Metric Dimension and Geodetic Set Parameterized by Vertex Cover

@STACS

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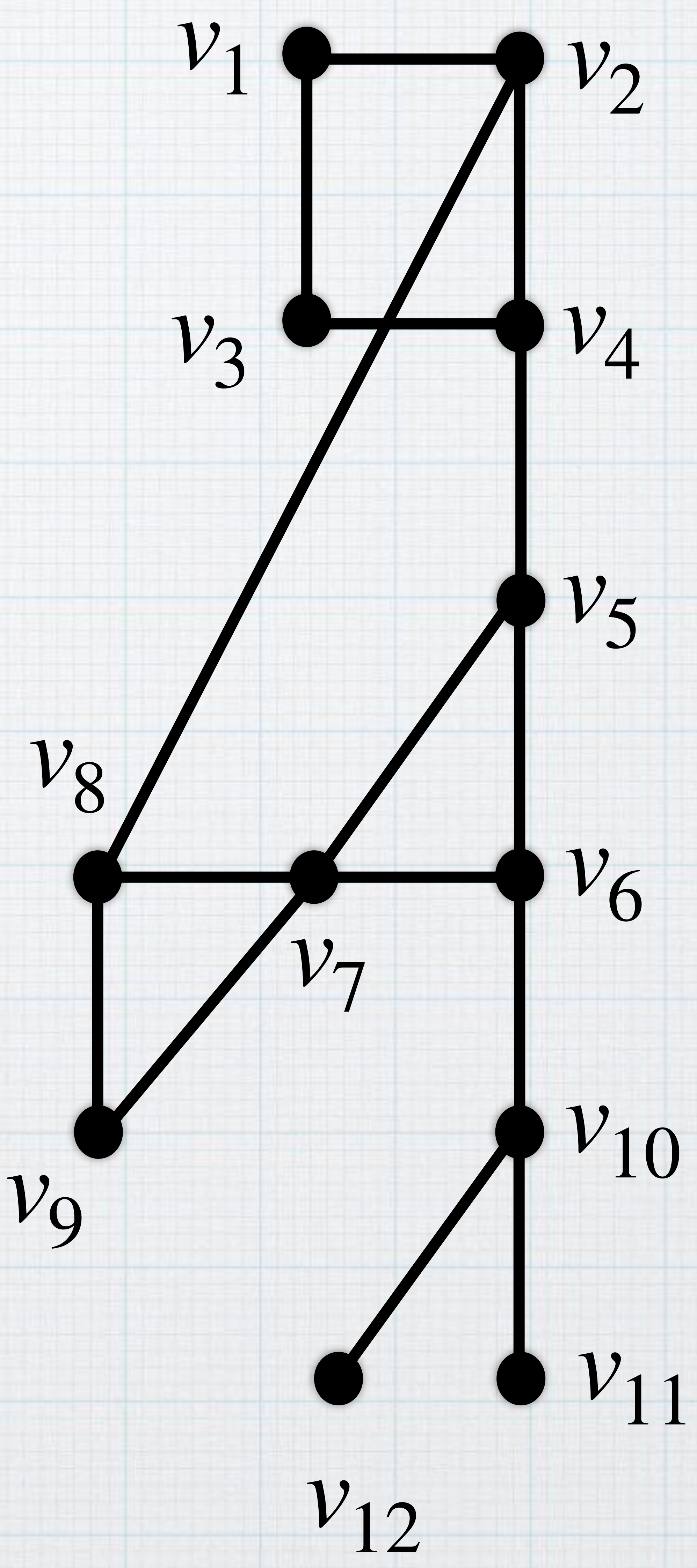
Prafullkumar Tale

Joint work with

Florent Foucaud, Esther Galby, Liana Khazaliya, Shaohua Li, Fionn Mc Inerney, Roohani Sharma

Catch the Spy



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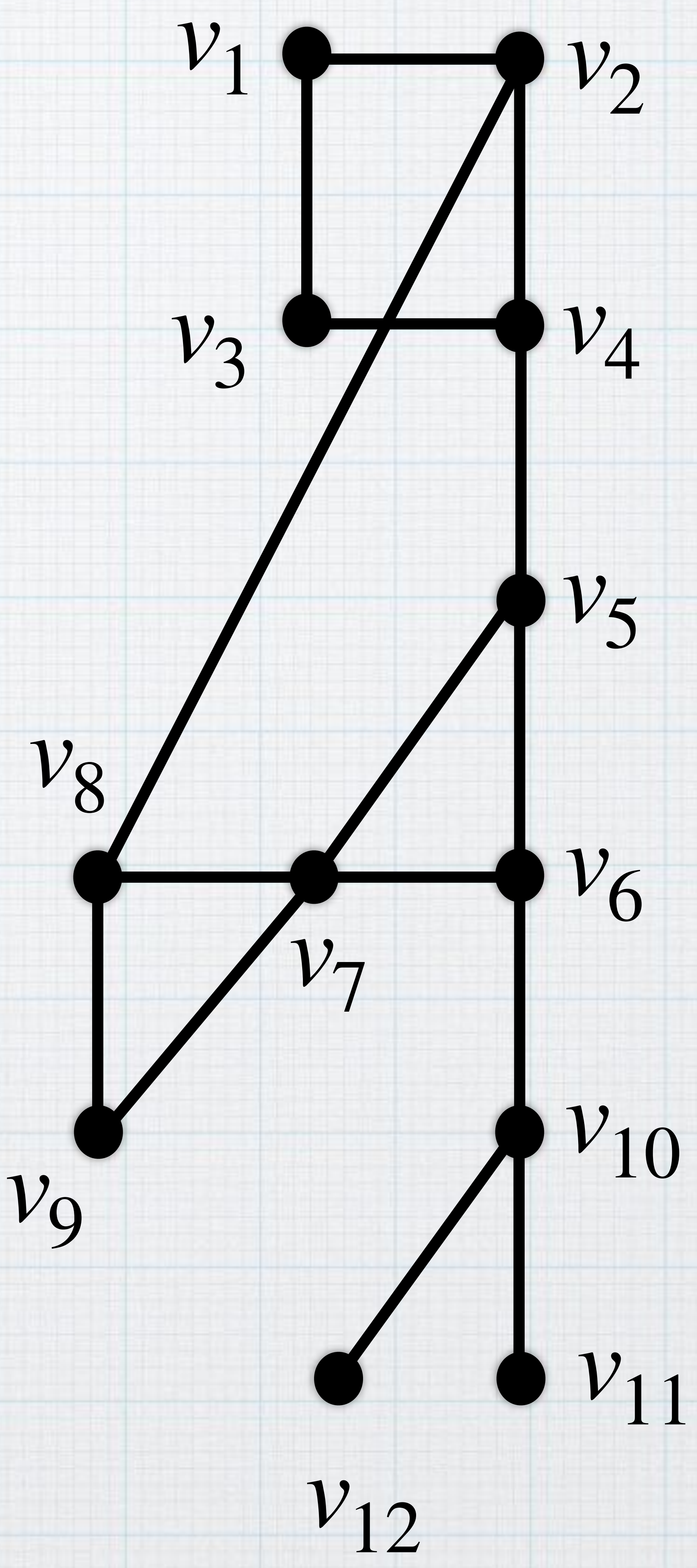

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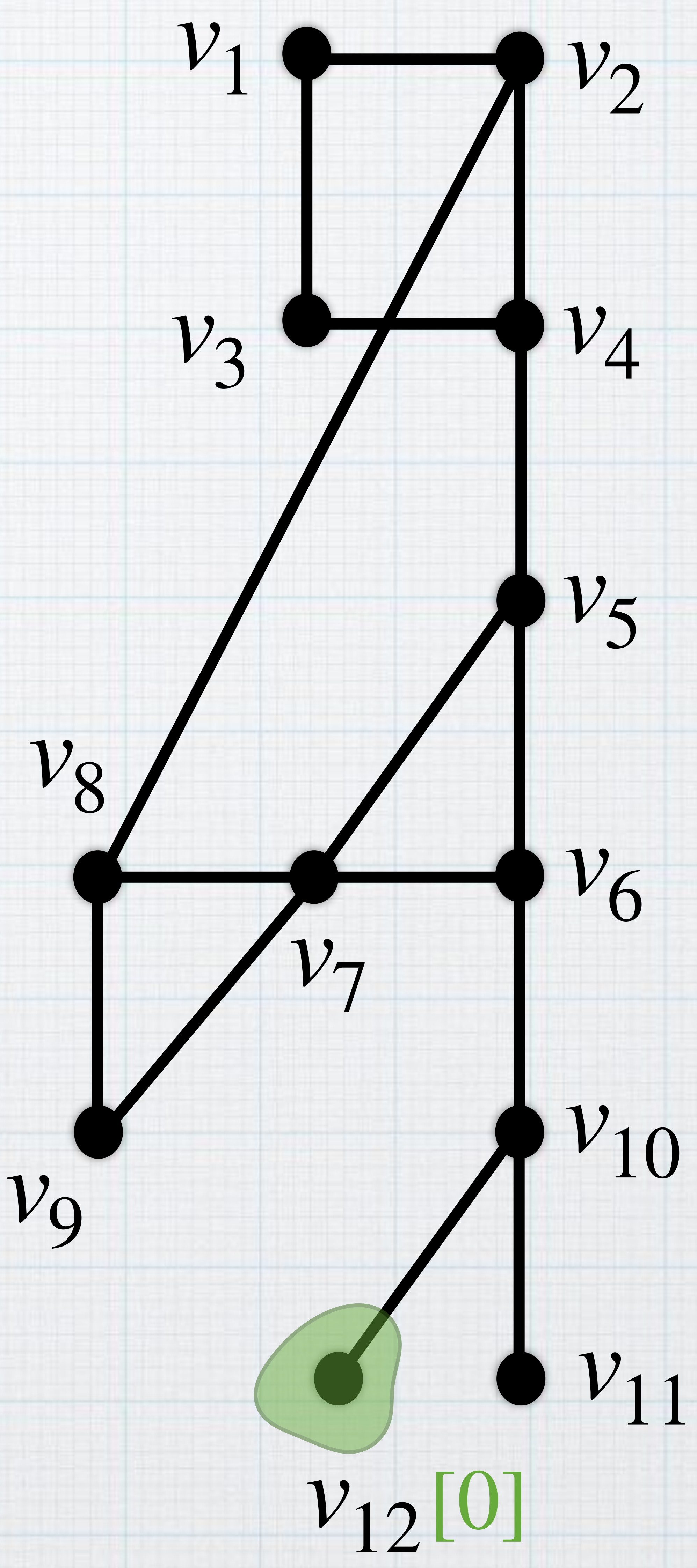

