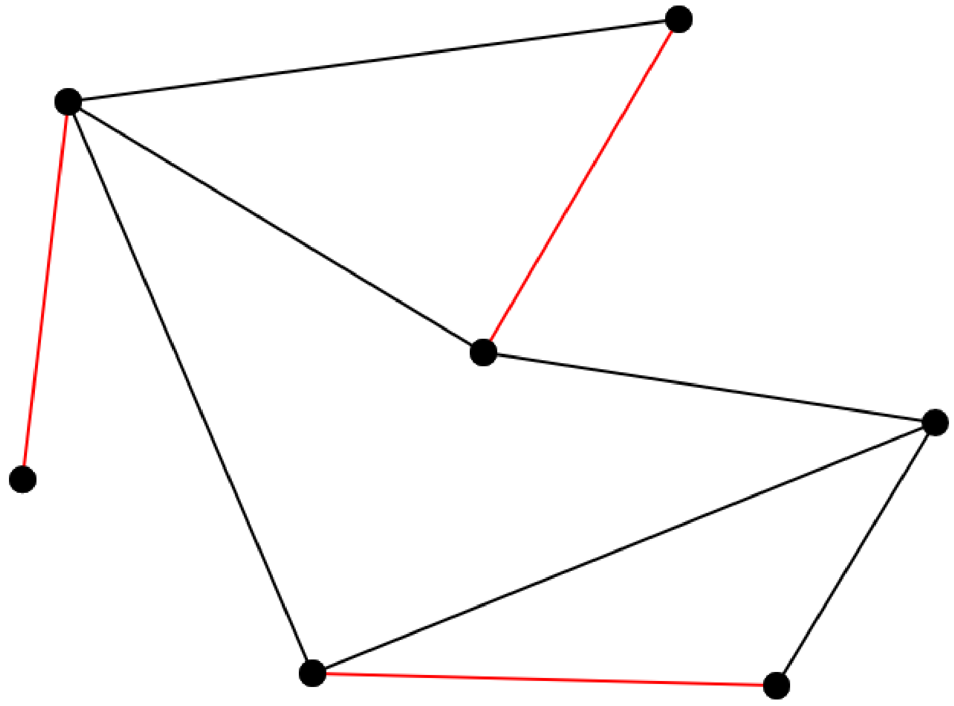

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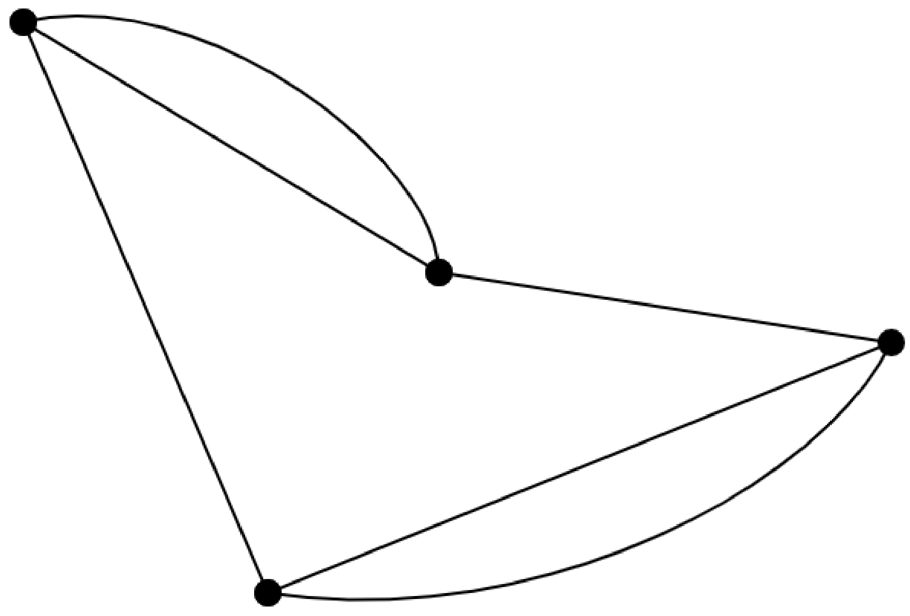

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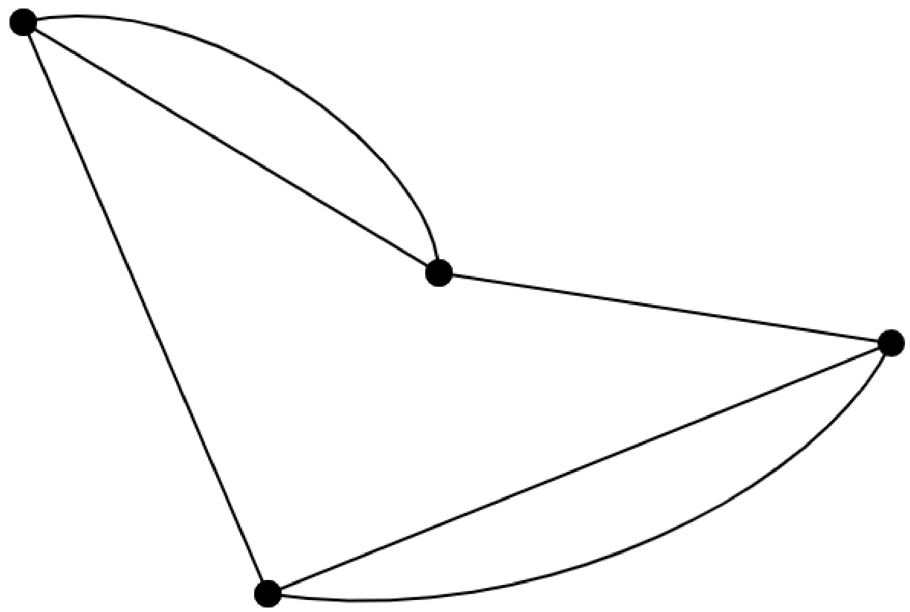

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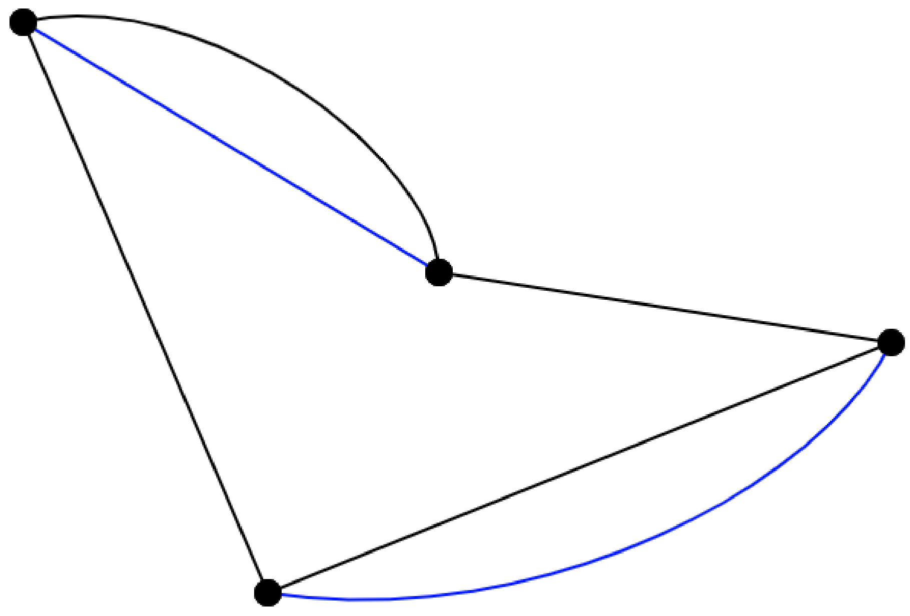

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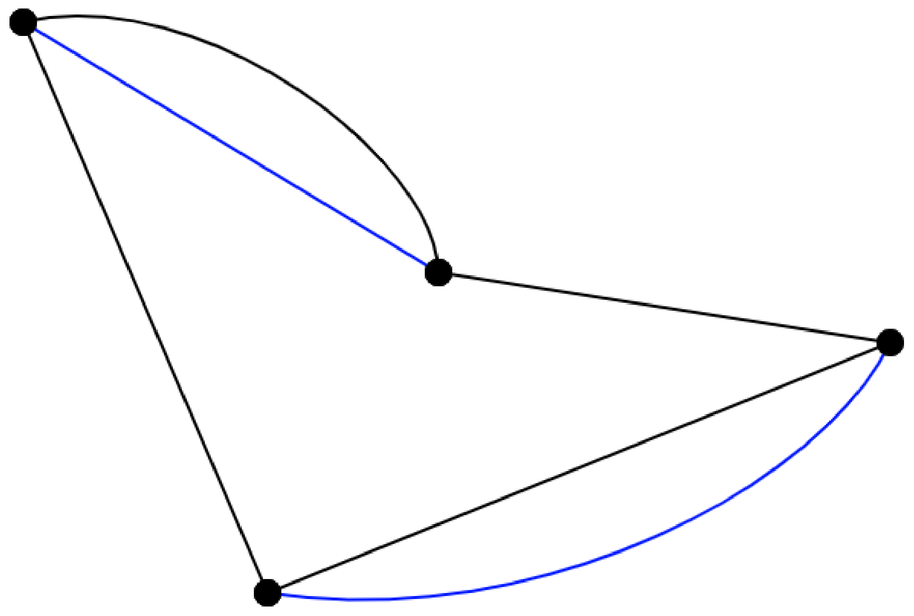
Return $M_1 \cup M_2$



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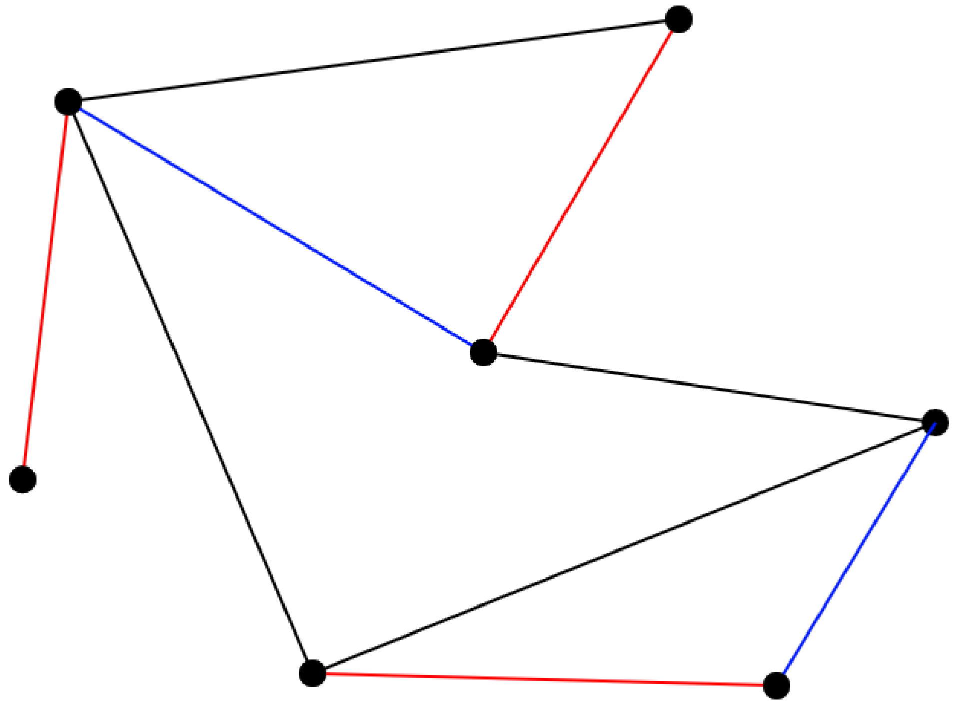

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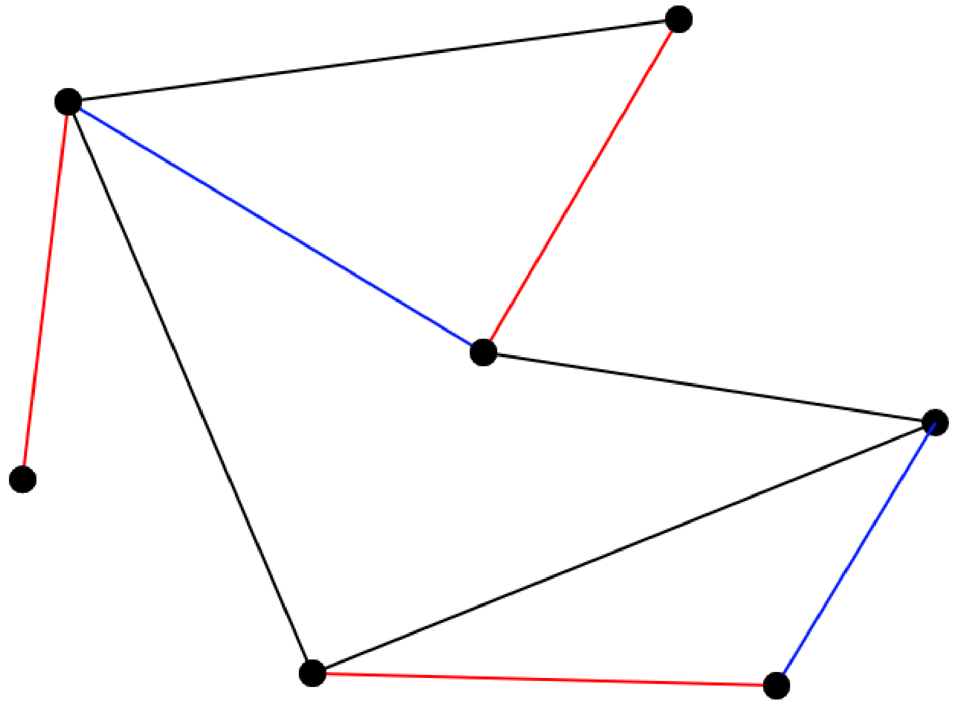

Theorem: $(2/3)$-approximation

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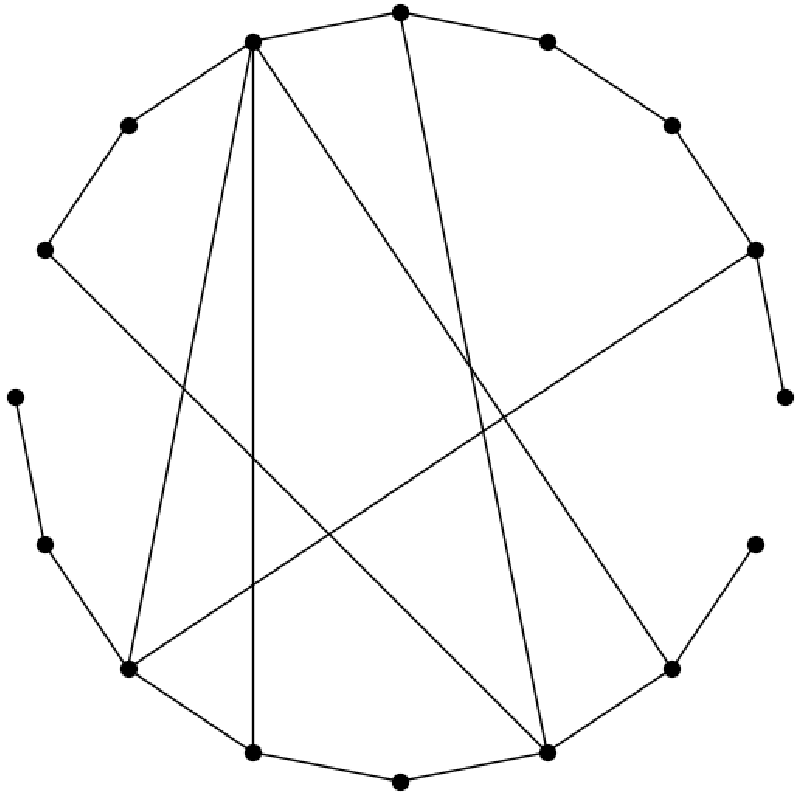

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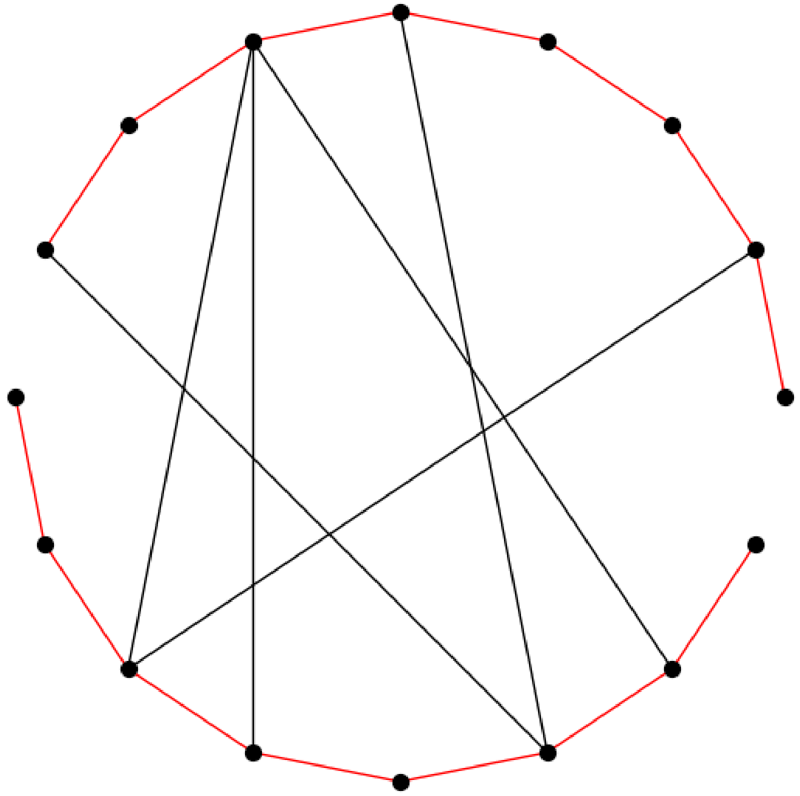
$P^\star$: MPC such that $|P^\star \cap M_1|$ is maximal



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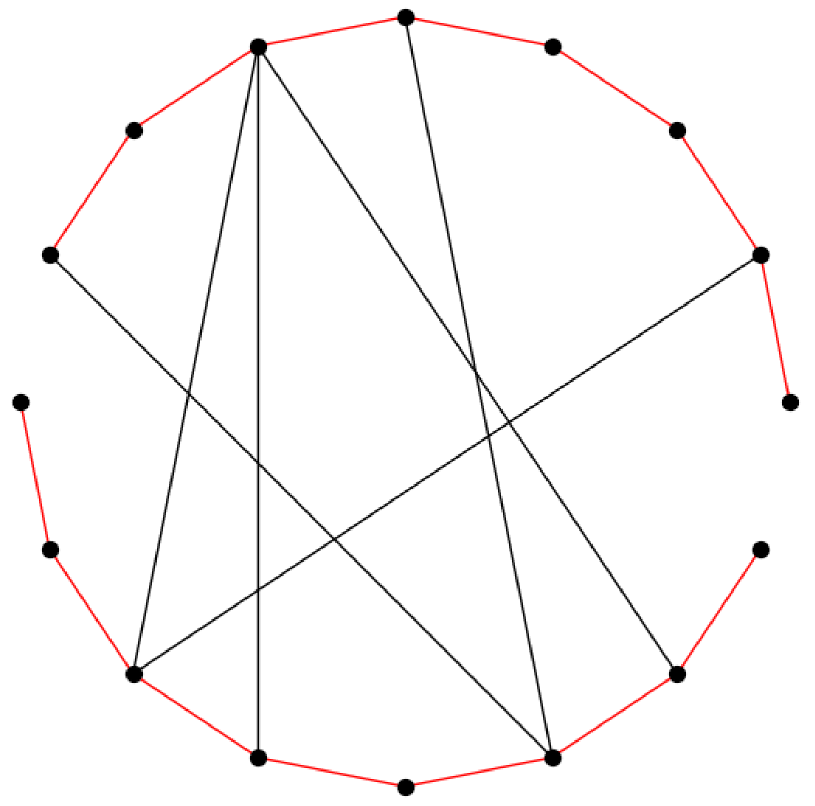
$G^\star : P^\star / M_1 \to$ very special graph (deg: 1,2 or 4)



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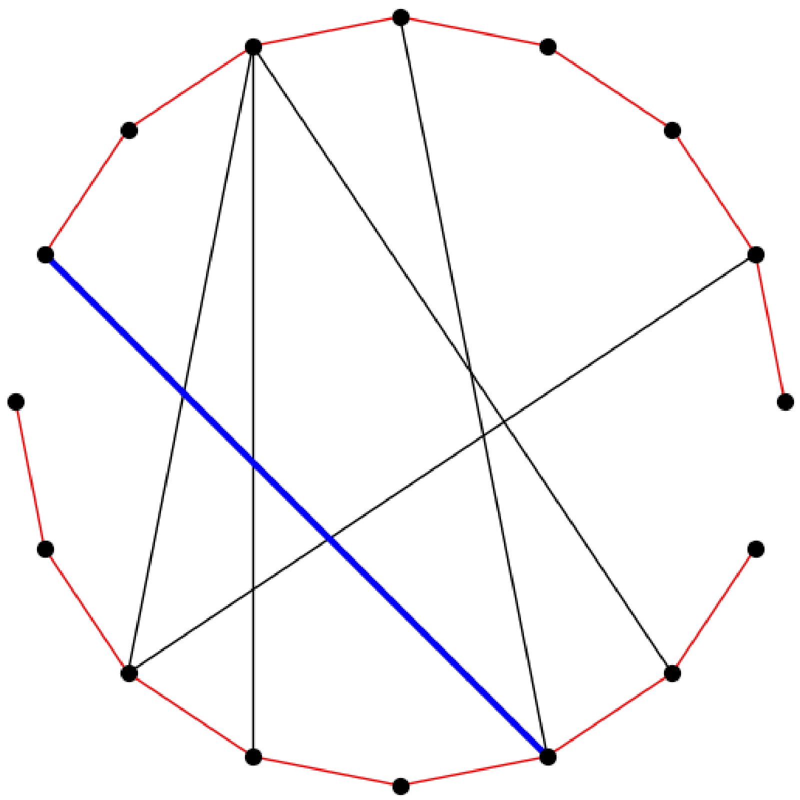

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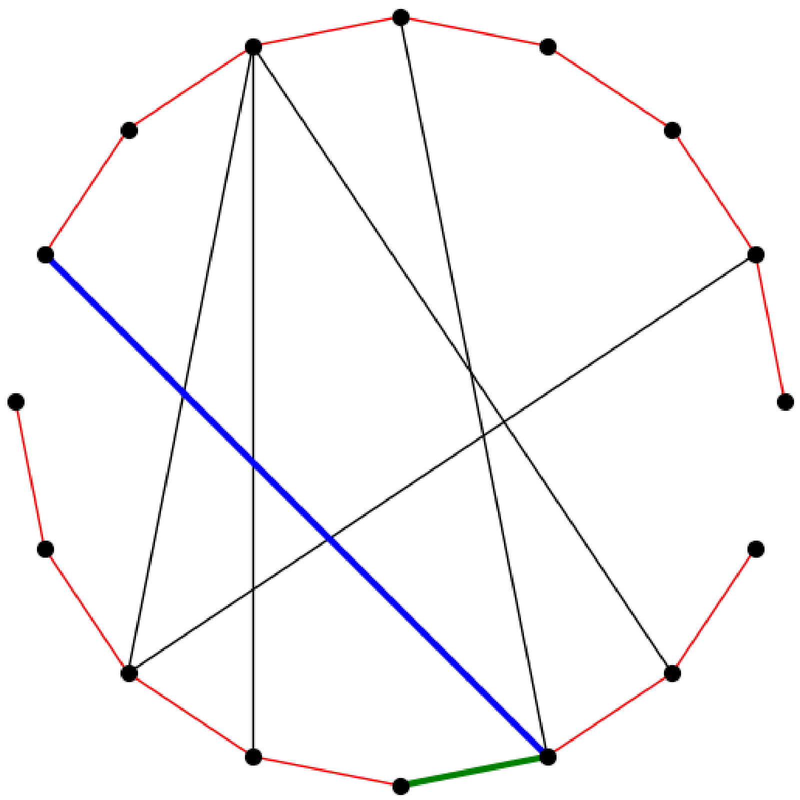

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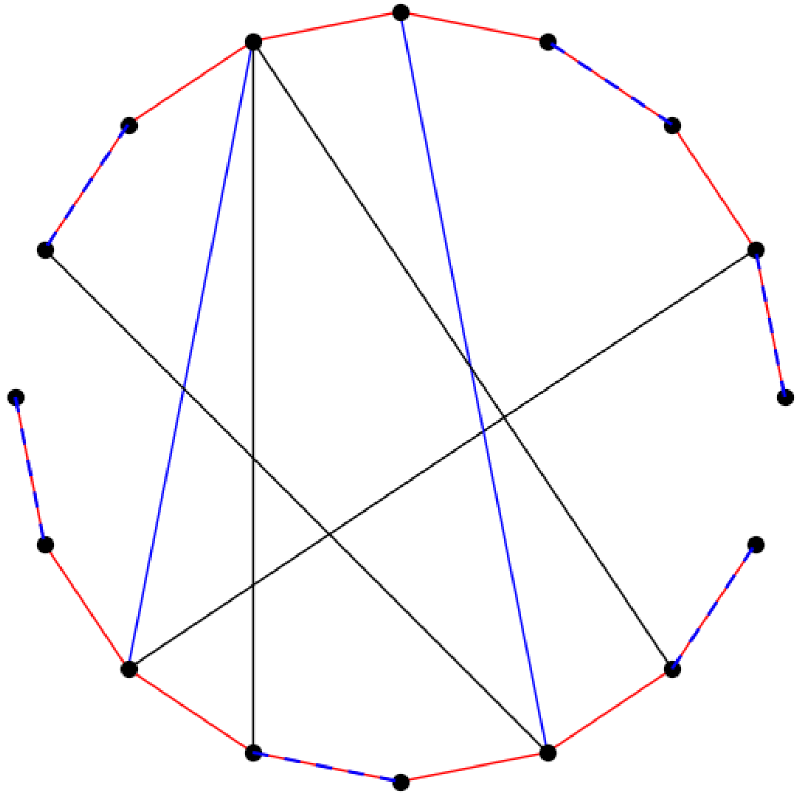

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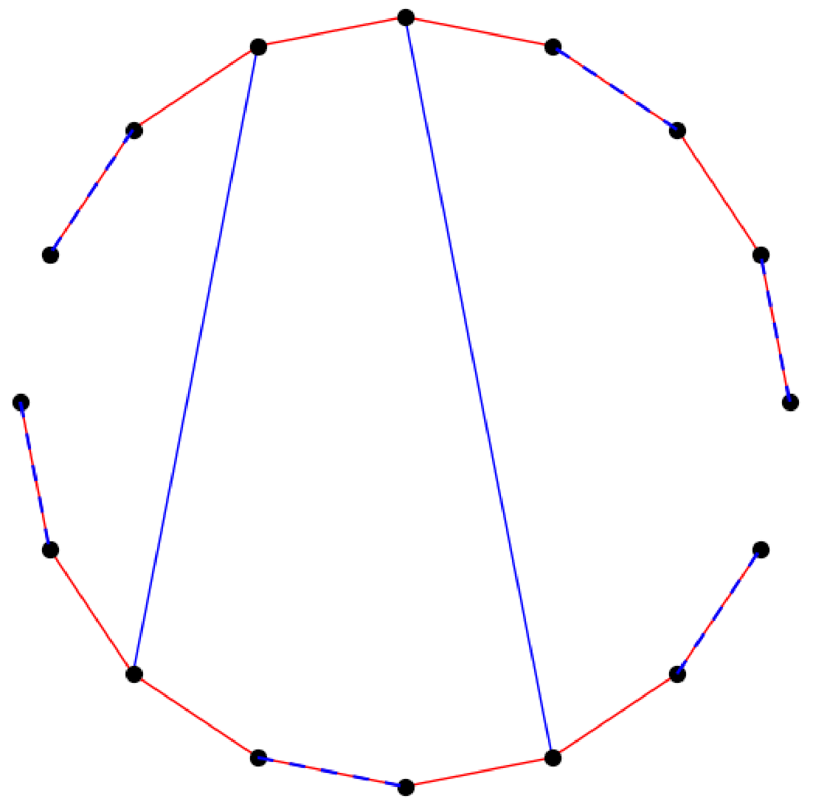

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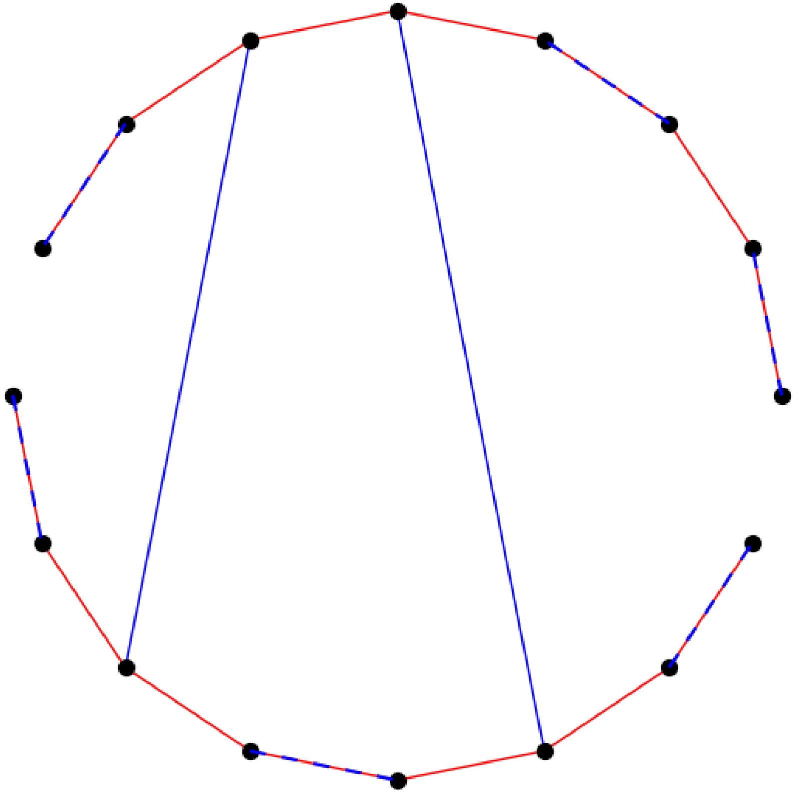