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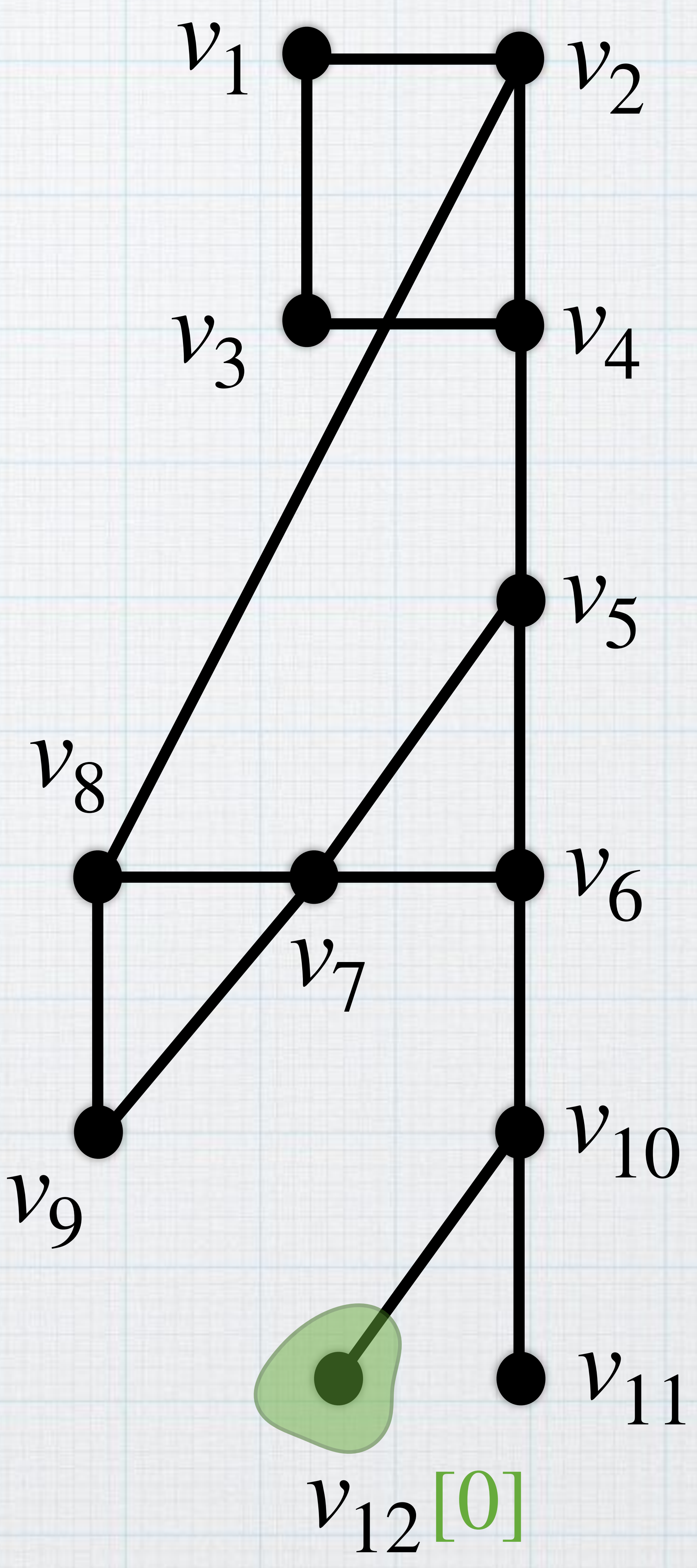

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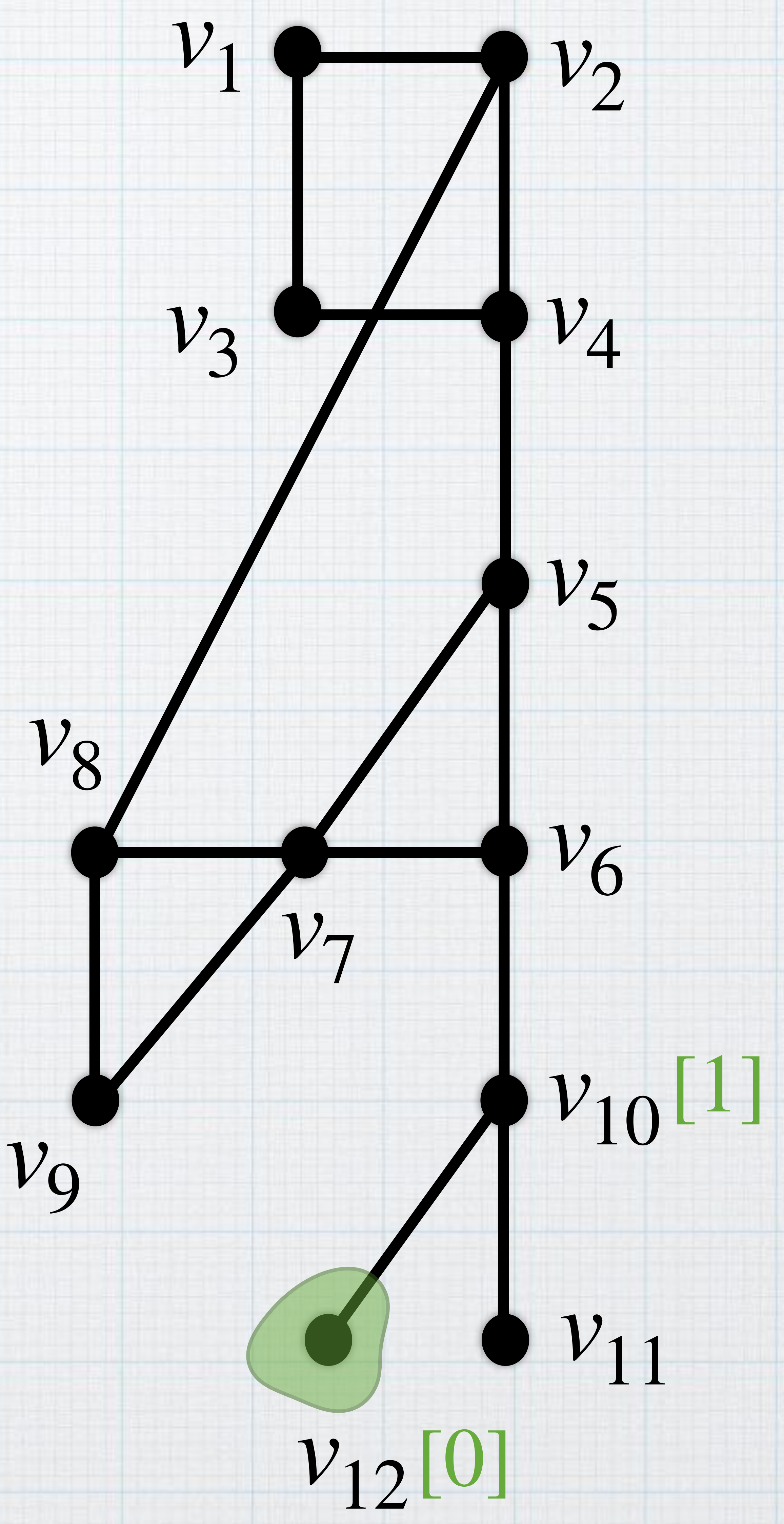

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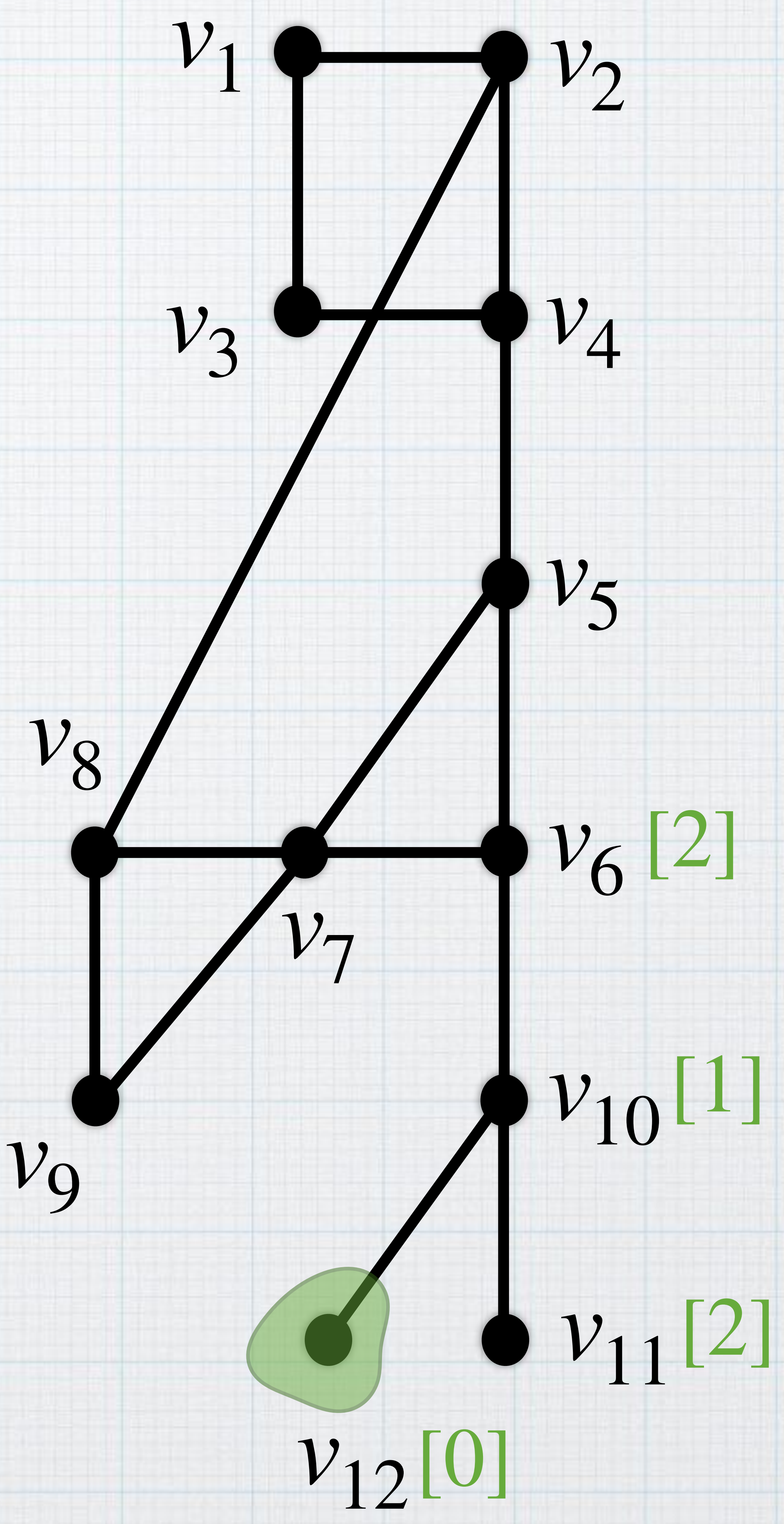