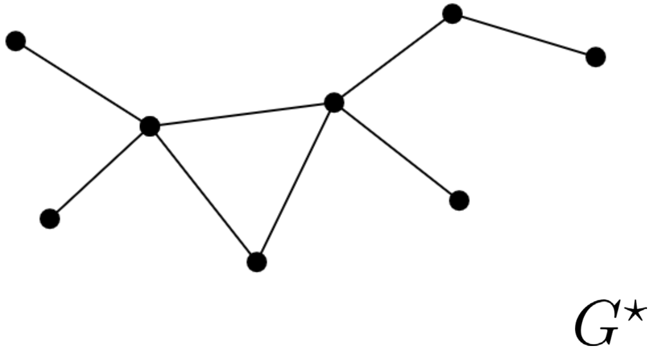

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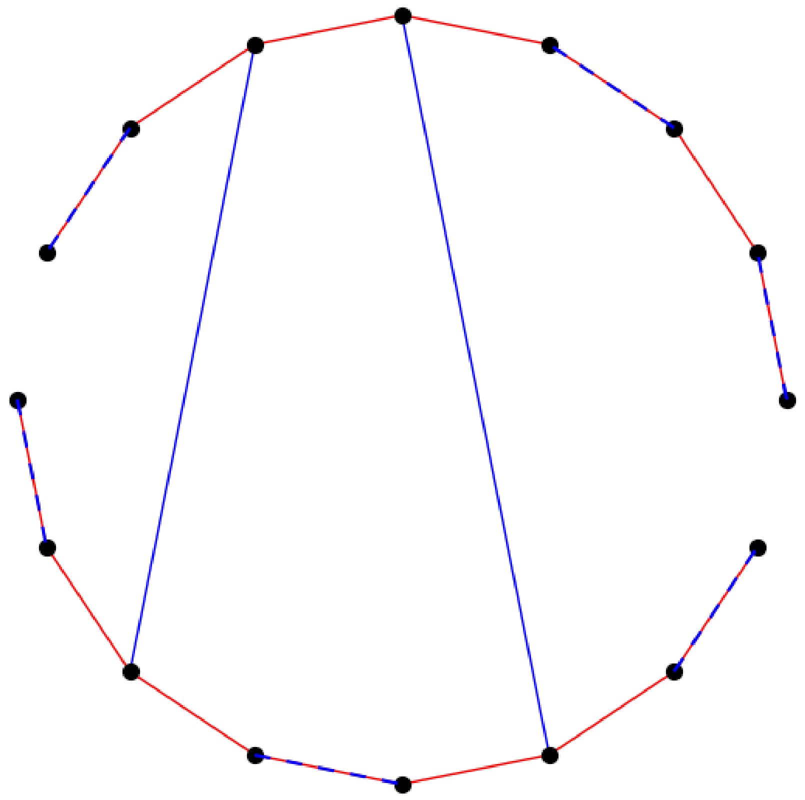
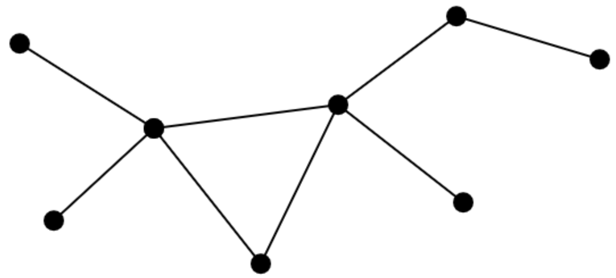
Lemma: $\mu(G^\star) \geq \frac{|E(G^\star)| - |V_4(G^\star)|}{3}$





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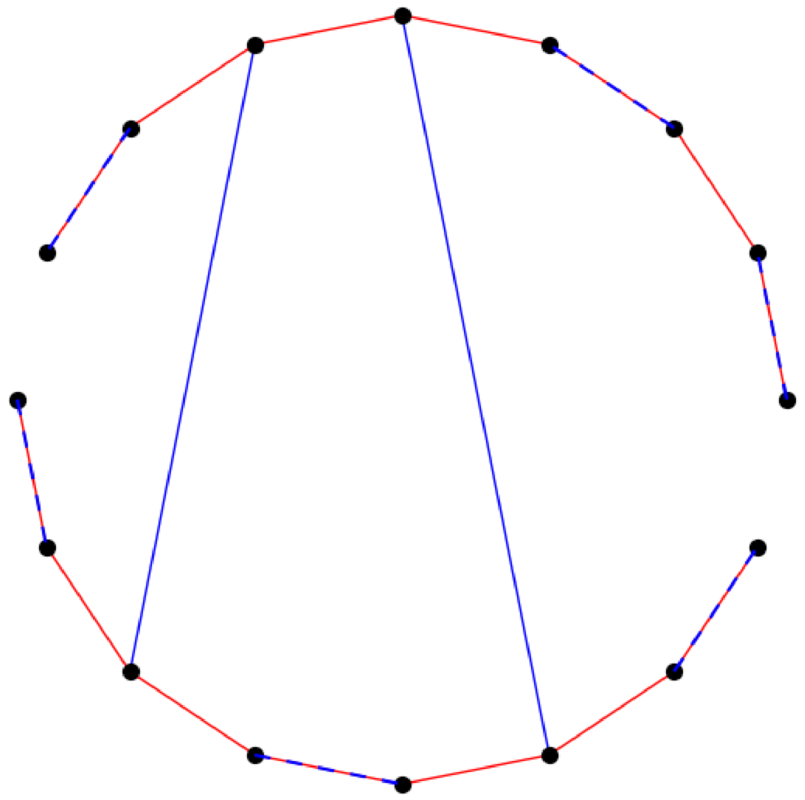
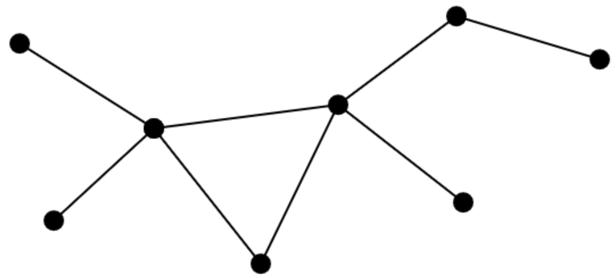
$= \frac{|P^\star|-|P^\star \cap M_1|-|M_1 \setminus P^\star|}{3}$





$G^\star$

Analysis:

$\rho(G)$:= size of the MPC on G

$\mu(G)$:= size of maximum matching of G

$P^\star$: MPC such that $|P^\star \cap M_1|$ is maximal

$G^\star$: $P^\star / M_1$ $\rightarrow$ very special graph (deg: 1,2 or 4)

Lemma: $\mu(G^\star) \geq \frac{|E(G^\star)| - |V_4(G^\star)|}{3}$

$$= \frac{|P^\star| - |P^\star \cap M_1| - |M_1 \setminus P^\star|}{3}$$

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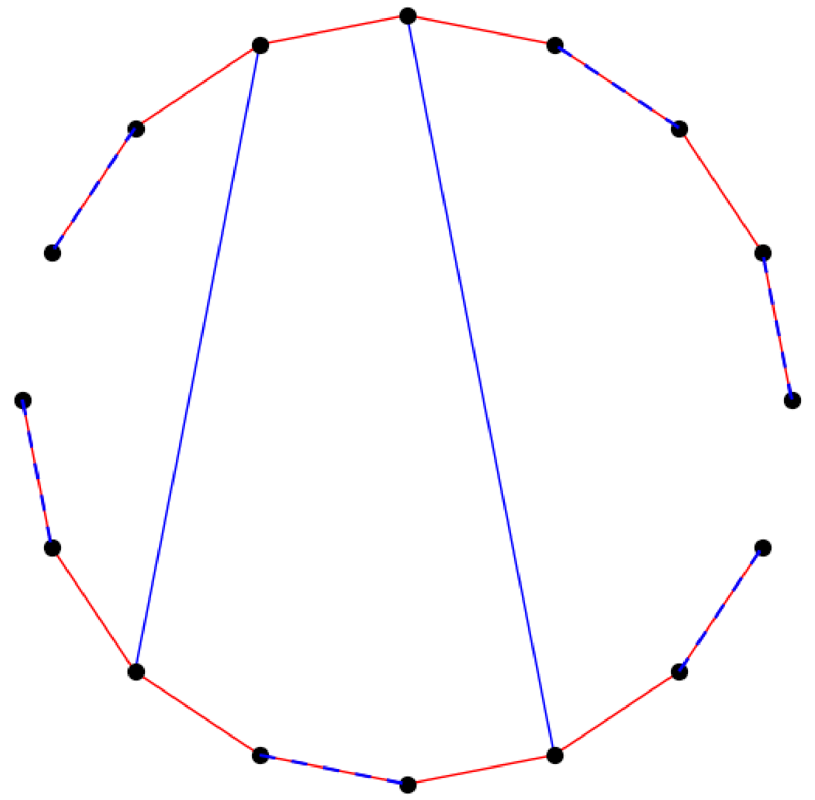

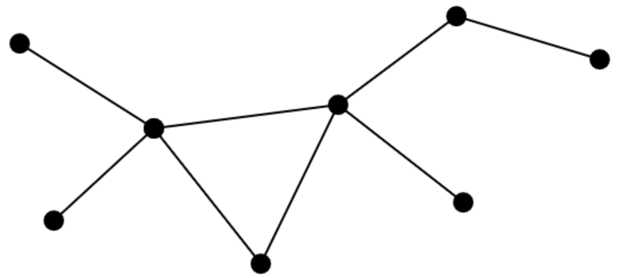

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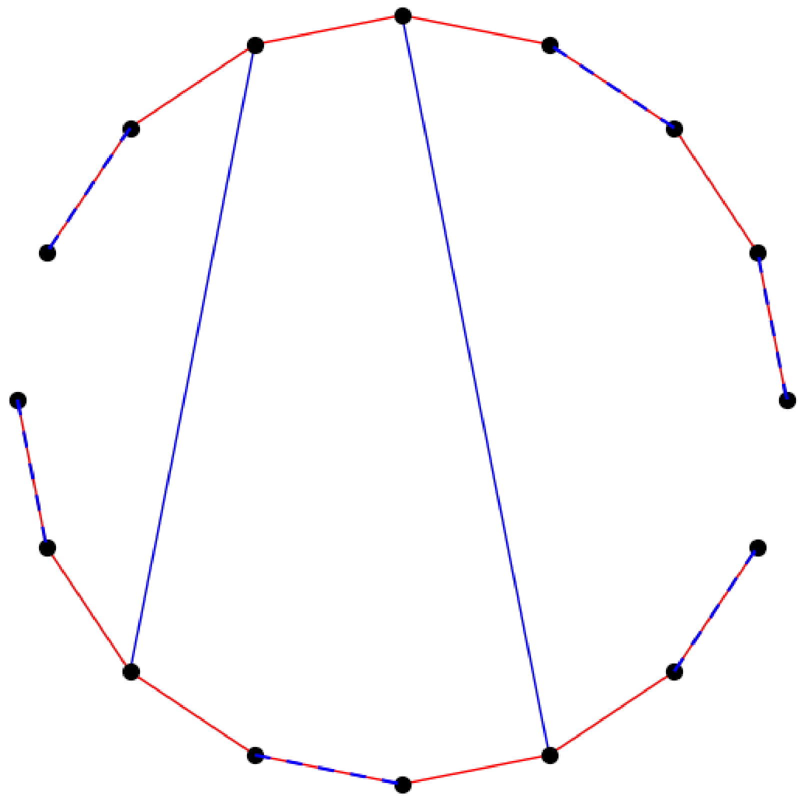
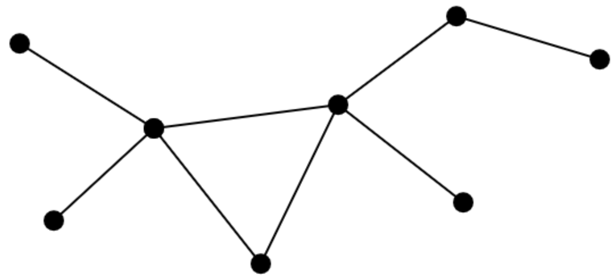
Analysis:

$$|M_1 \cup M_2| = |M_1| + |M_2|$$





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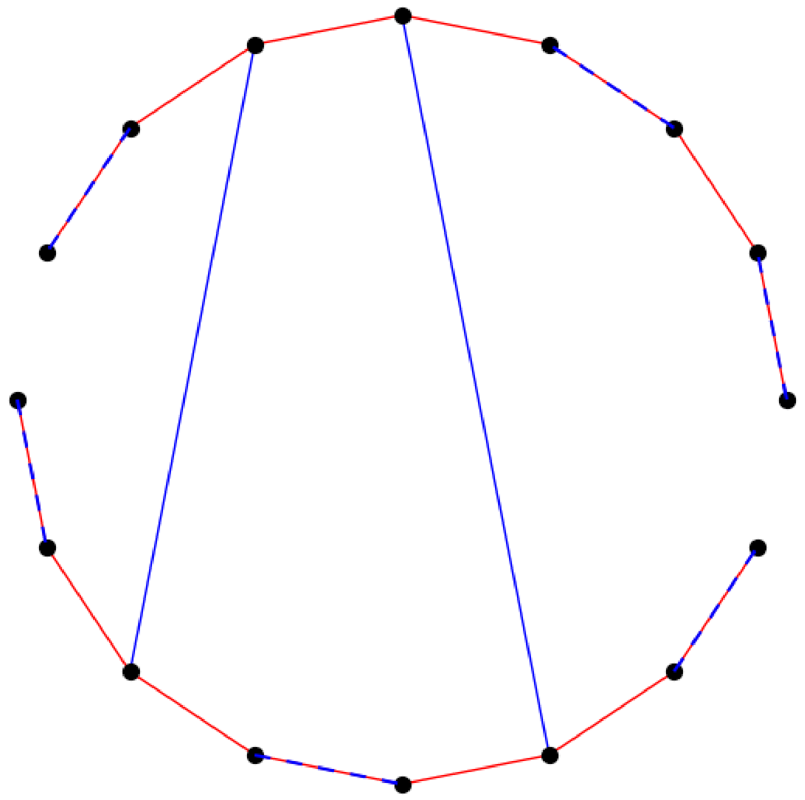

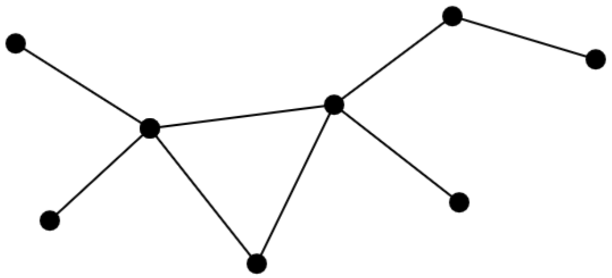

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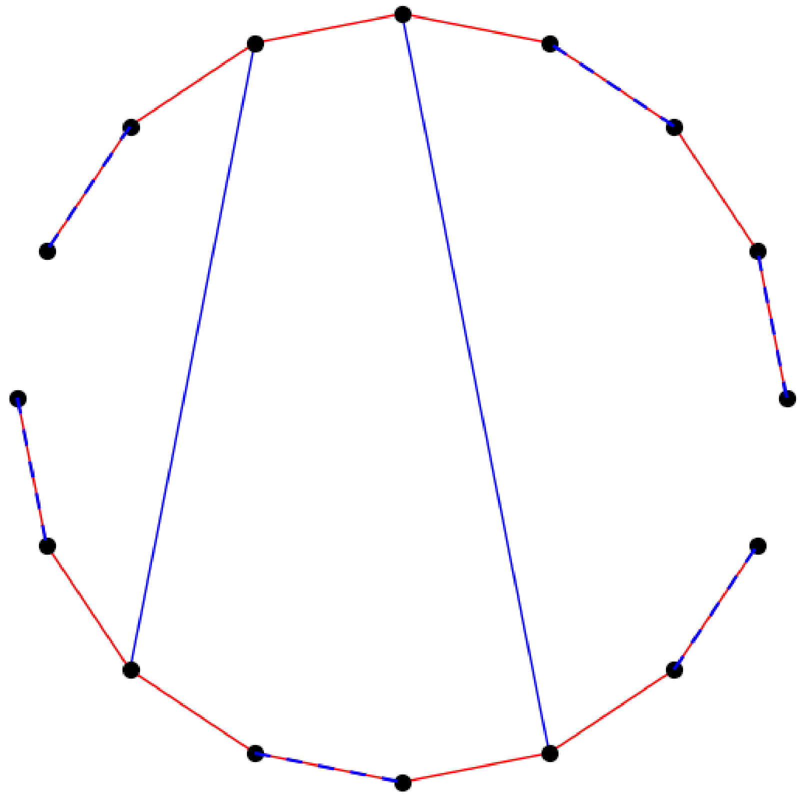