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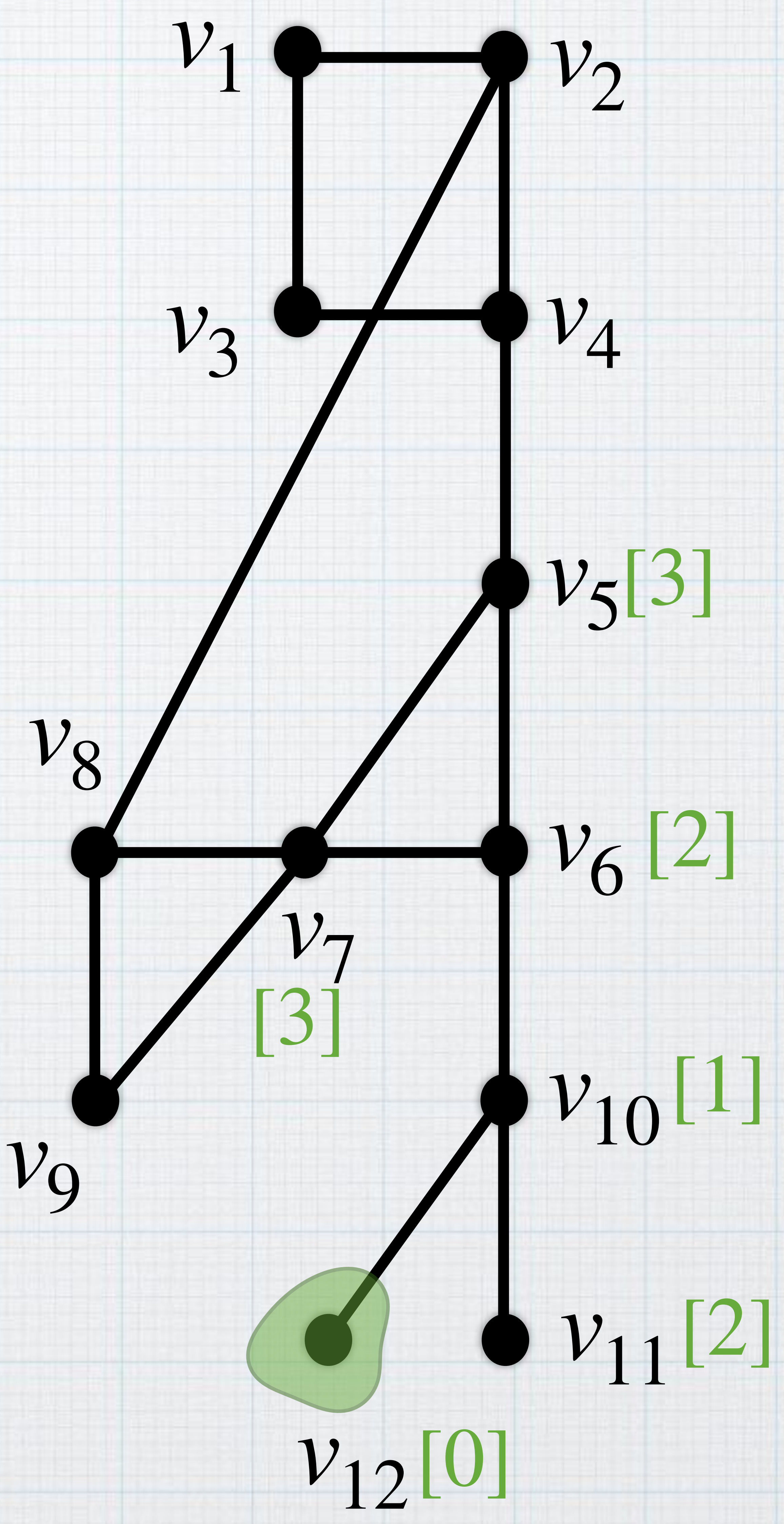

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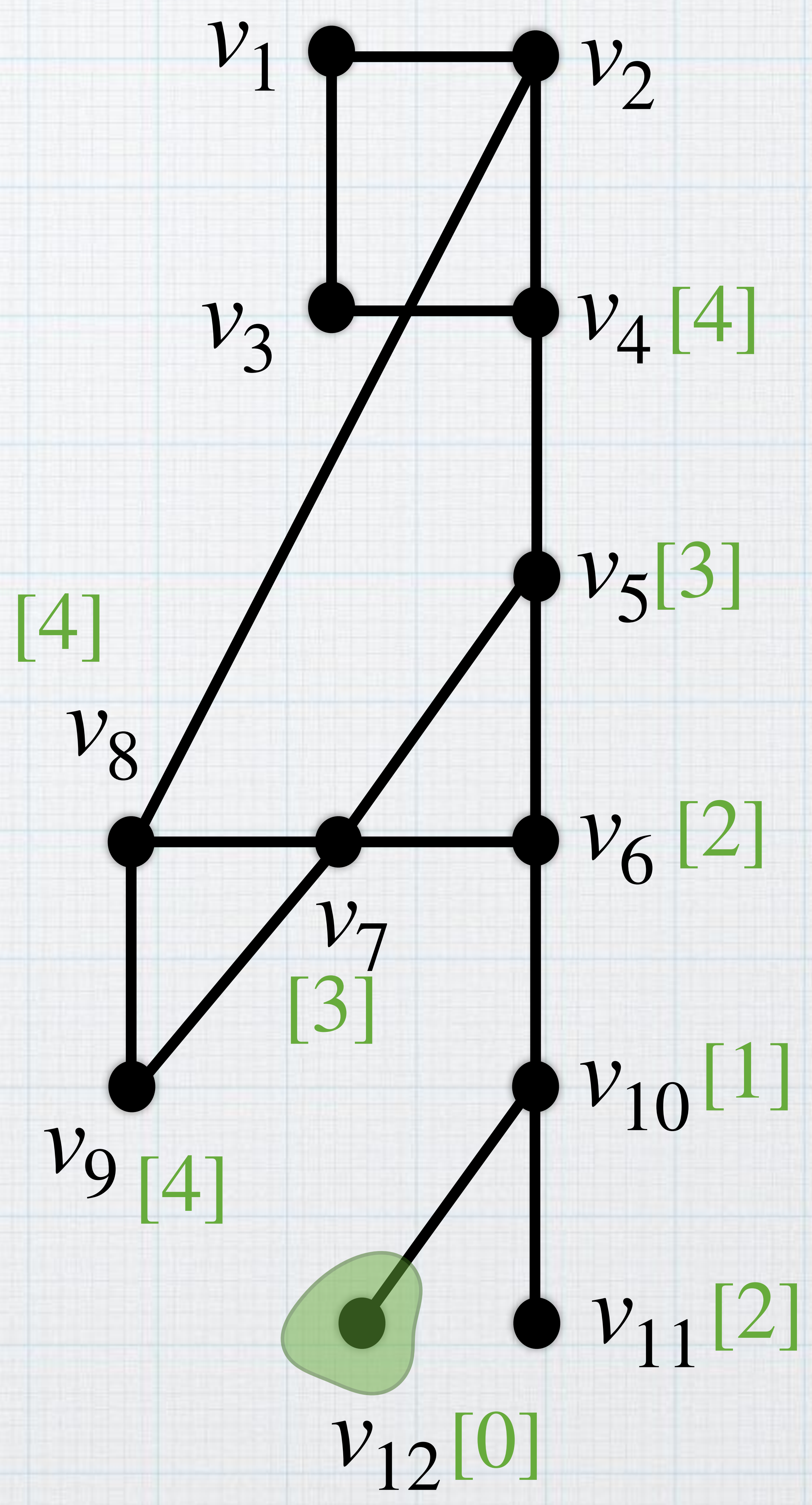

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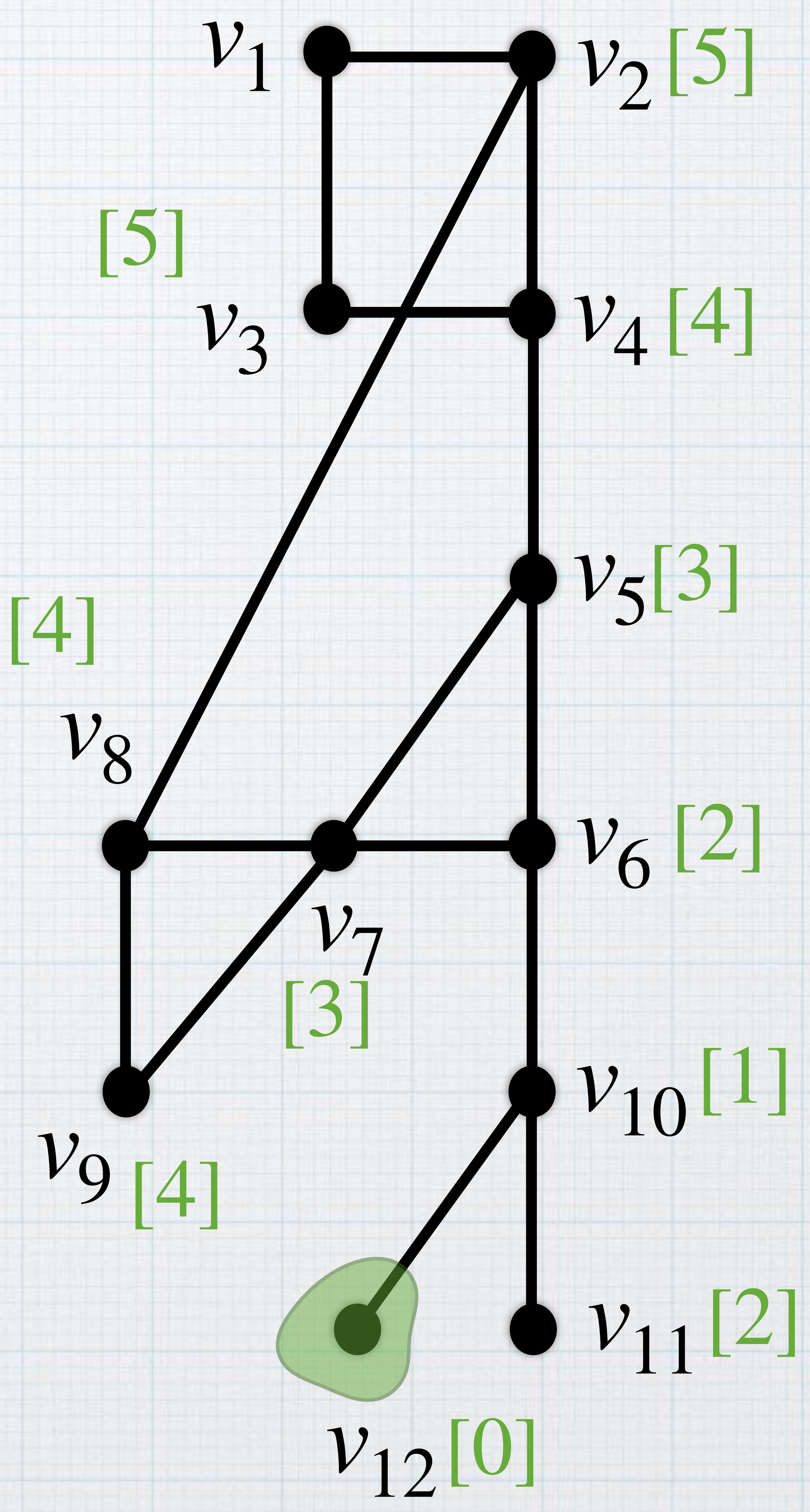