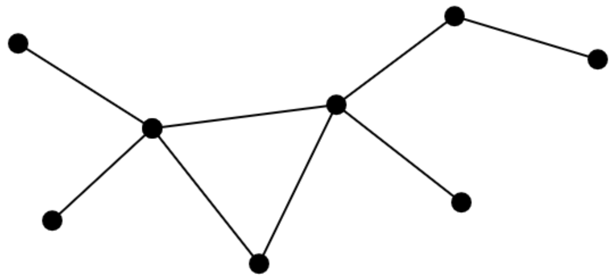

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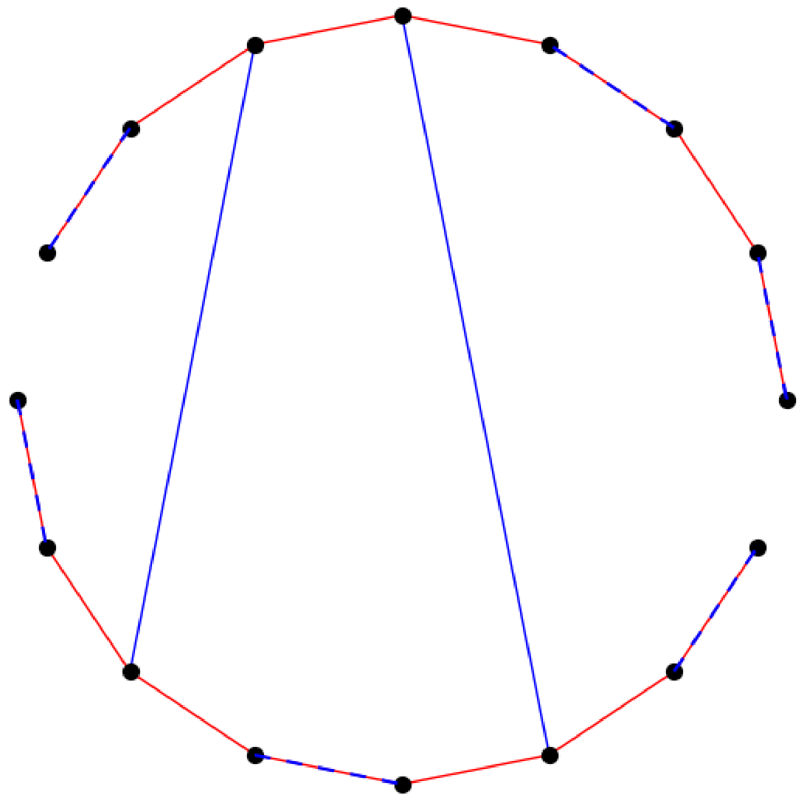
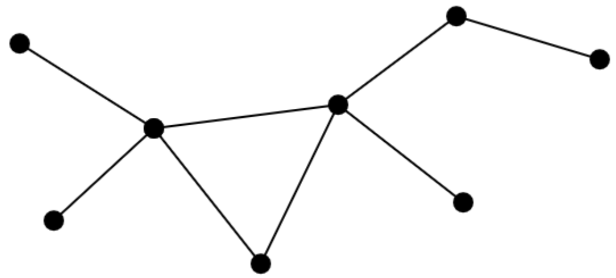
approx $\geq 2/3$





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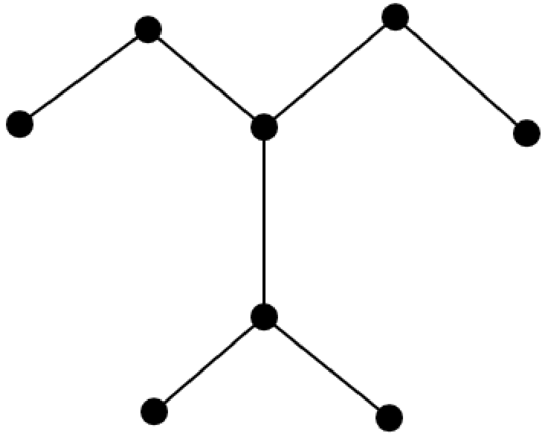

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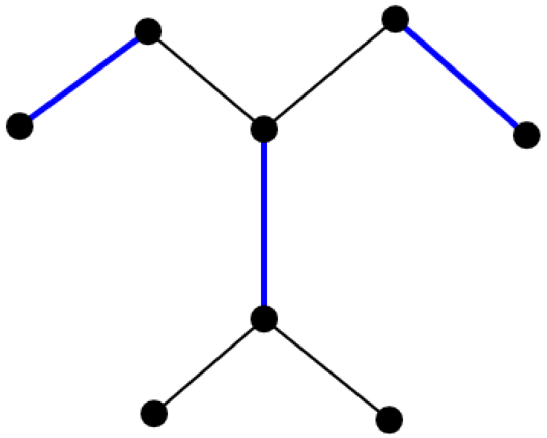

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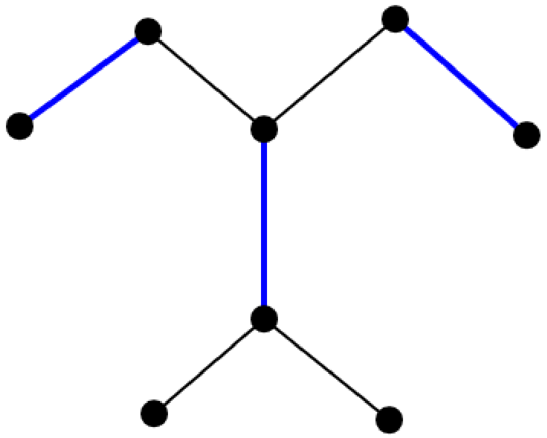

$|M_1| = 3$

Analysis:

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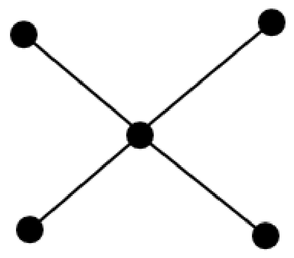

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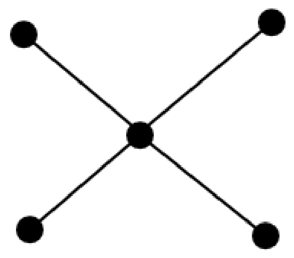

$|M_1| = 3 \quad |M_2| = 1$

Analysis:

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|M_1 \cup M_2| &= |M_1| + |M_2| \\
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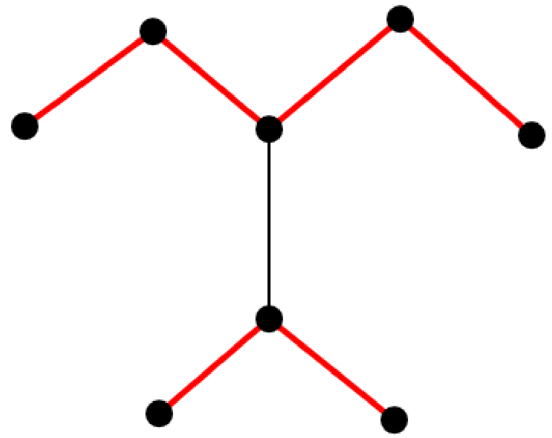

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Analysis:

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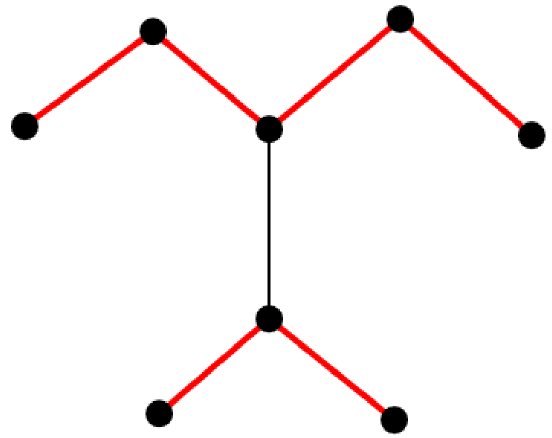

$|M_1| = 3 \quad |M_2| = 1$

$|P^\star| = 6$

Analysis:

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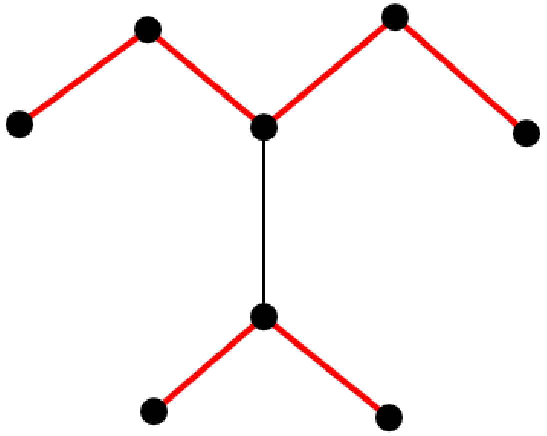
\end{aligned}
$$

approx $\geq 2/3$

approx $\leq 2/3$



$|M_1| = 3 \quad |M_2| = 1$

$|P^\star| = 6$

Conclusion:

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Maximum Matching

$(1-\epsilon)$-approx

Conclusion:

Maximum Matching

$(1-\epsilon)$-approx

Maximum Path Cover

$(2/3-\epsilon)$-approx

Conclusion:

Maximum Matching

$(1-\epsilon)$-approx

Maximum Path Cover

$(2/3-\epsilon)$-approx

$(1,2)$-TSP

$(4/3+\epsilon)$-approx

Max-TSP:

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G: complete weighted graph

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Find a Hamiltonian cycle with maximum weight

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Algorithm:

Same algorithm with weighted matching

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Find a Hamiltonian cycle with maximum weight

Algorithm:

Same algorithm with weighted matching

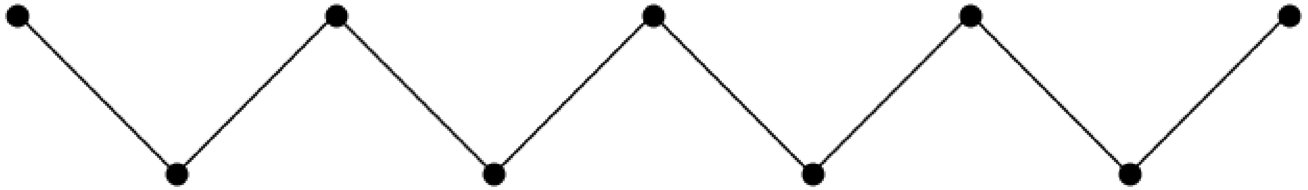
Theorem:

(7/12)-approximation

Analysis:

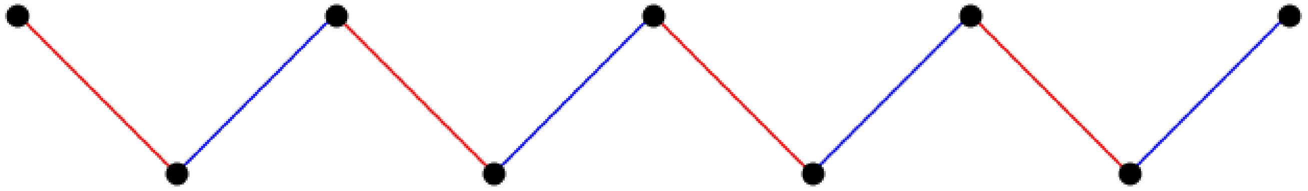
Analysis:

Observation:



Analysis:

Observation:



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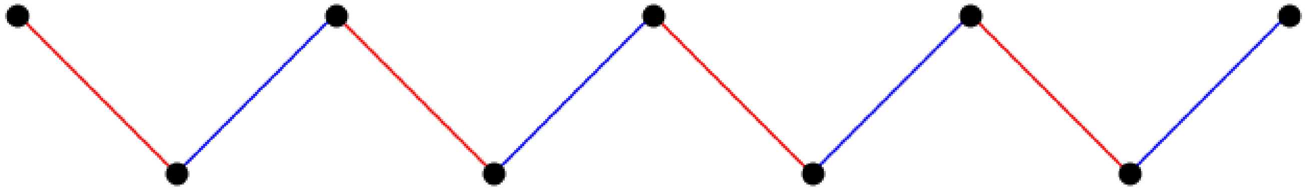
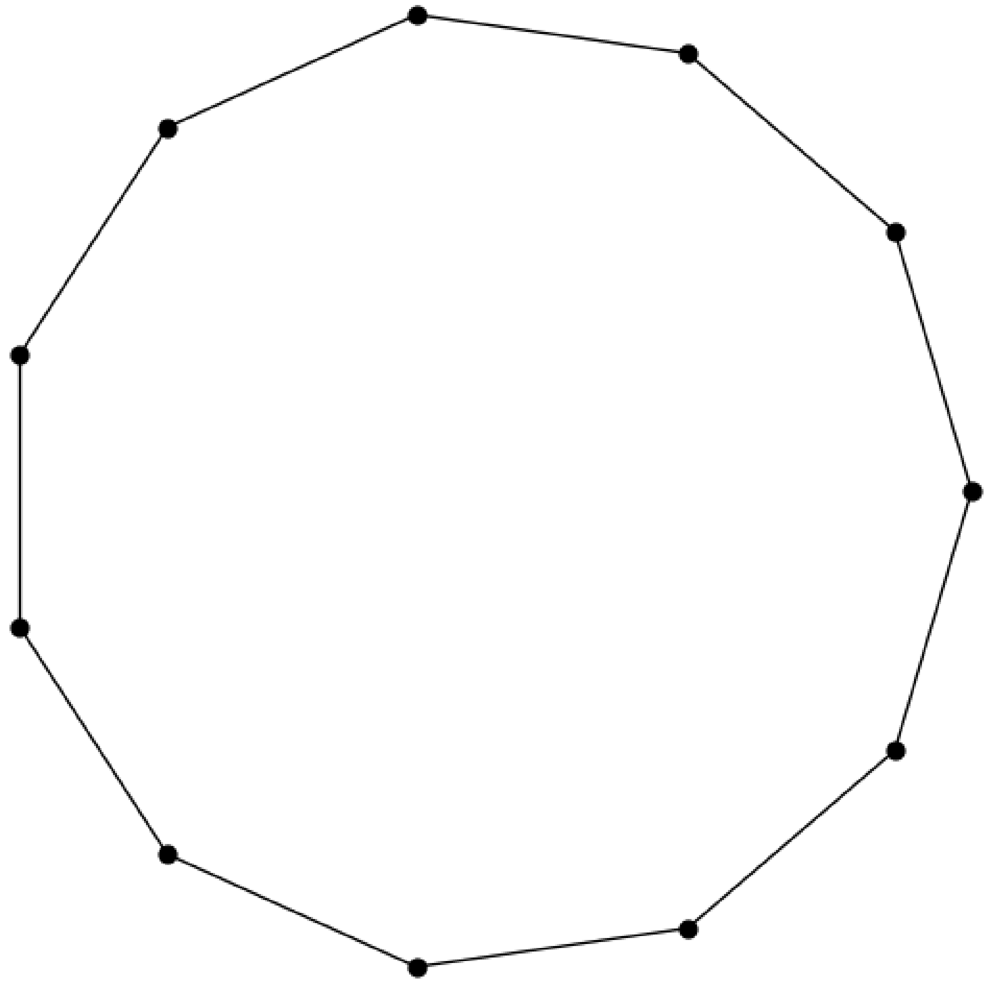


$\exists M \subseteq P \text{ s.t. } w(M) \geq w(P)/2$

Analysis:

Observation:



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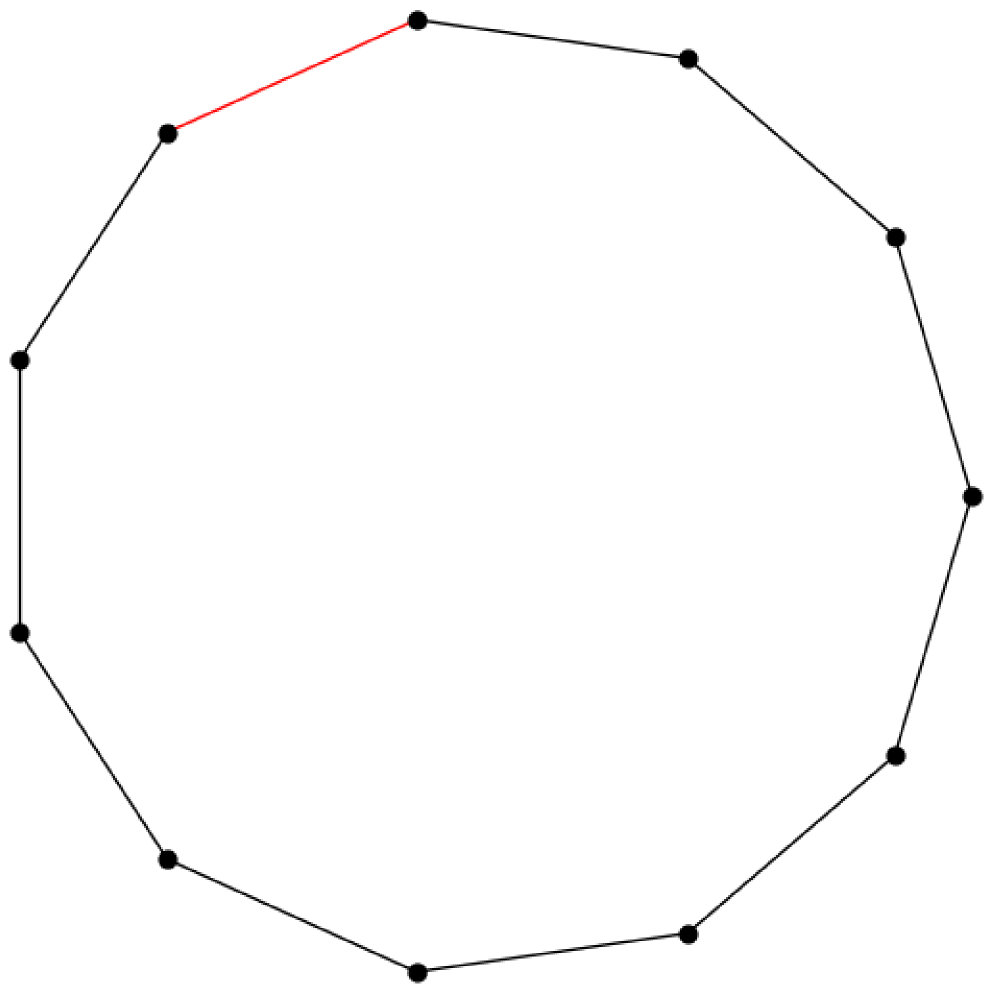

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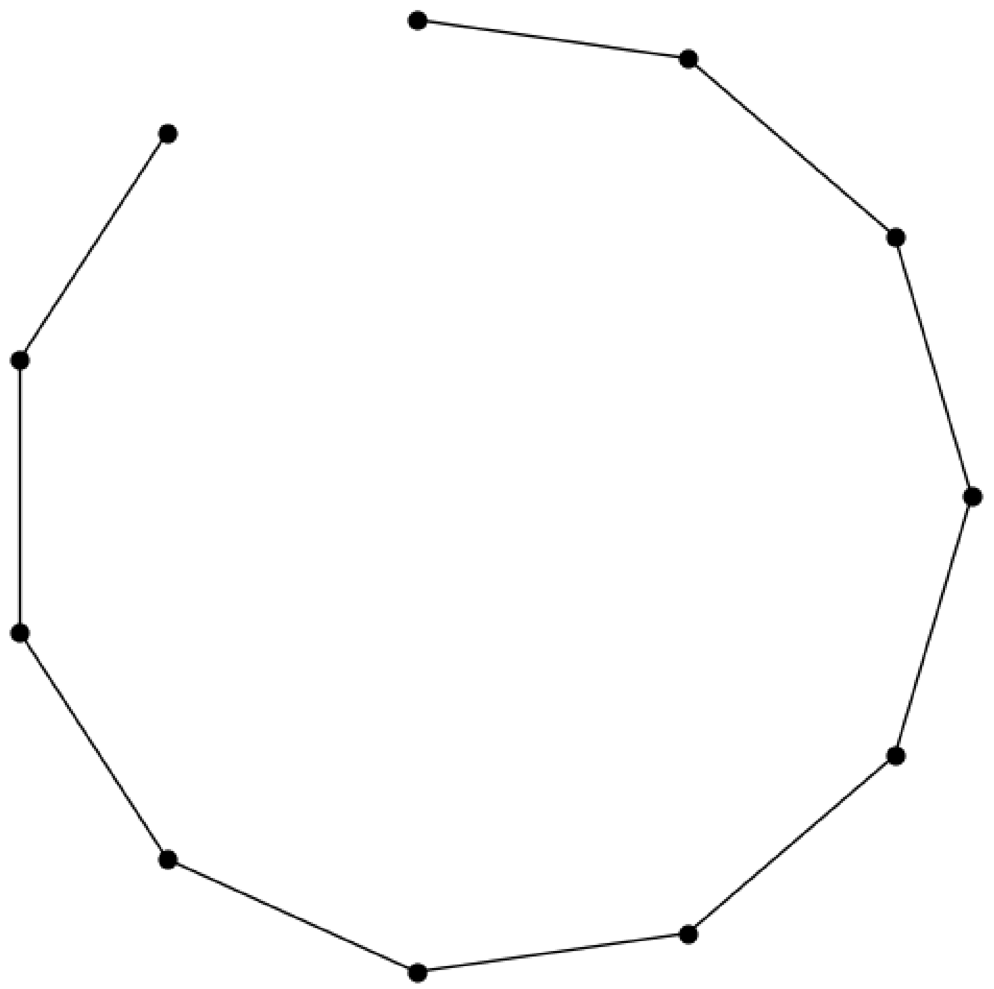

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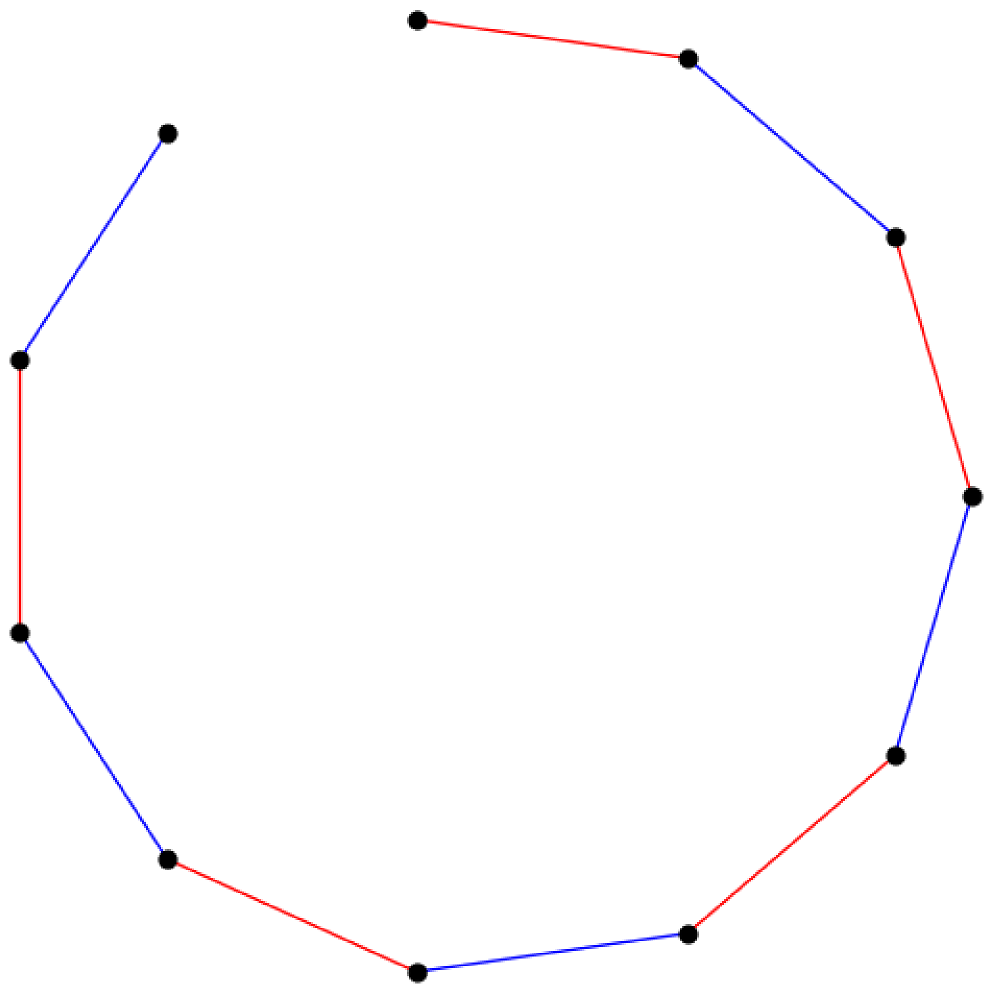
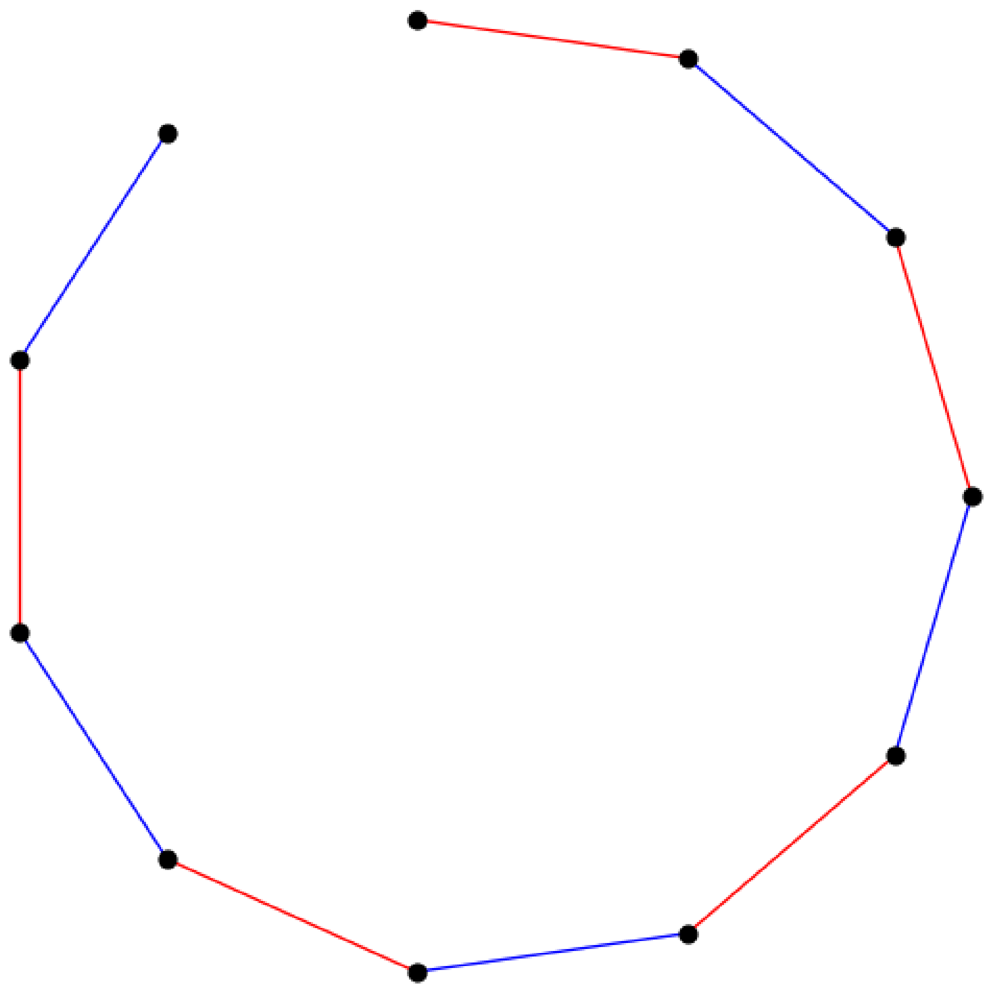


Analysis:

Observation:

$\exists M \subseteq C$ s.t.

$$w(M) \geq (1 - 1/\ell) \cdot w(C)/2$$



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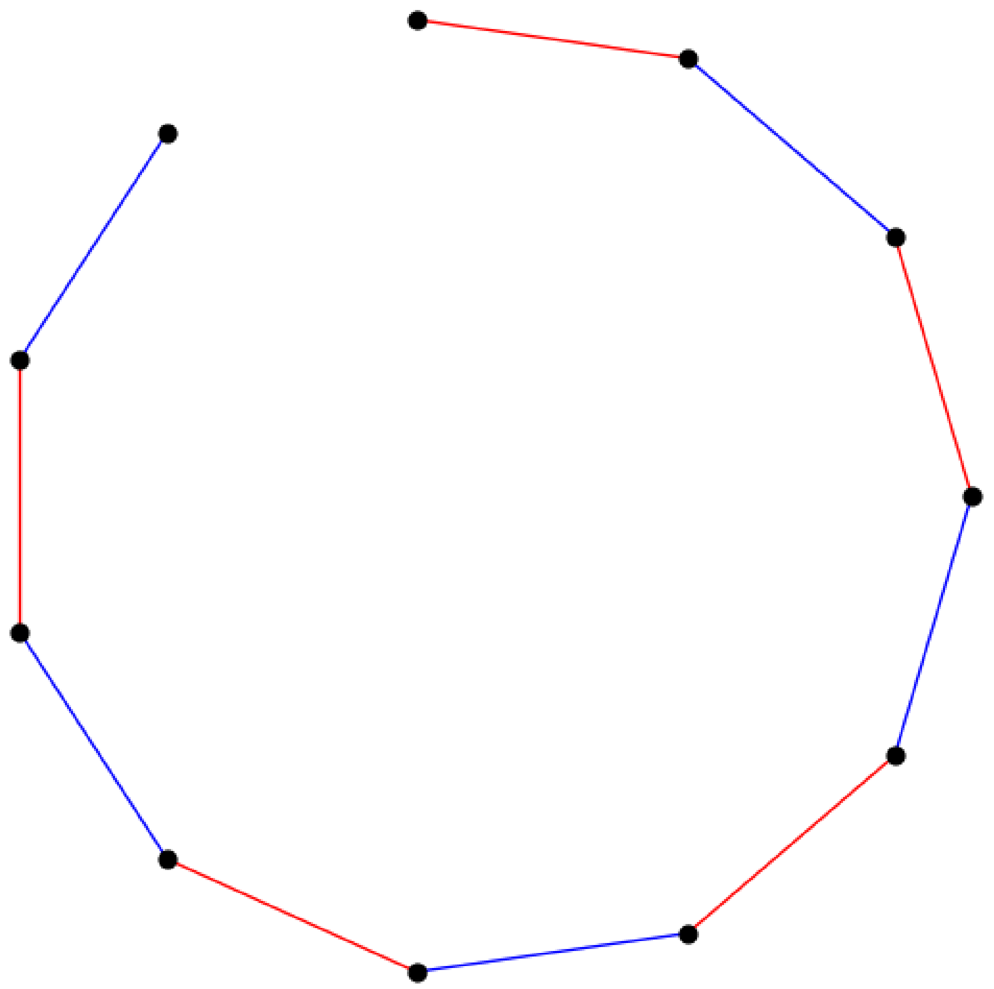
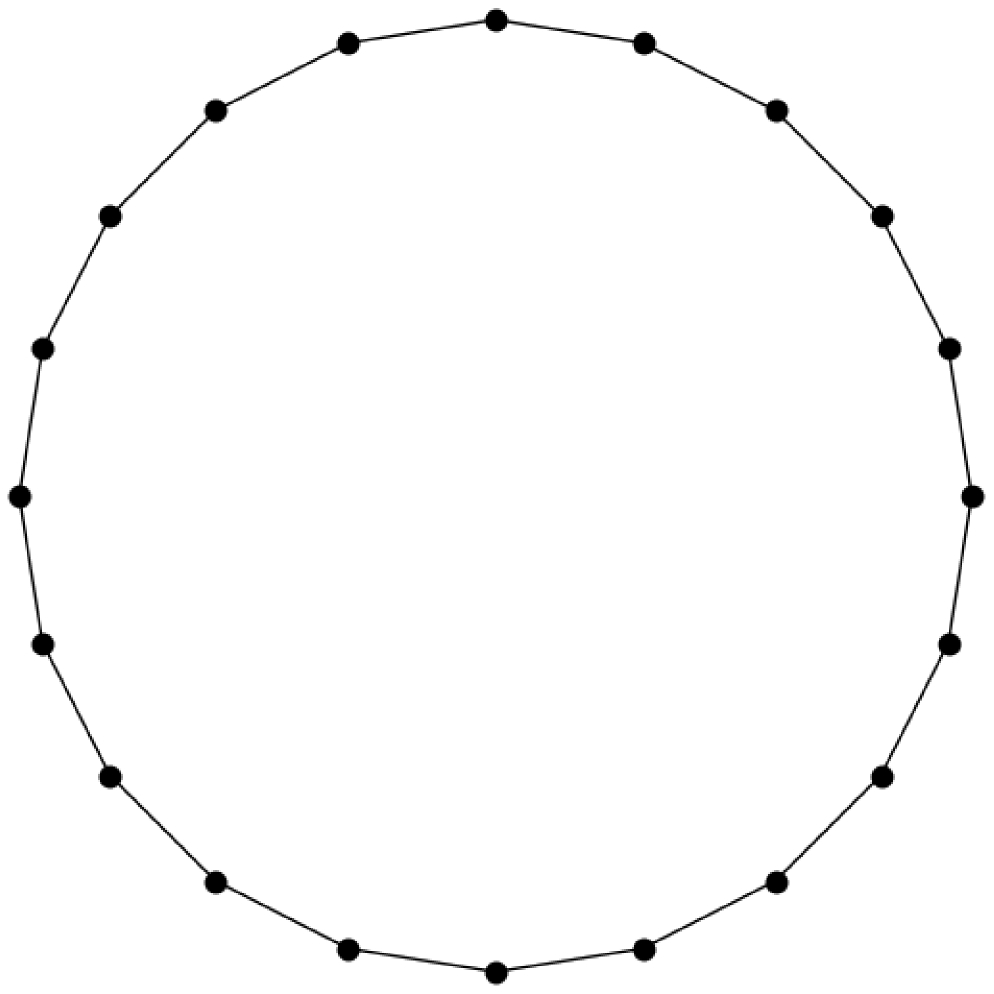
worst case: $w(M) \geq w(C)/3$



Analysis:

$C^\star$: Max-TSP



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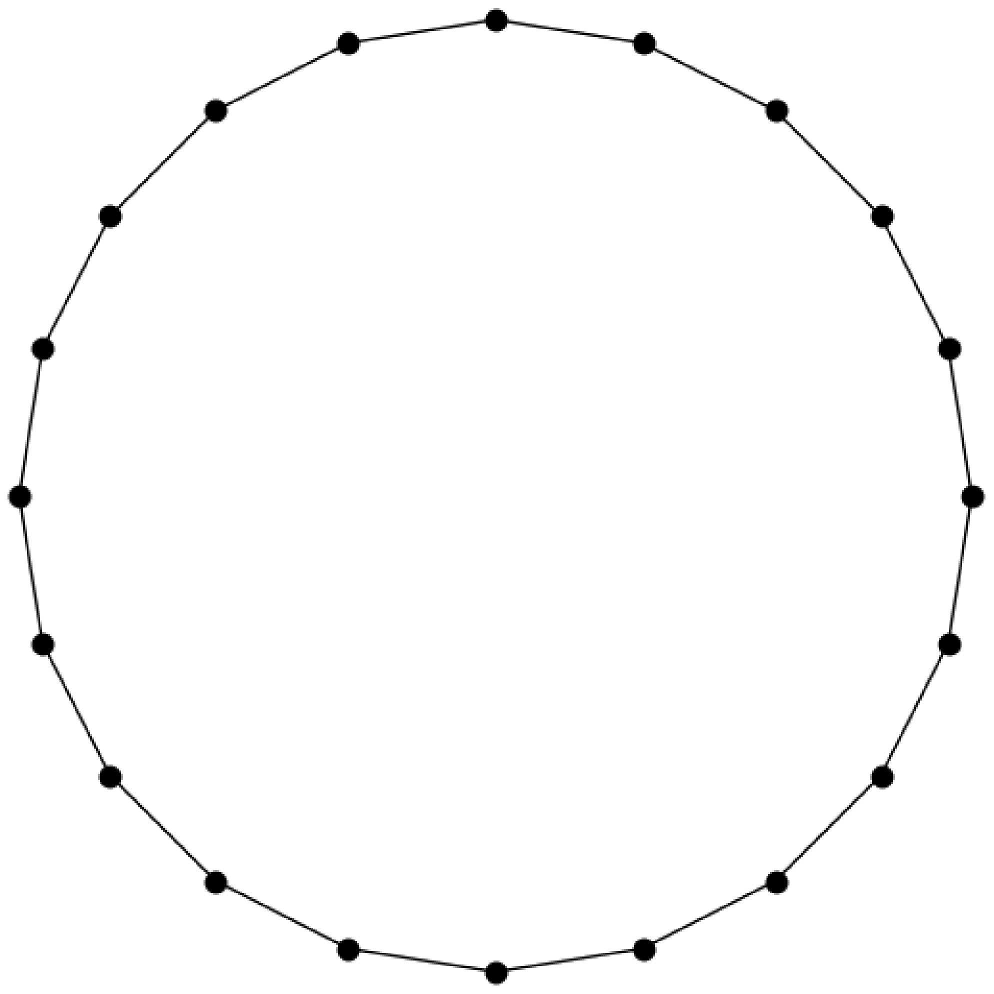

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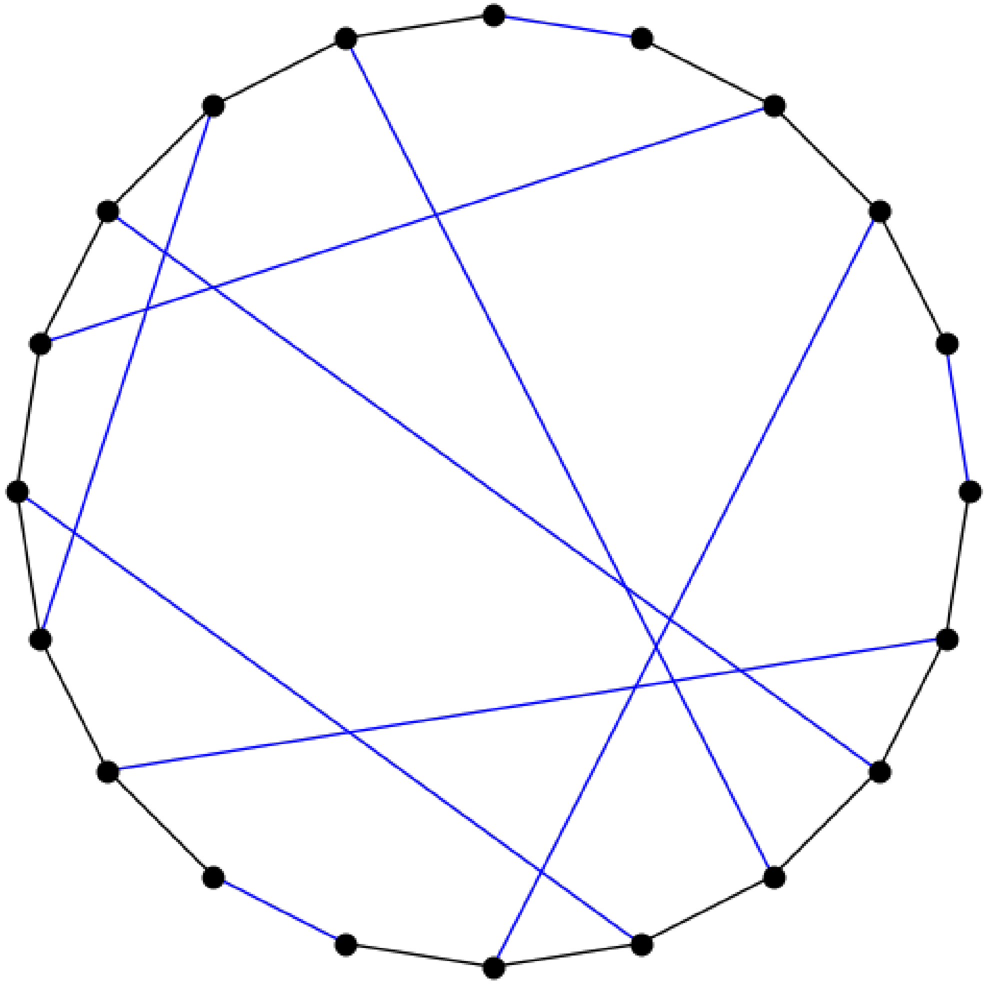

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$C^\star$: Max-TSP

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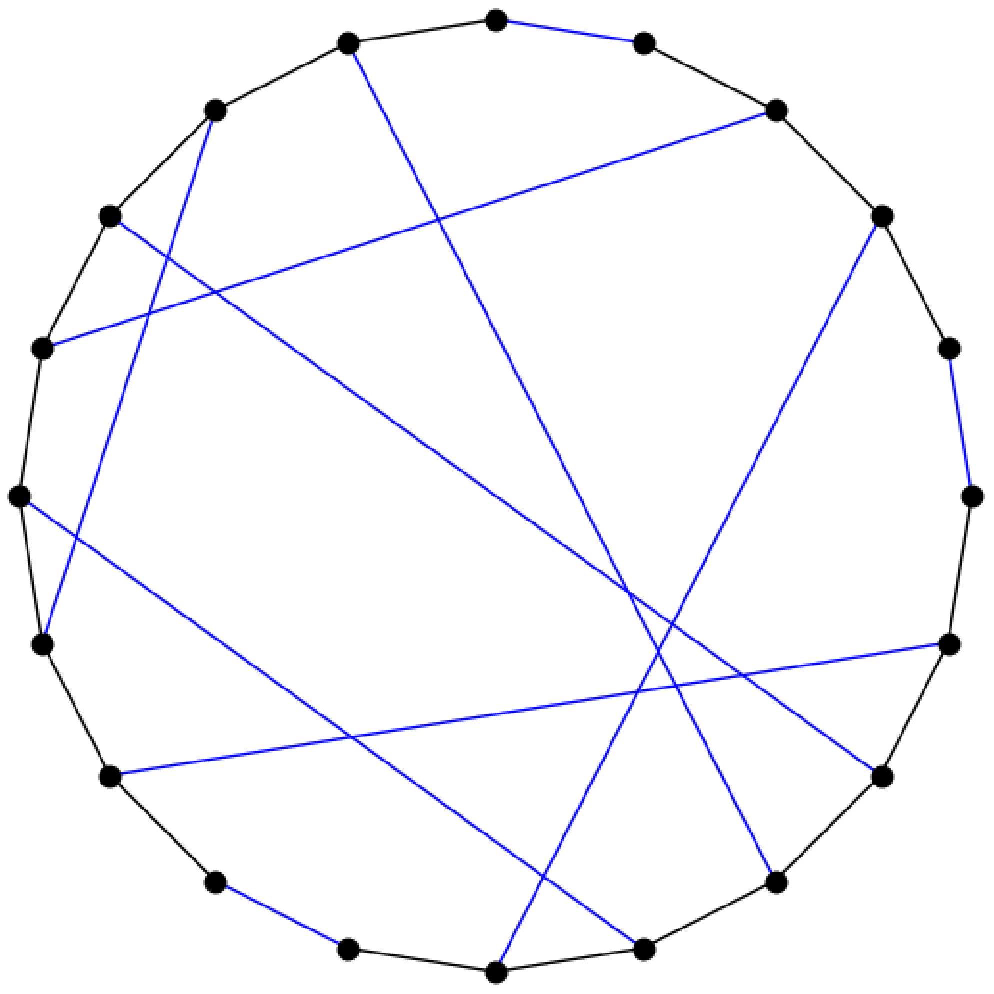
Consider $C = C^\star / (M_1 \cap C^\star)$



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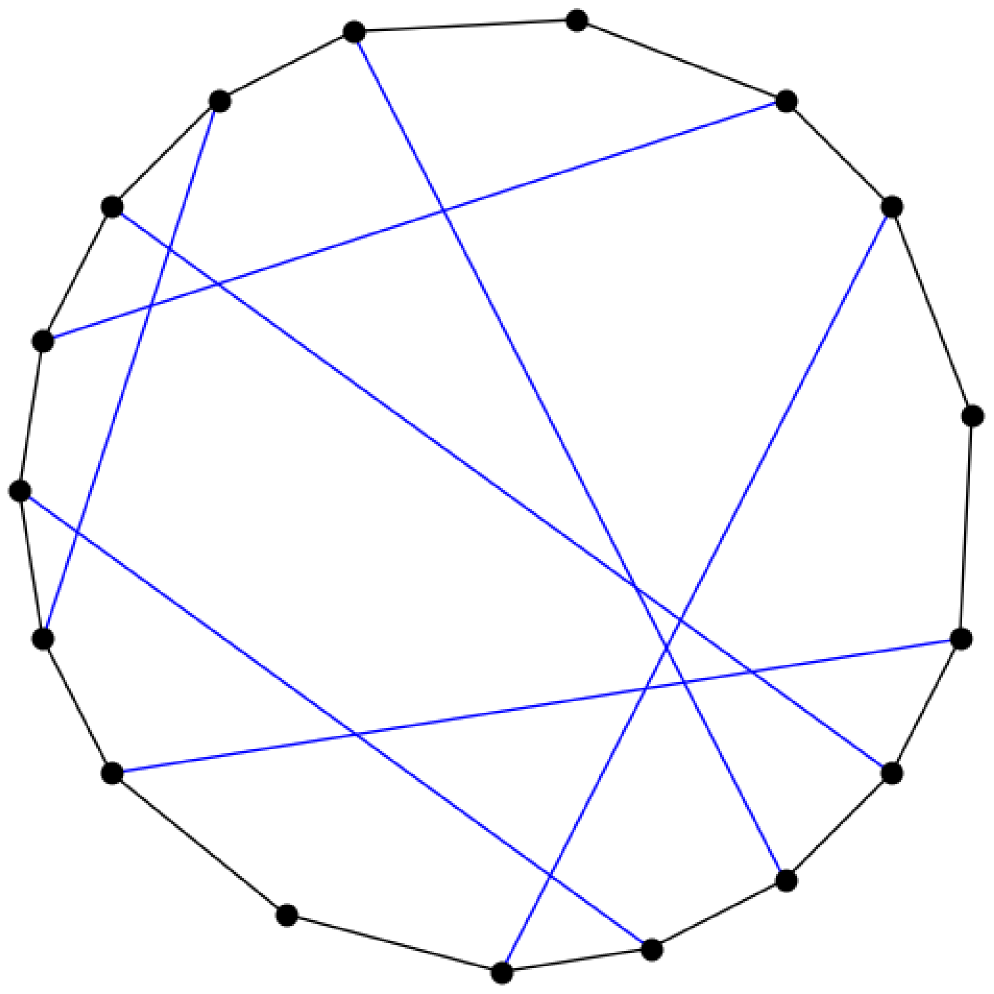