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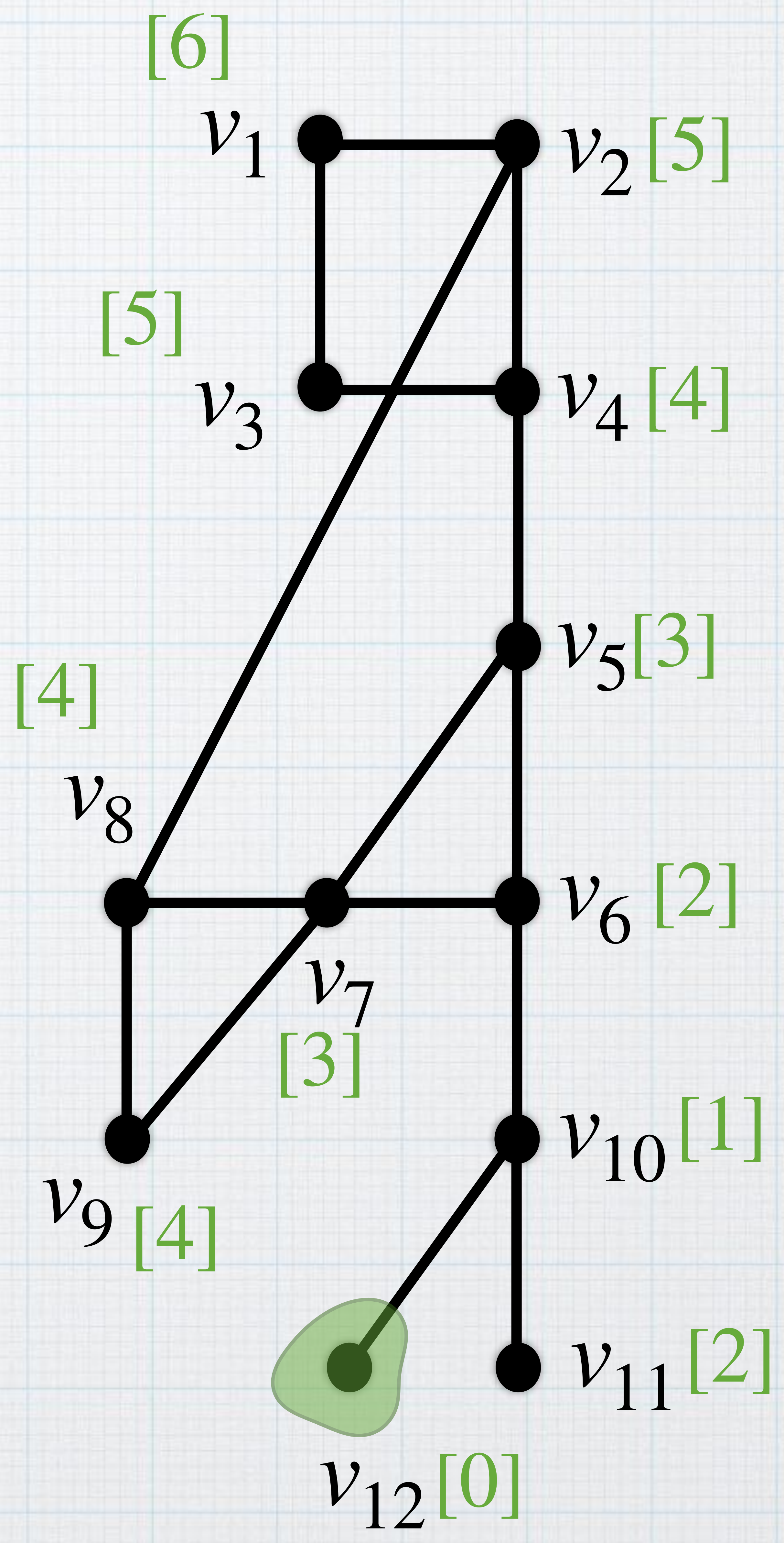

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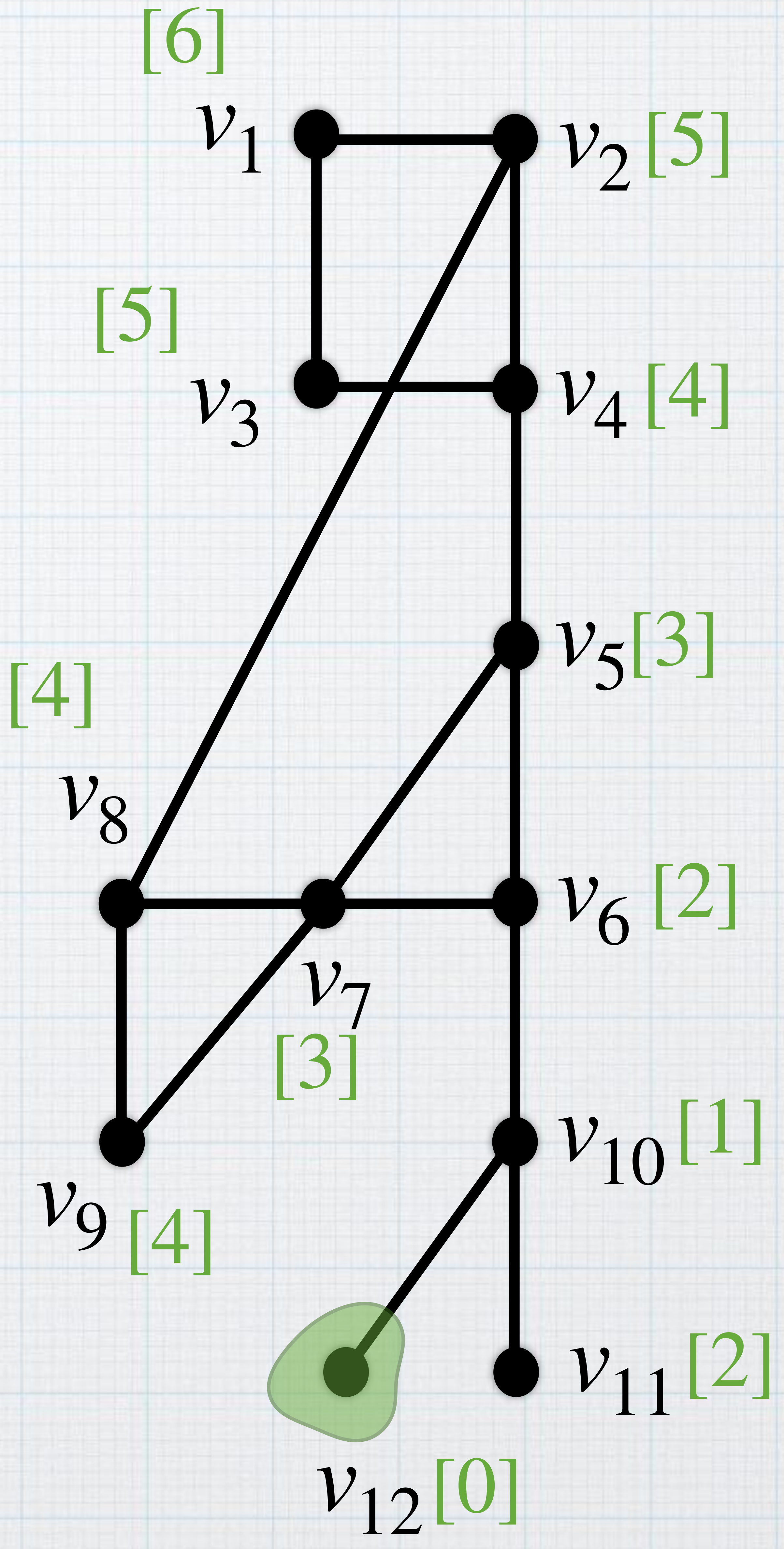

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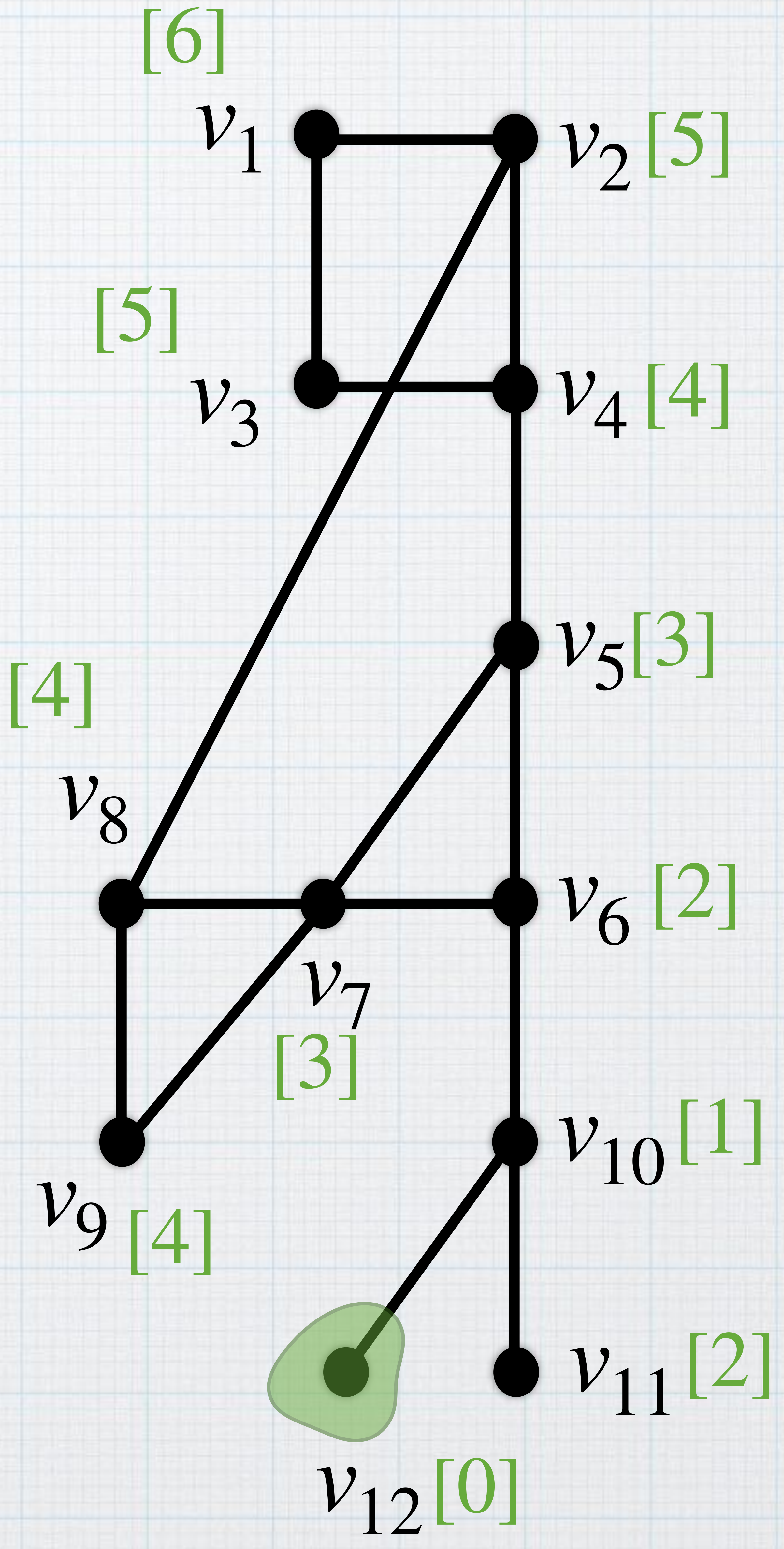

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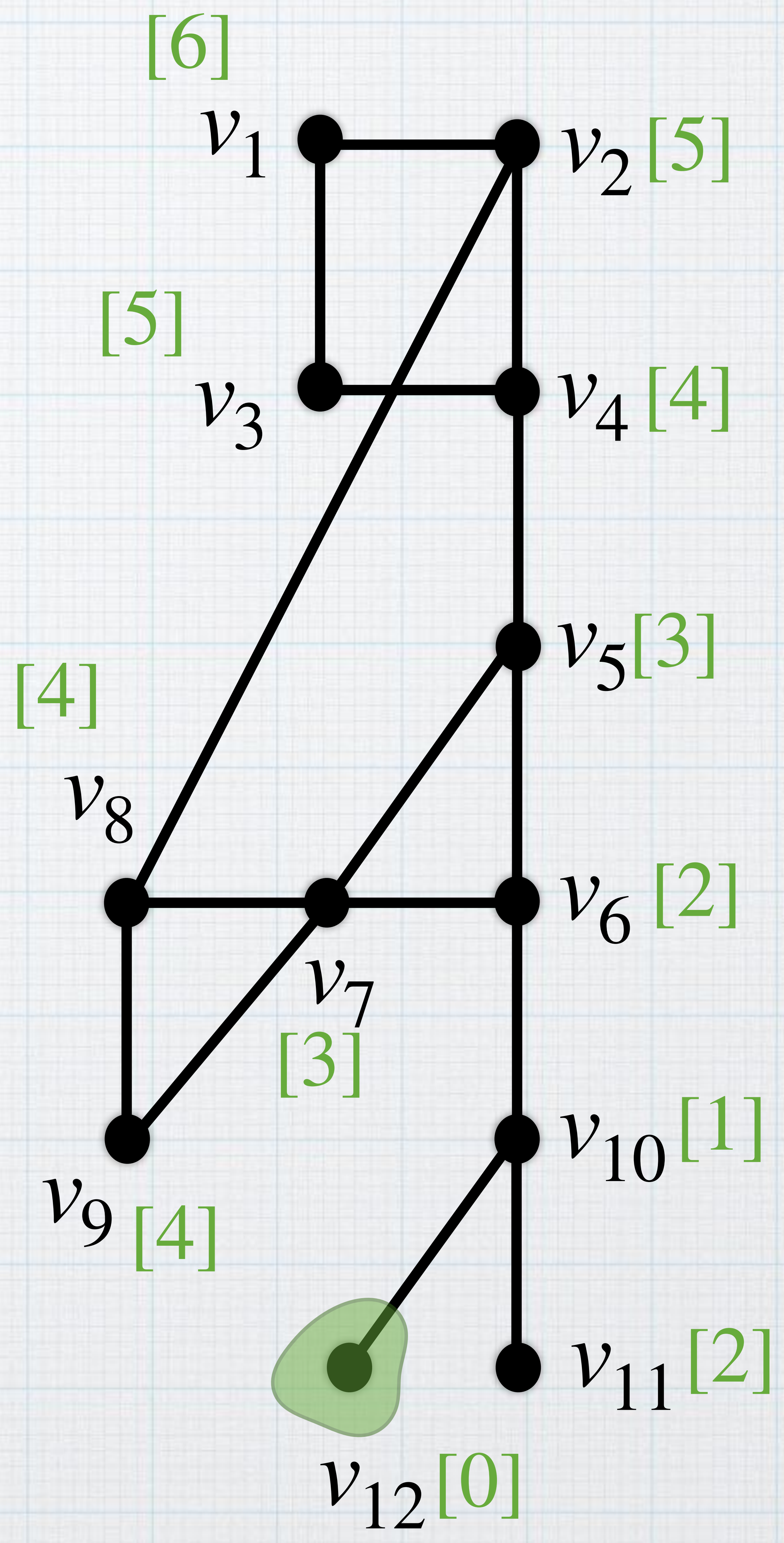

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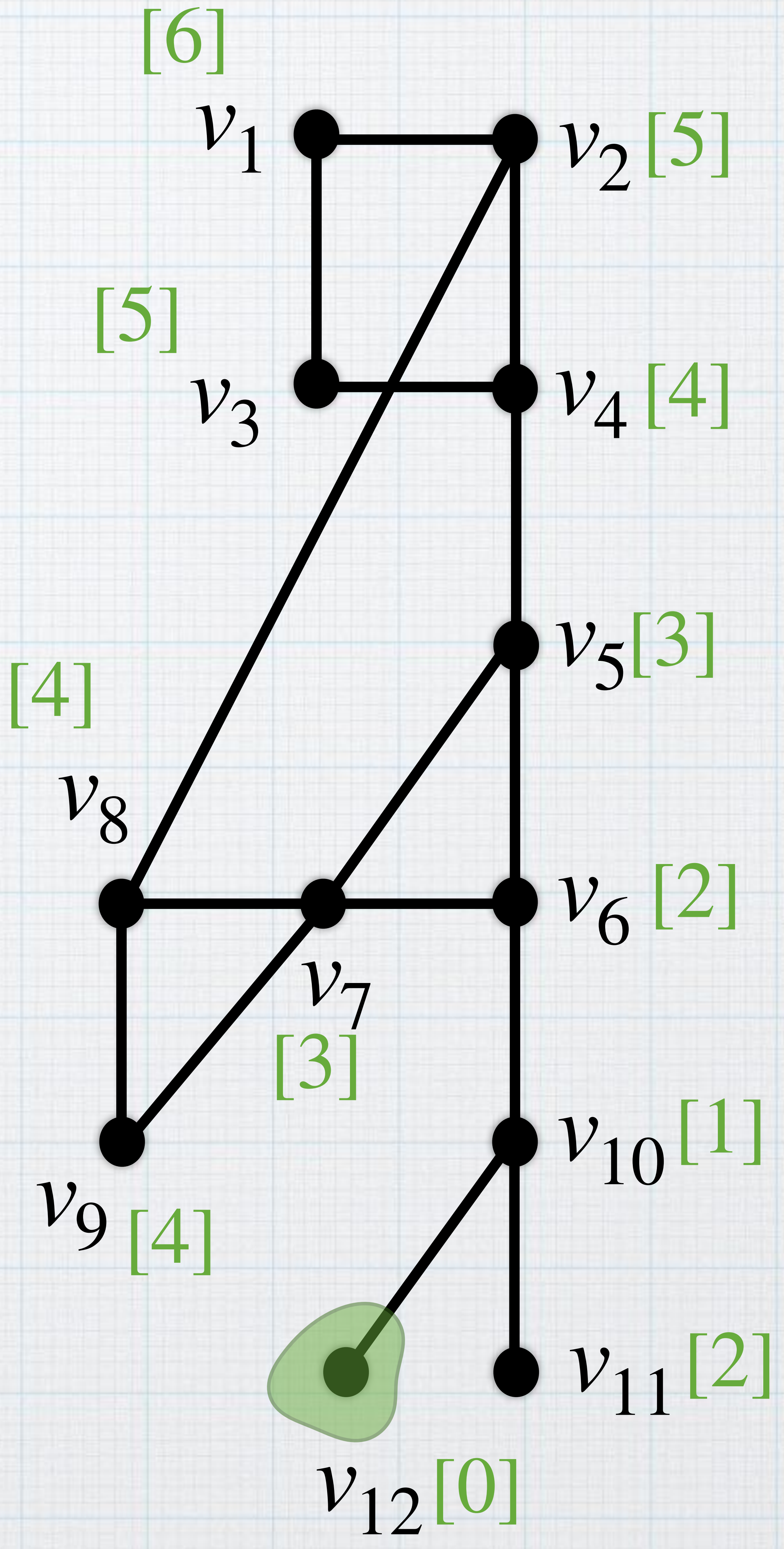
Q: Minimum nr of messages to catch the spy?



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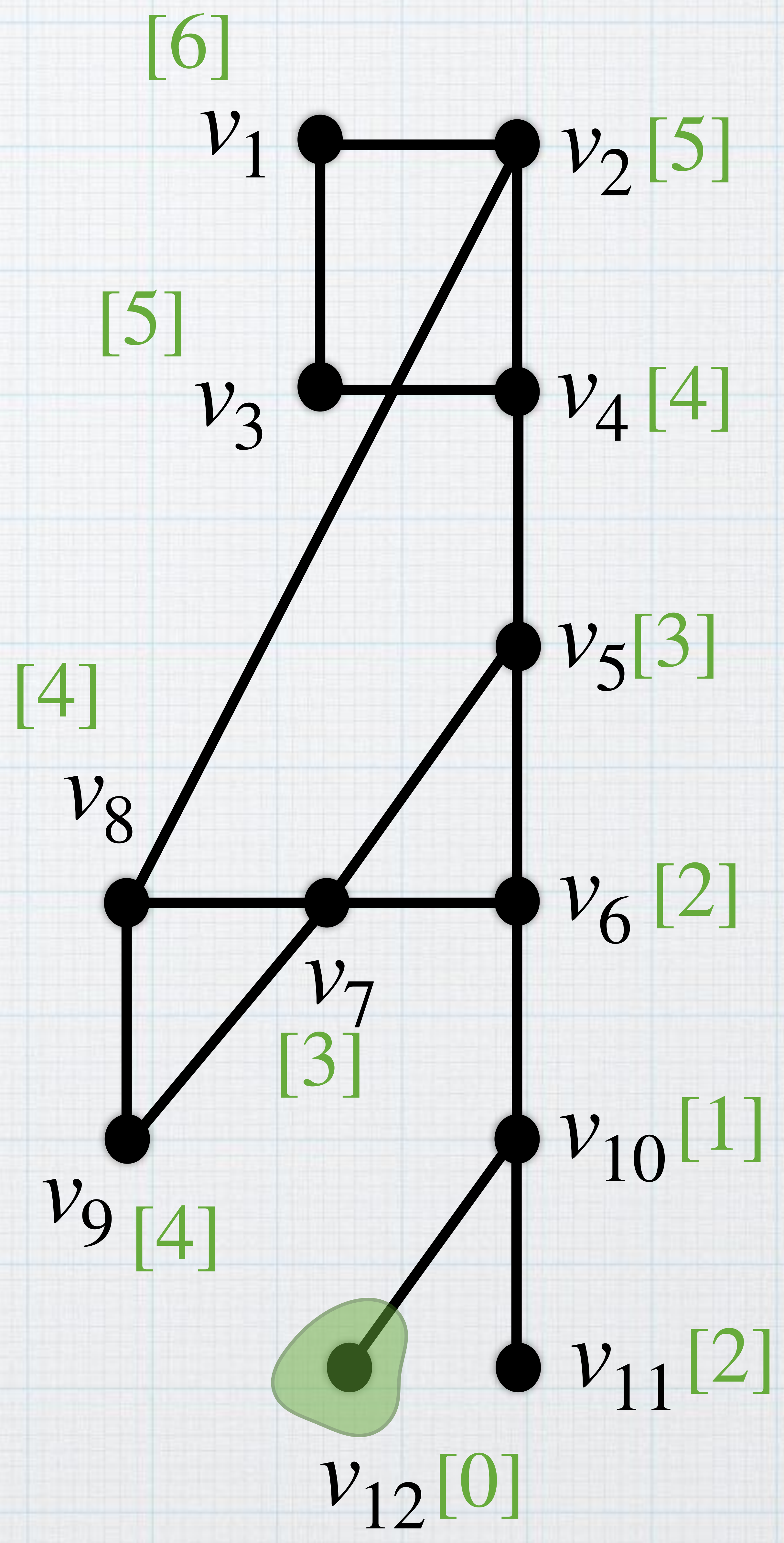