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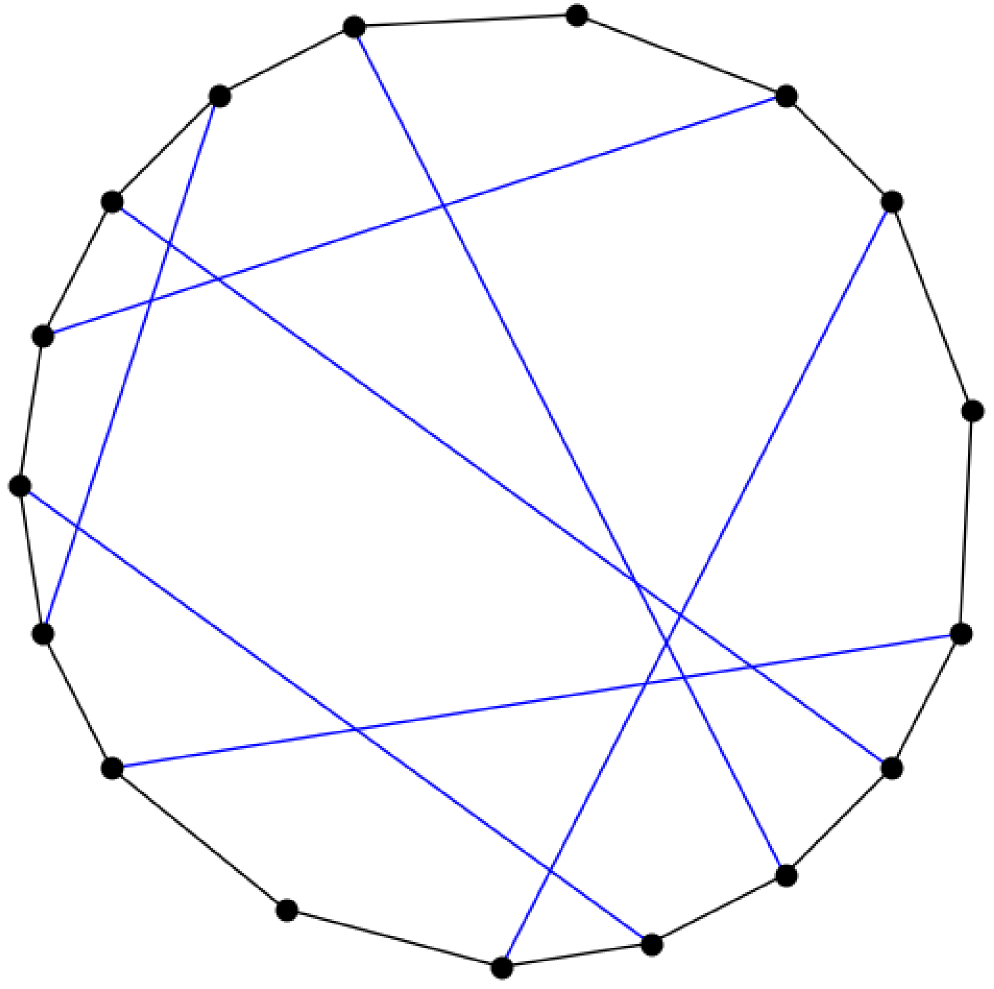
$\exists M \subseteq C$ s.t. $w(M) \approx w(C)/2$



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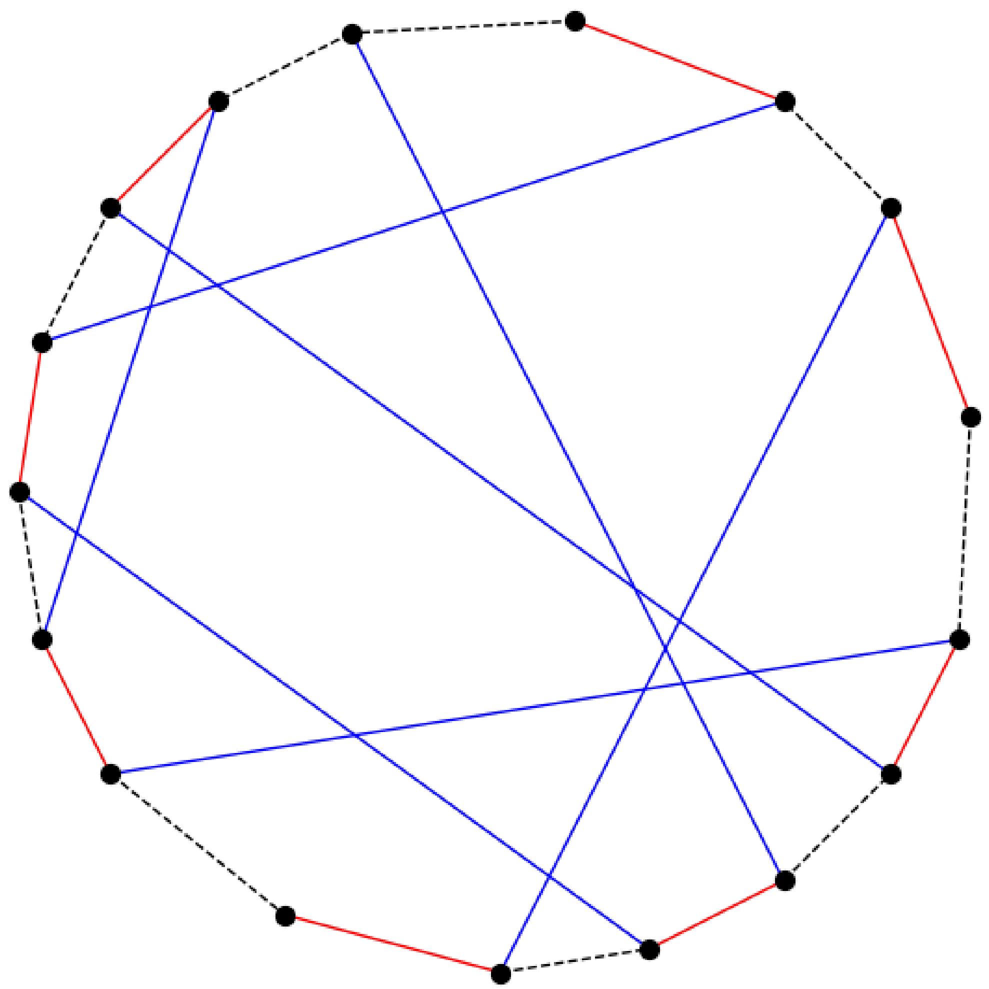

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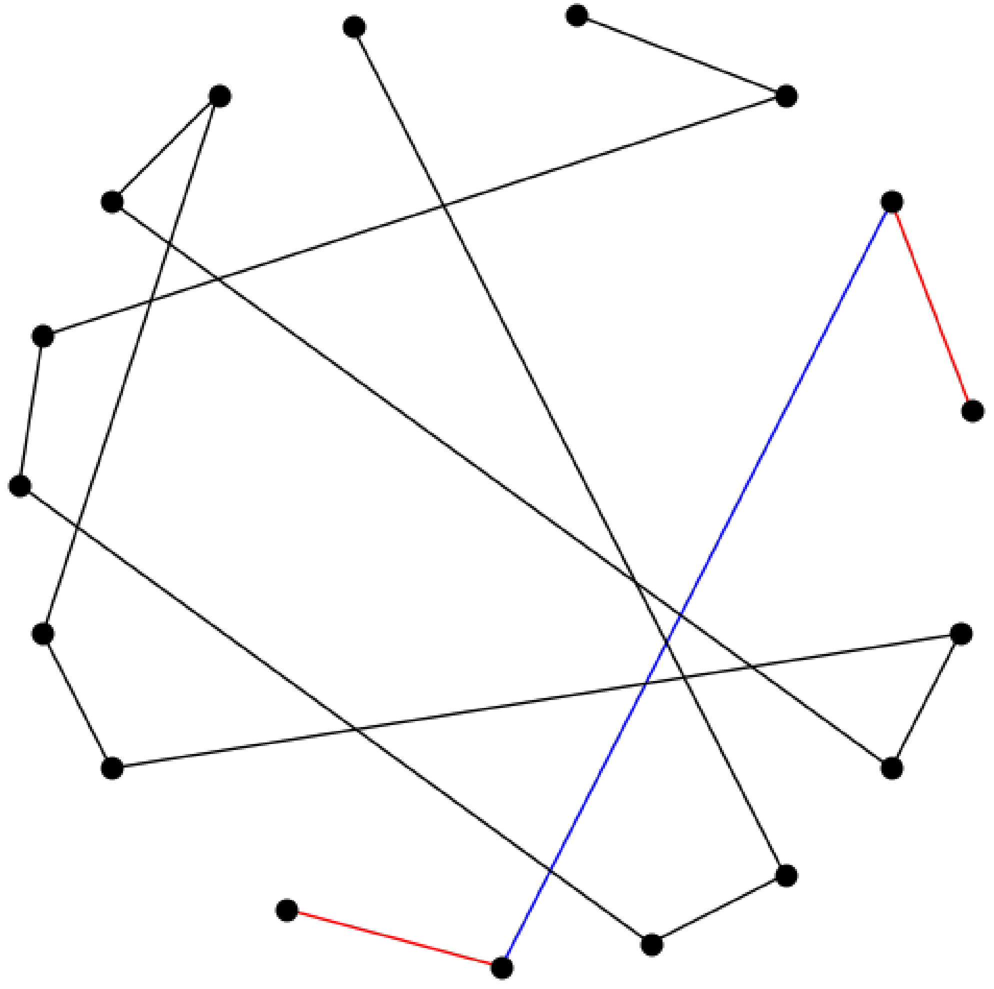

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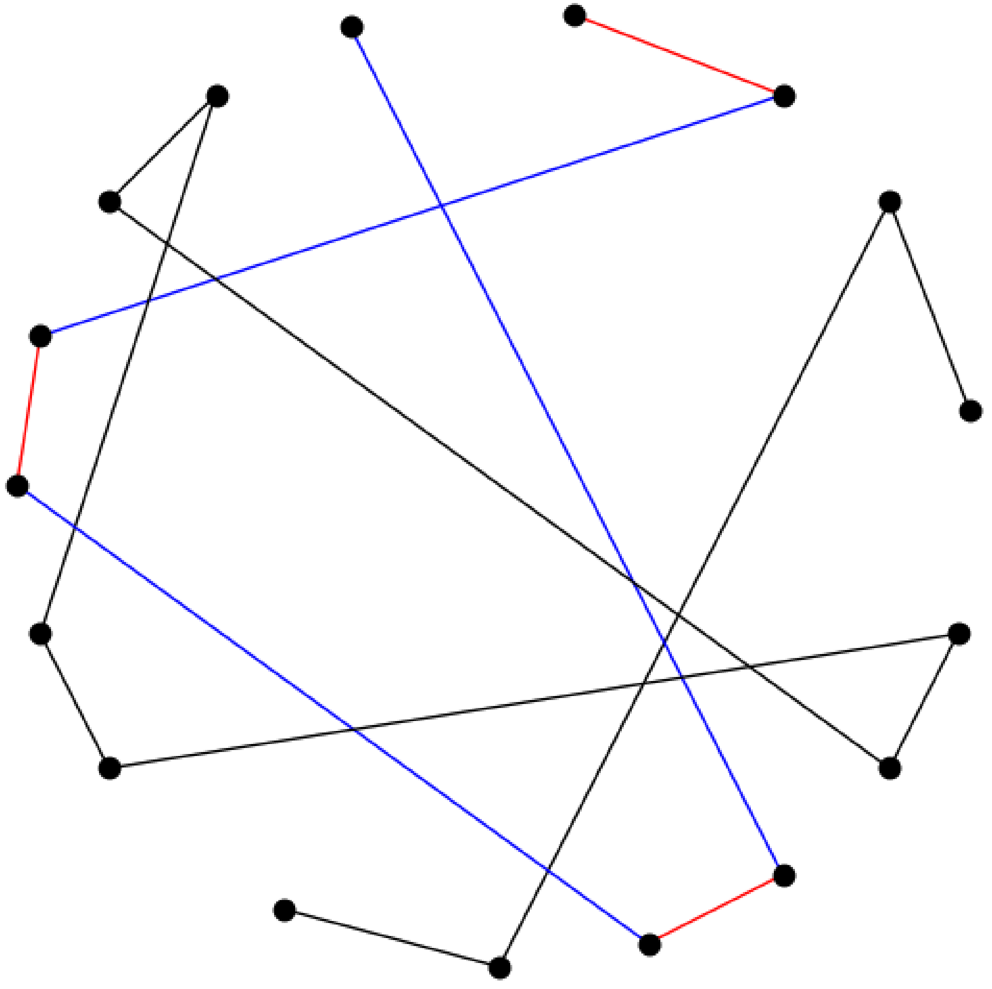

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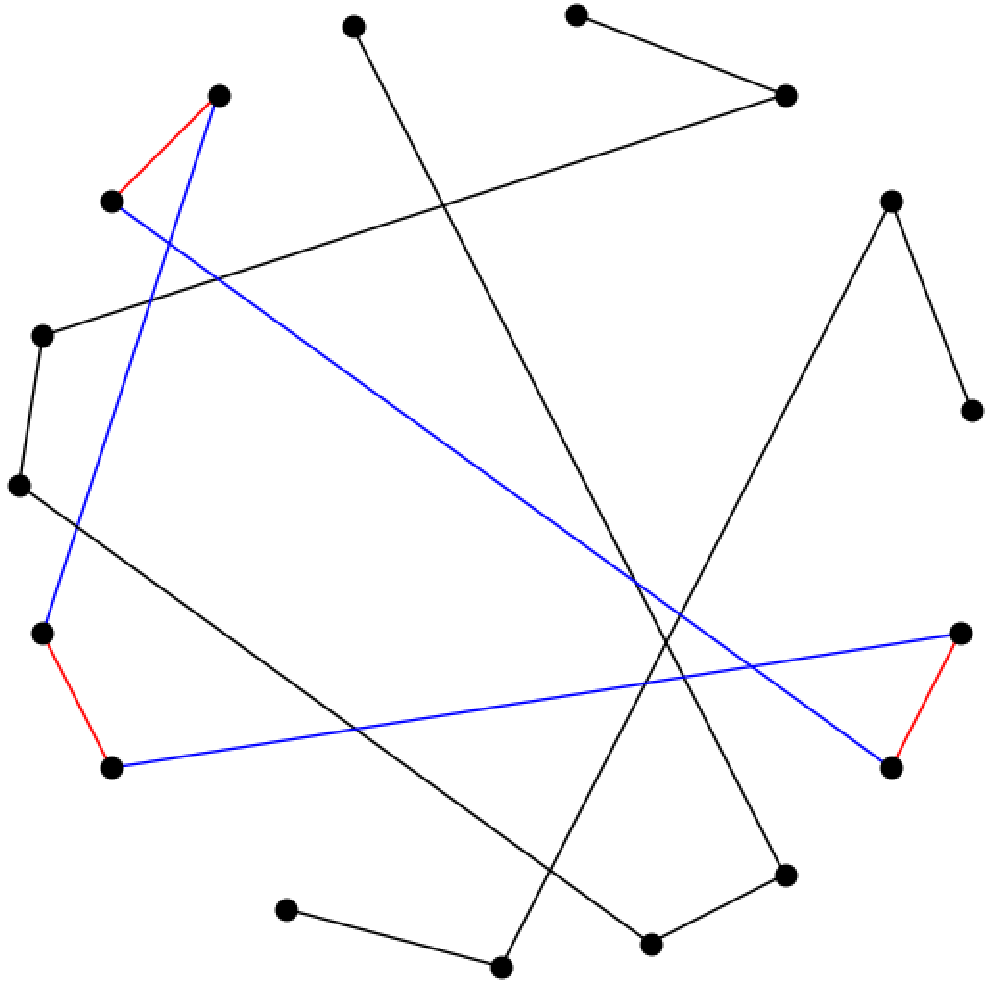

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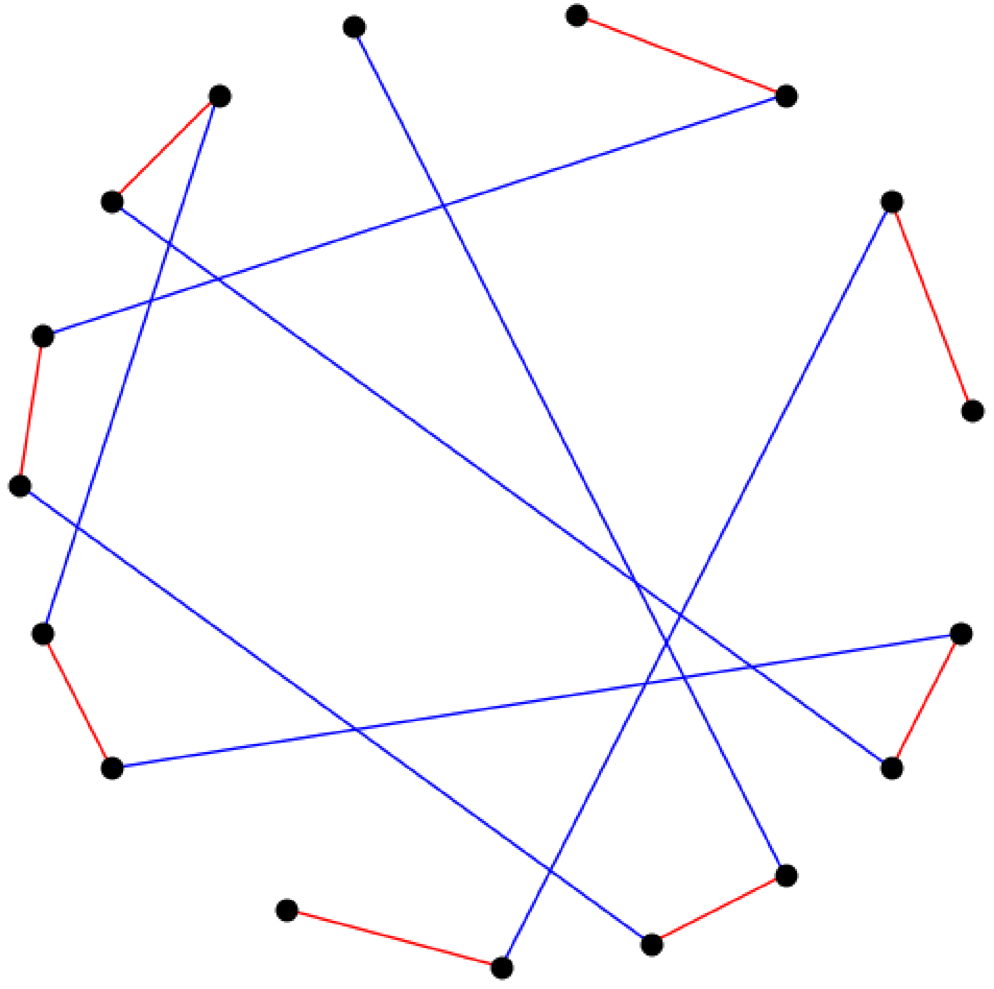

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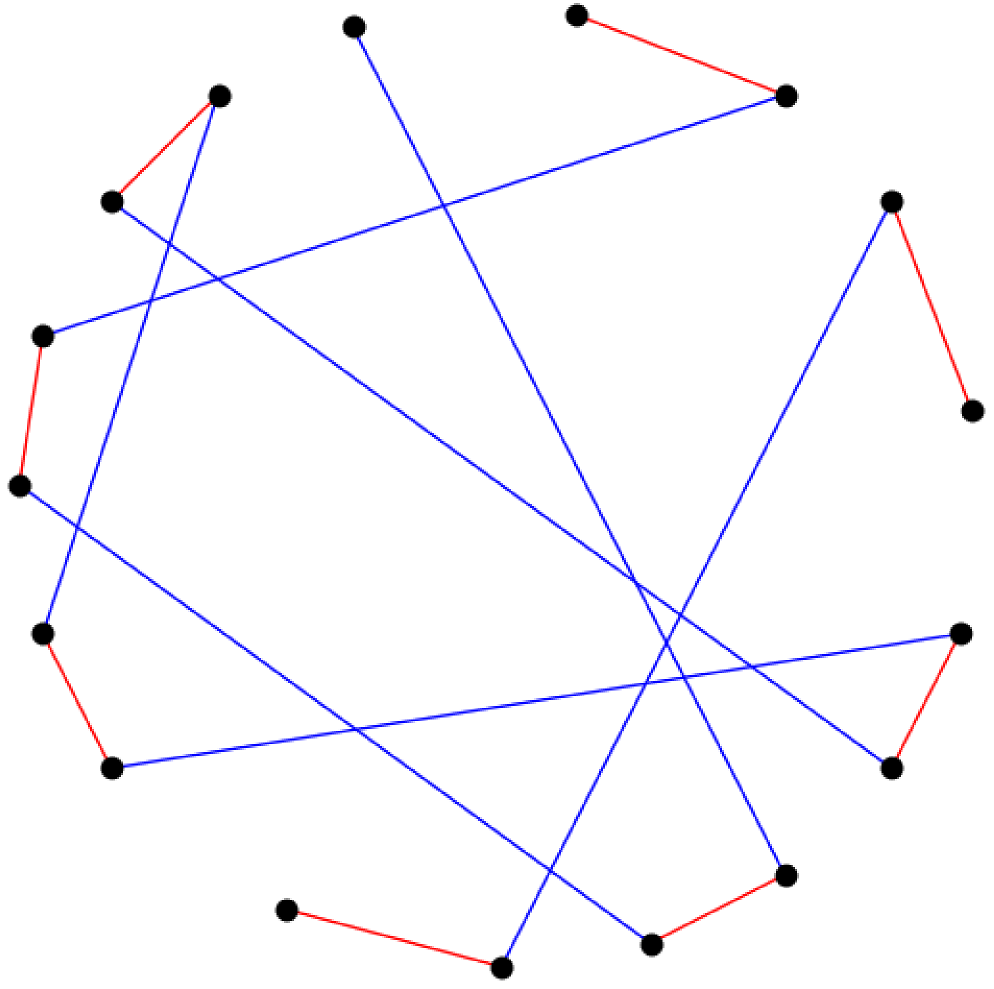
M / M_1



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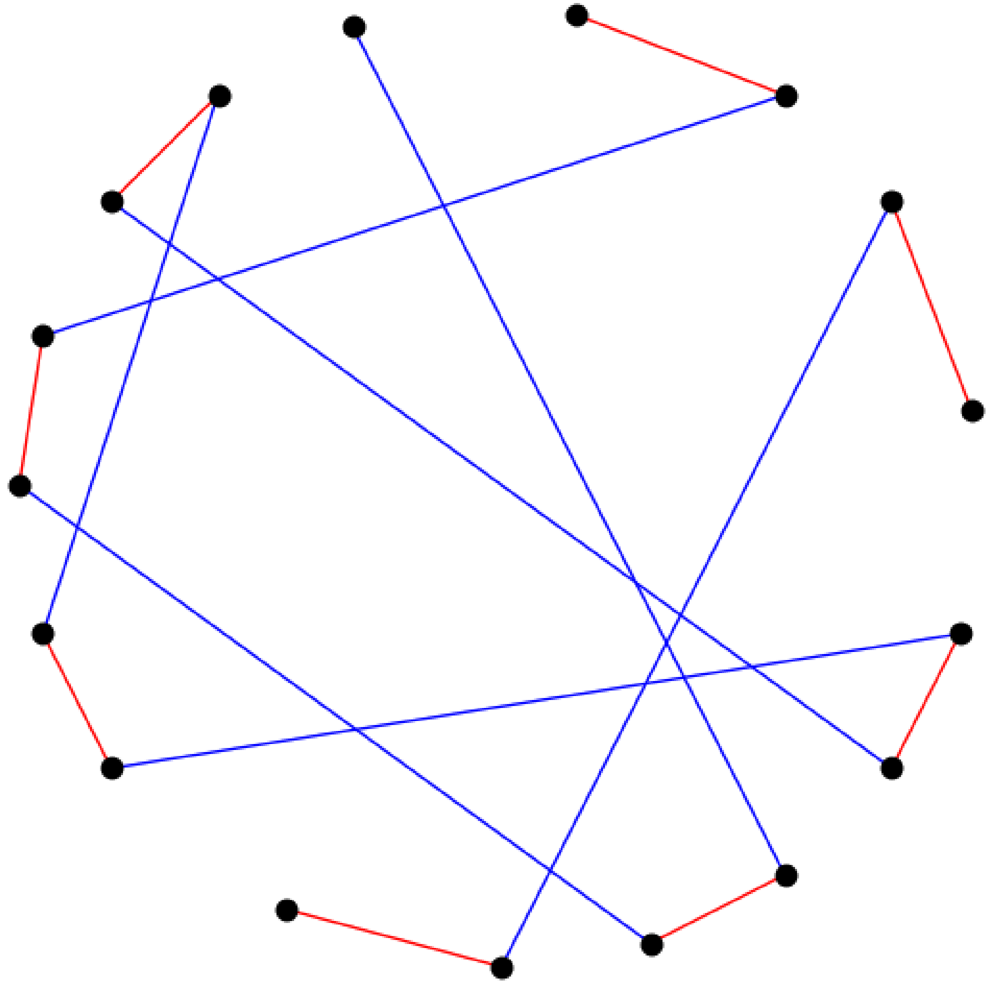
M / M_1

disjoint union of paths and cylces



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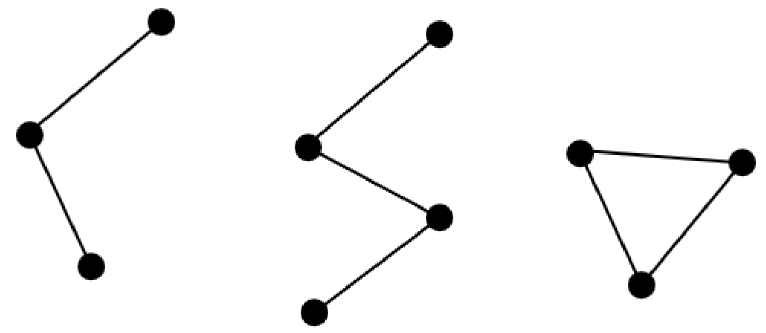

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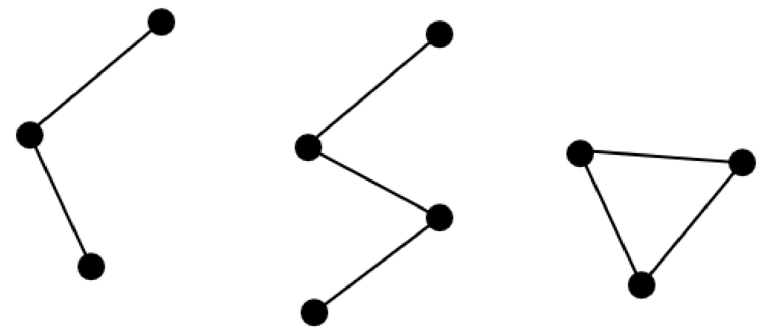
M / M_1

disjoint union of paths and cylces

$\exists M_2 \subseteq M$ s.t. $w(M_2) \geq w(M)/3 \approx w(C)/6$



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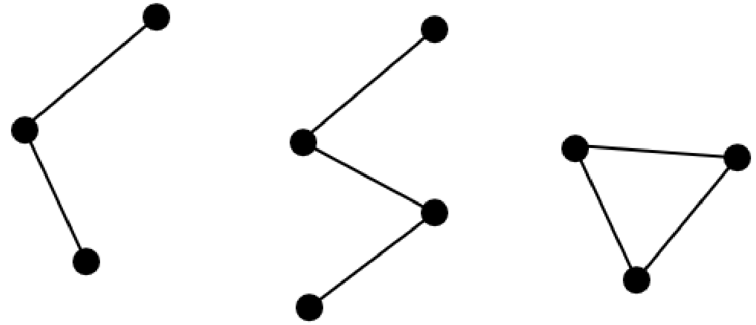

M/M_1

disjoint union of paths and cylces

$\exists M_2 \subseteq M$ s.t. $w(M_2) \geq w(M)/3 \approx w(C)/6$

$w(M_1) + w(M_2) \geq w(M_1) + \frac{w(C^\star)-w(M_1)}{6} \geq (7/12) \cdot w(C^\star)$

Conclusion:

Maximum Weighted Matching

$(1 - \epsilon)$-approx

Conclusion:

Maximum Weighted Matching

$(1-\epsilon)$-approx

Max-TSP

$(7/12-\epsilon)$-approx

Open Questions:

Better analysis for Max-TSP

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What happens if we repeat the process to find M_3, ...

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Better analysis for Max-TSP

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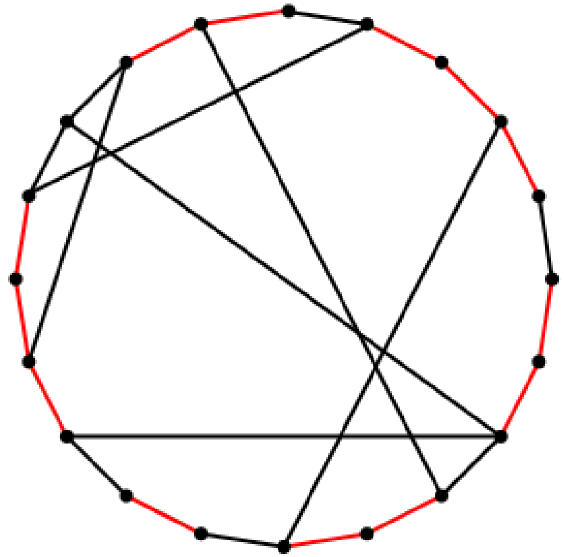
Open Questions:



Better analysis for Max-TSP

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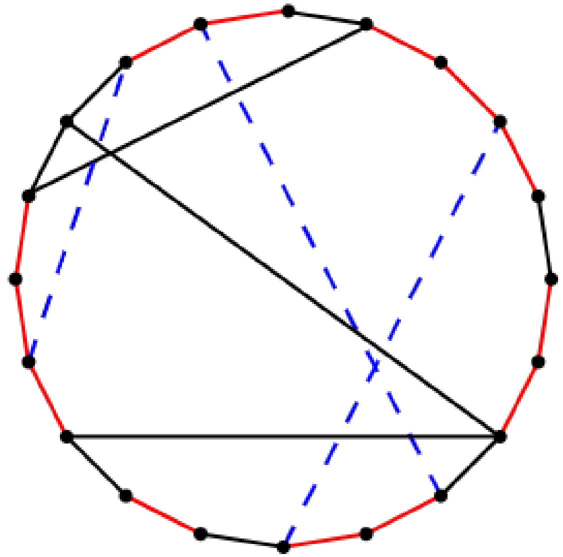
Open Questions:



Better analysis for Max-TSP

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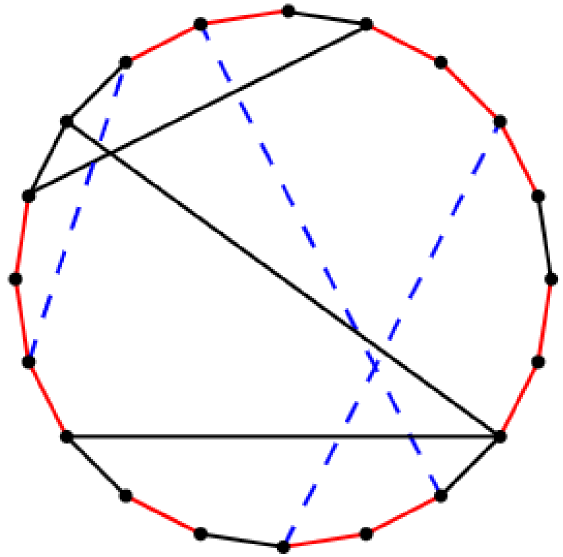
Bottleneck: $M_1 \cup M_2 \cup M_3 \cup \cdots$ is not necessarily a path cover

Open Questions:



Better analysis for Max-TSP

What happens if we repeat the process to find M_3, ...

Bottleneck: $M_1 \cup M_2 \cup M_3 \cup \cdots$ is not necessarily a path cover

Maintain feasibality: remove some edges

Thanks for Listening!