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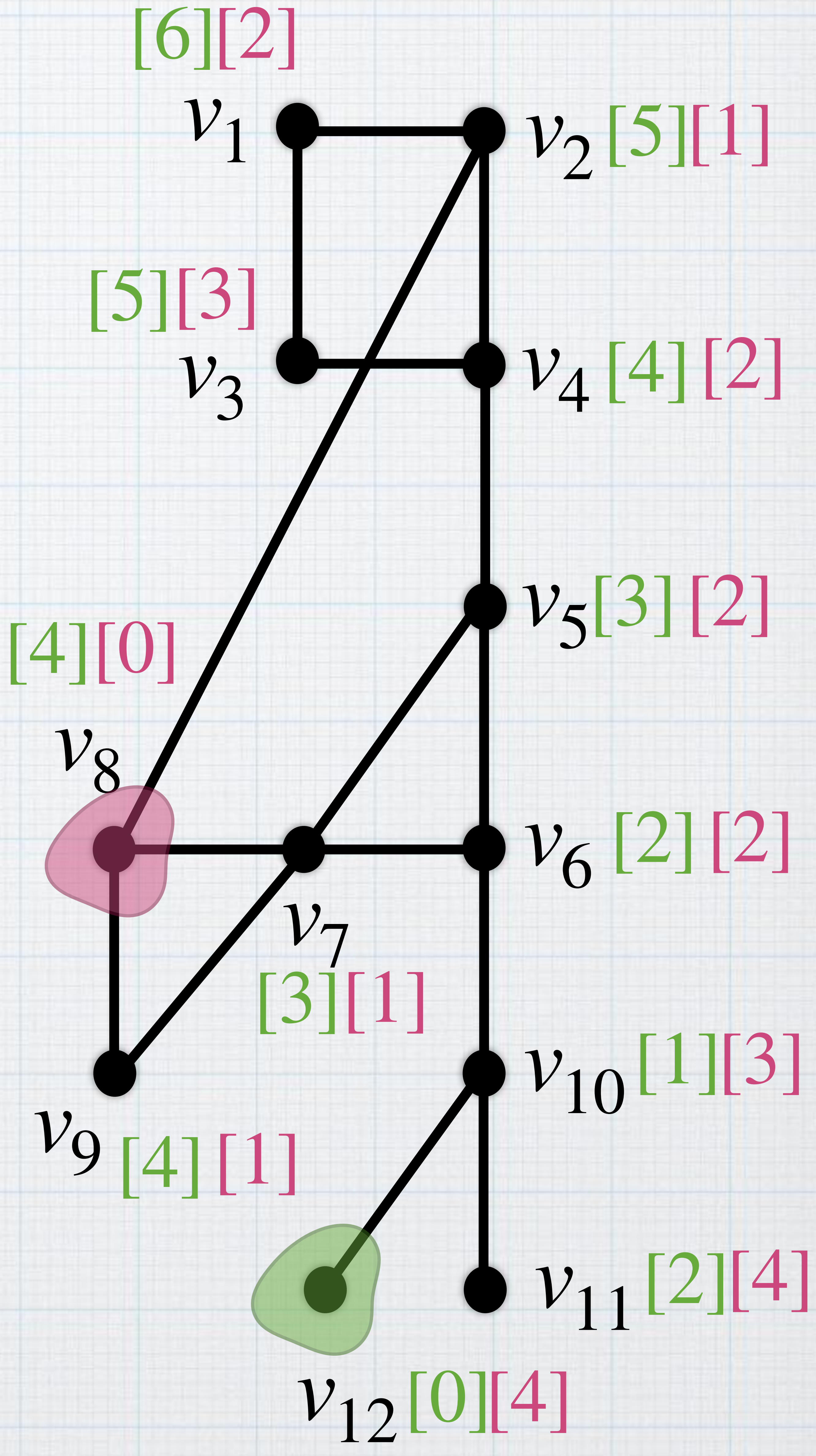

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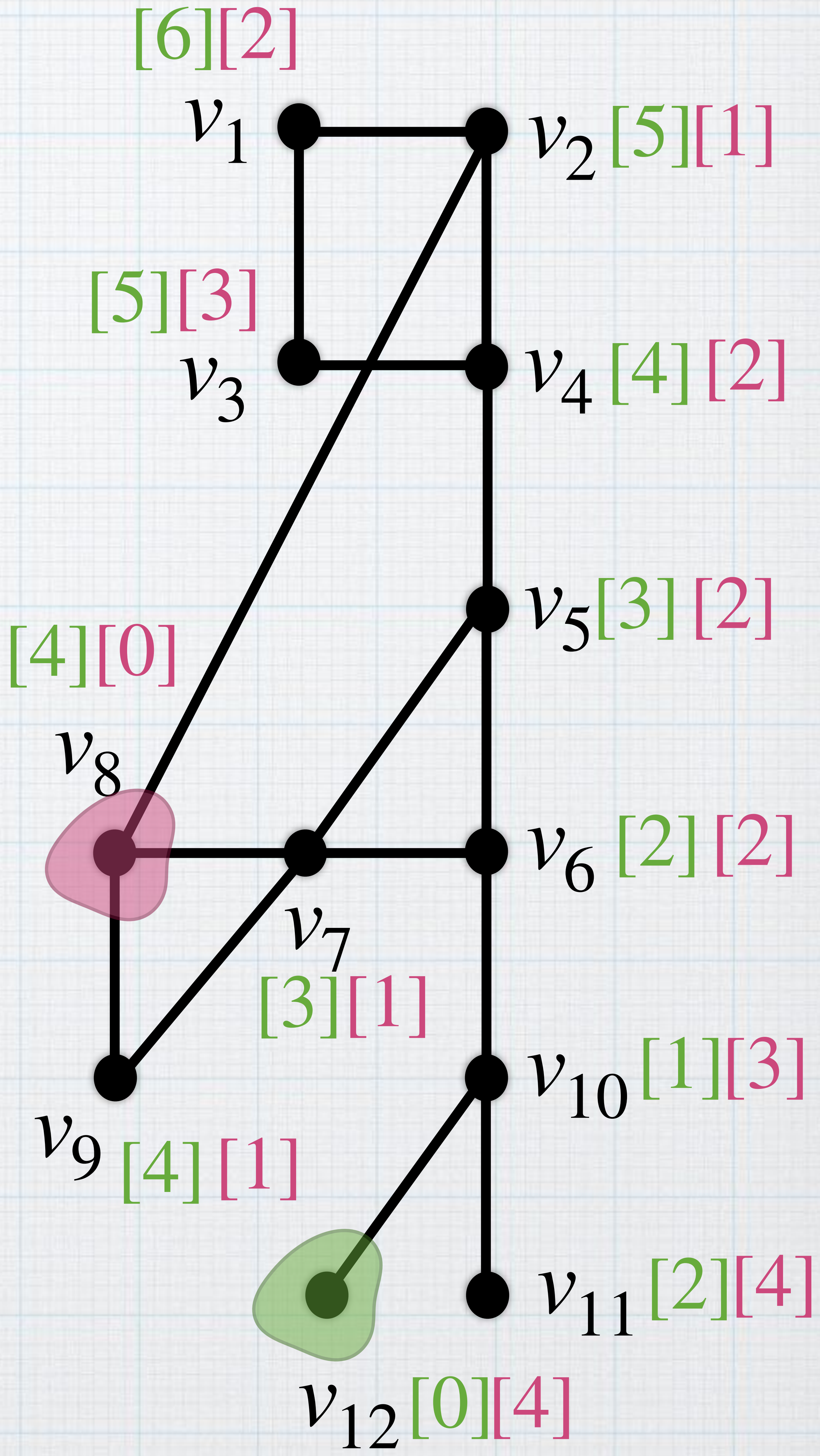

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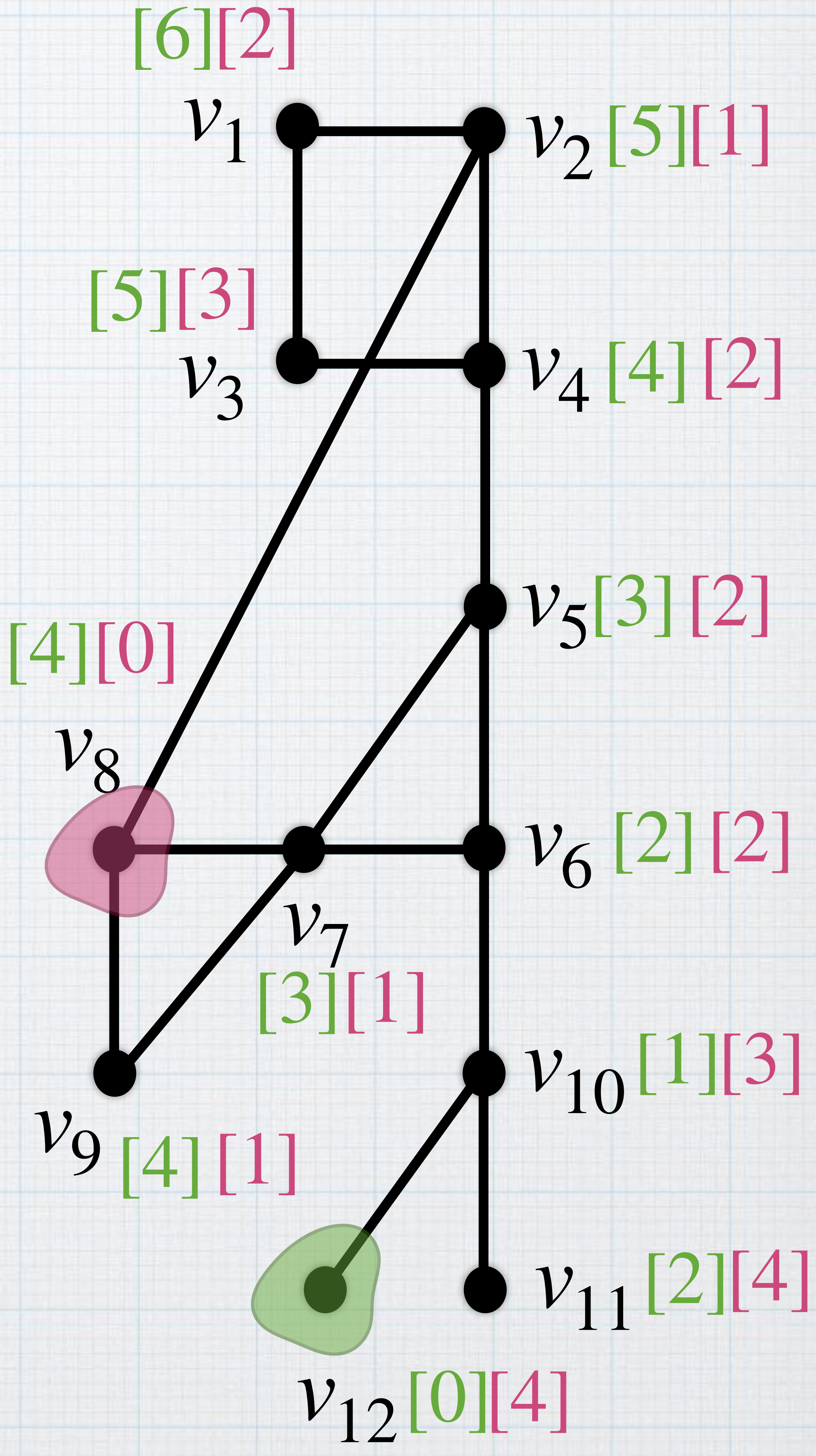

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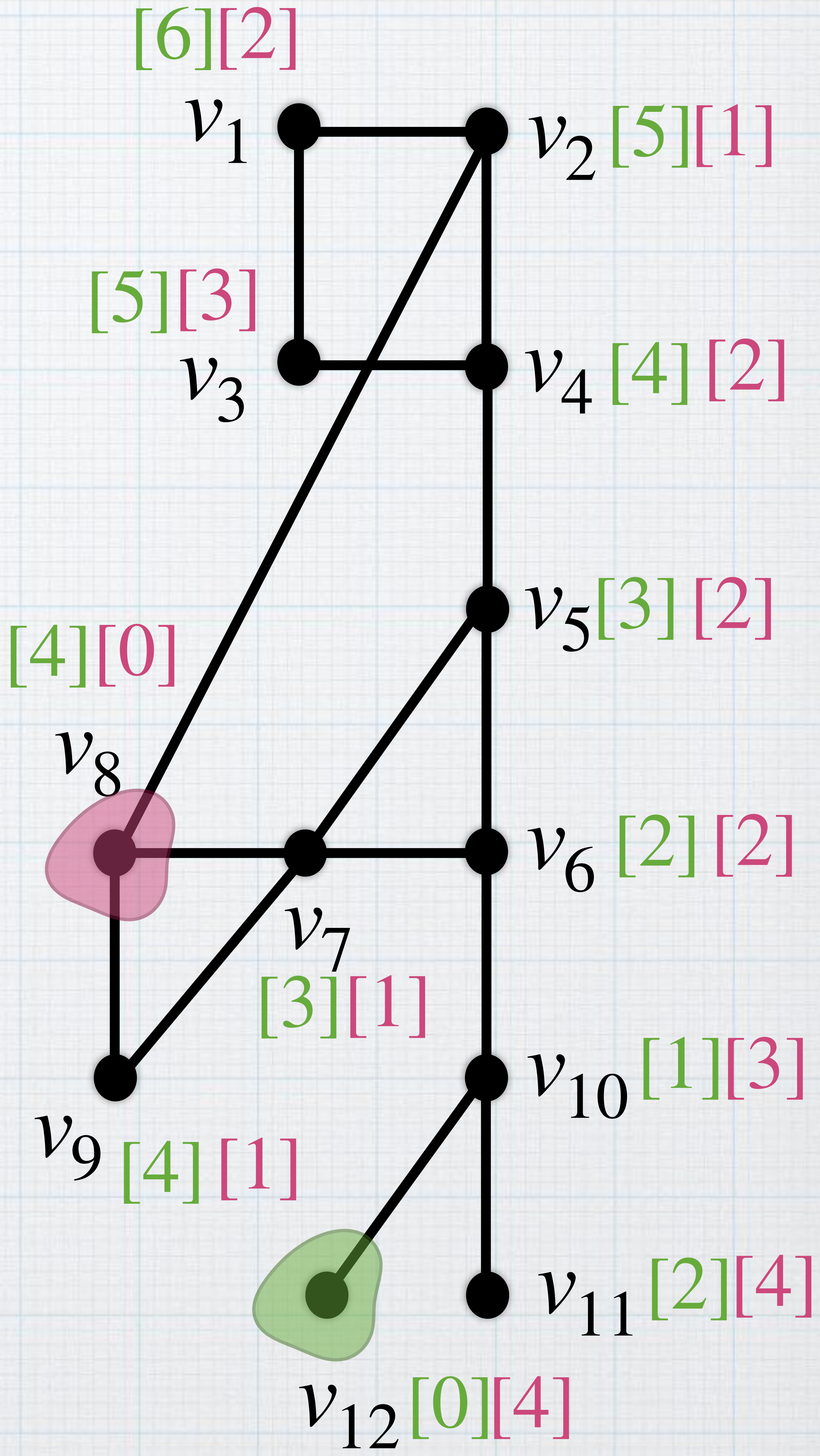

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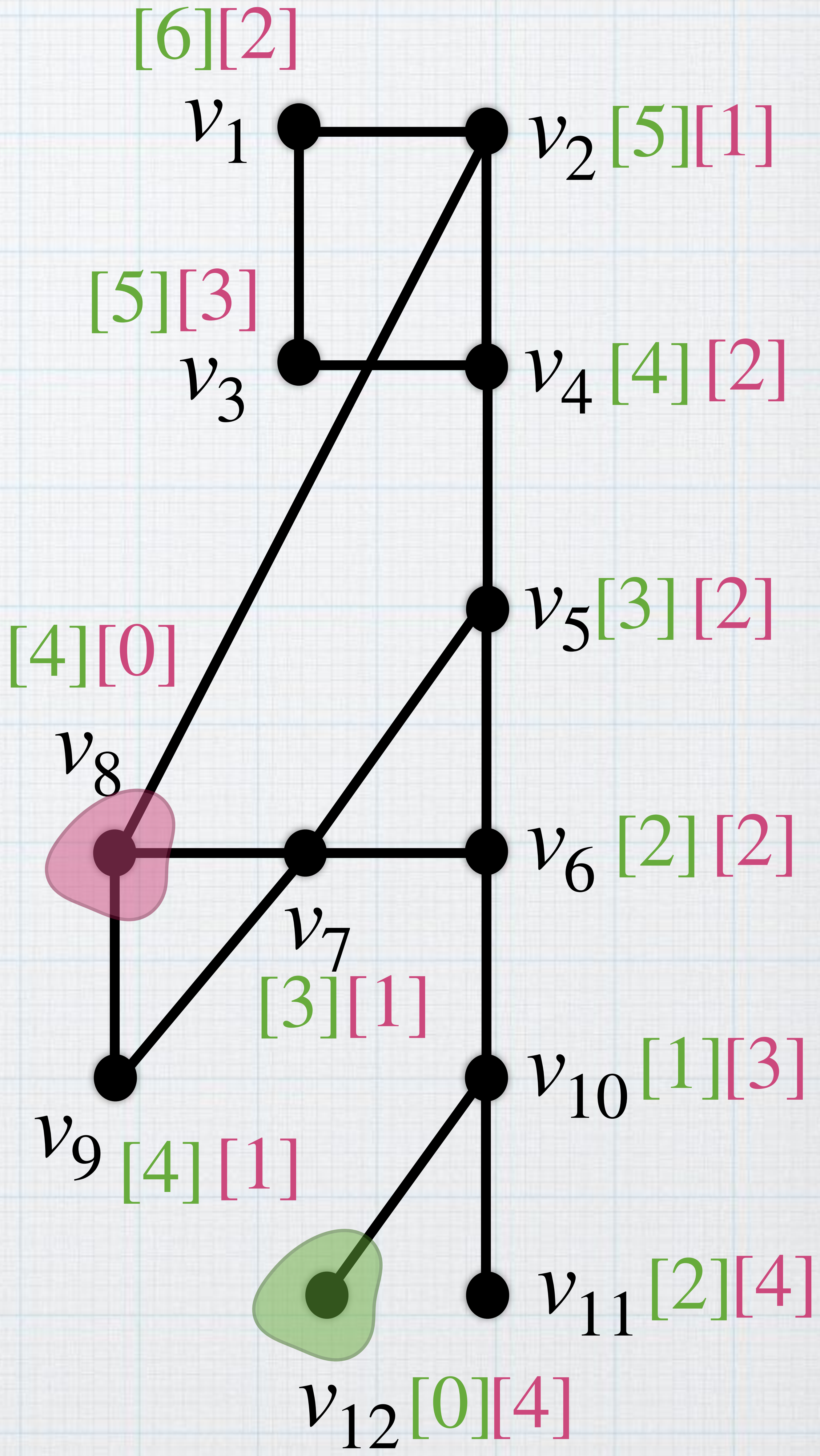

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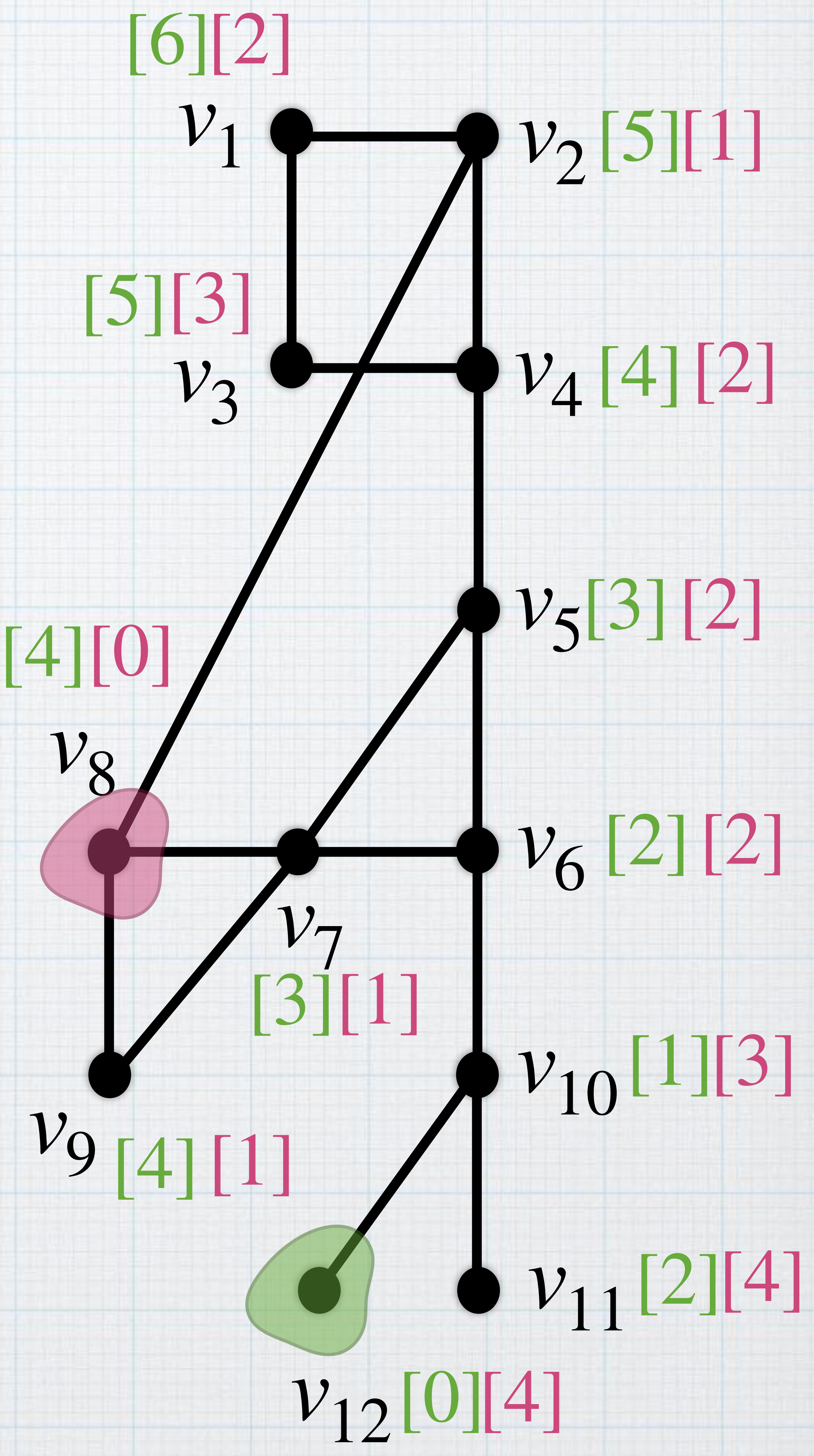
- A resolving set is an ordered set $S = \{s_1, s_2, ..\} \subseteq V(G)$ s.t. any $u \neq v \in V(G)$, $\langle d(u, s_1);\ d(u, s_2);\ ..\rangle \neq \langle d(v, s_1);\ d(v, s_2);\ ..\rangle$



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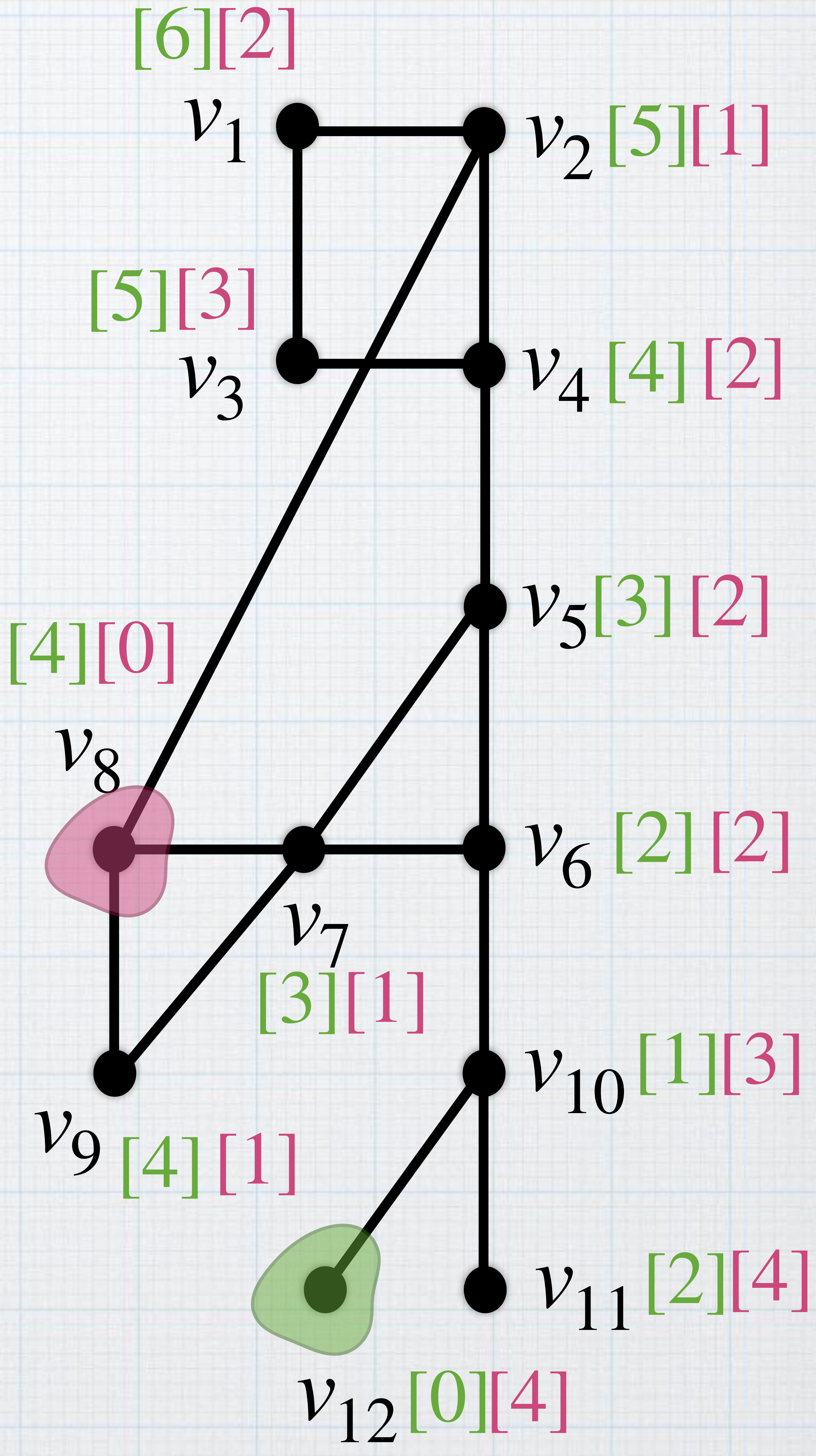
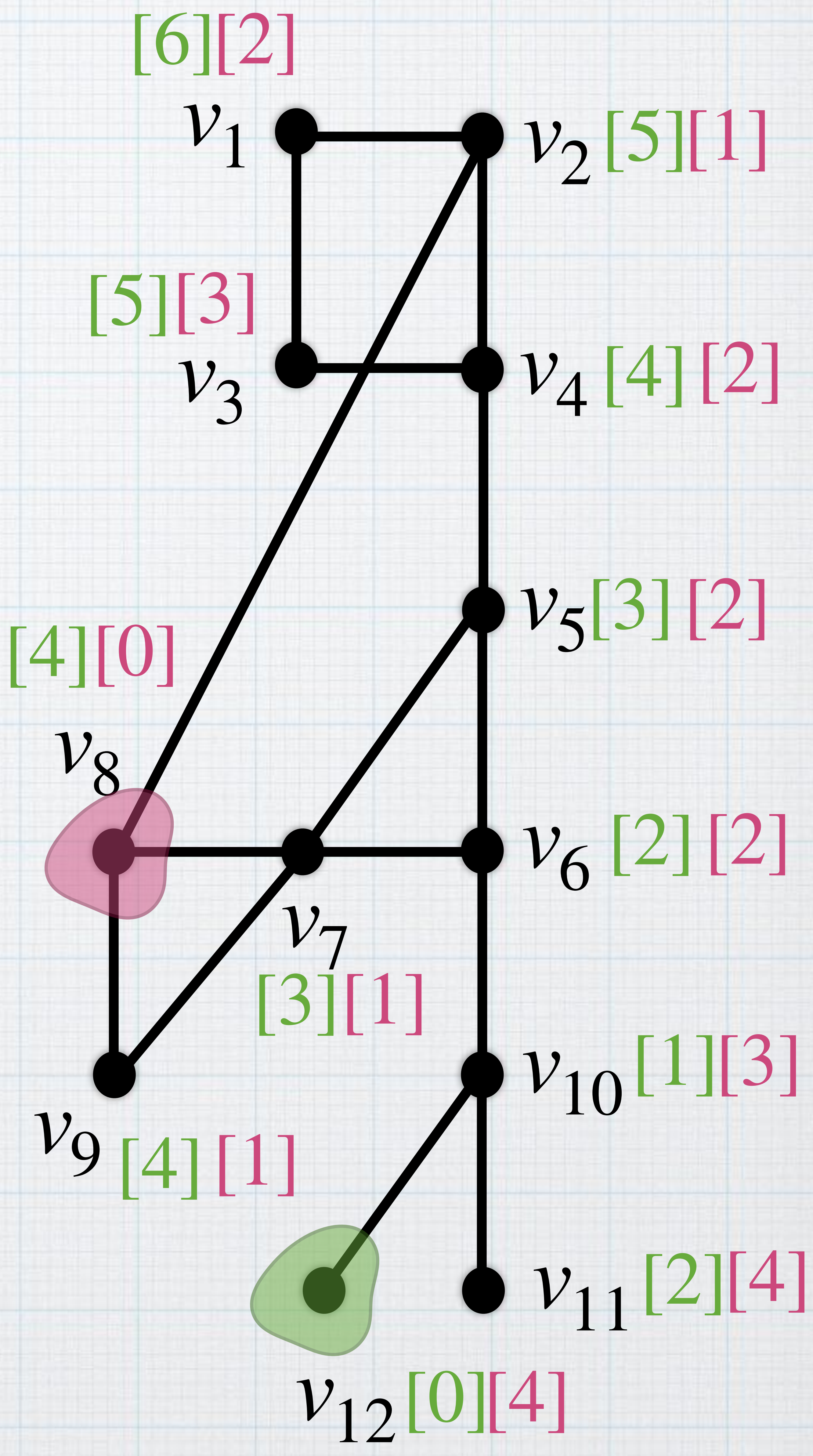
- $d(u, s_i)$ = shortest distance from u to s_i

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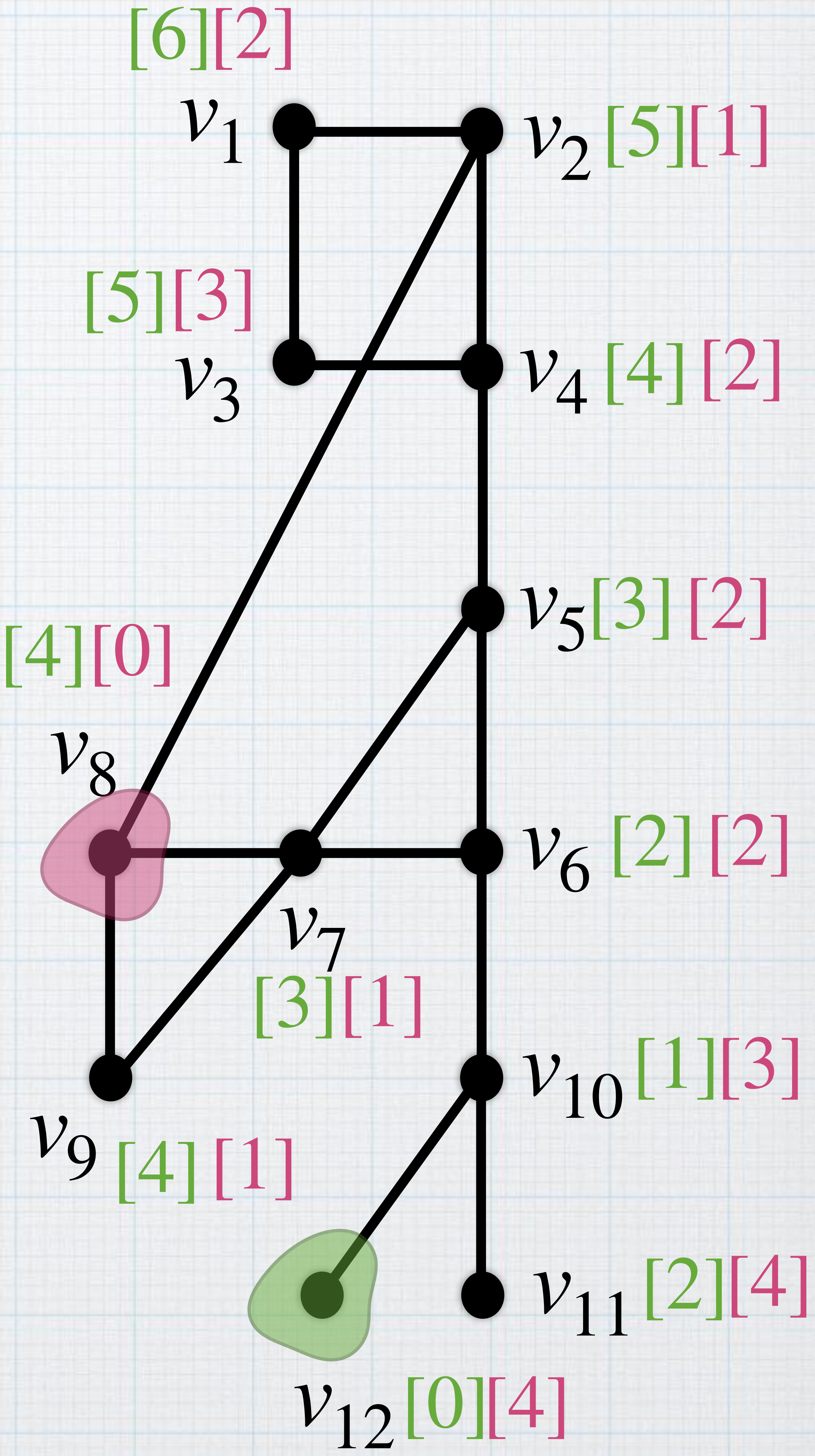

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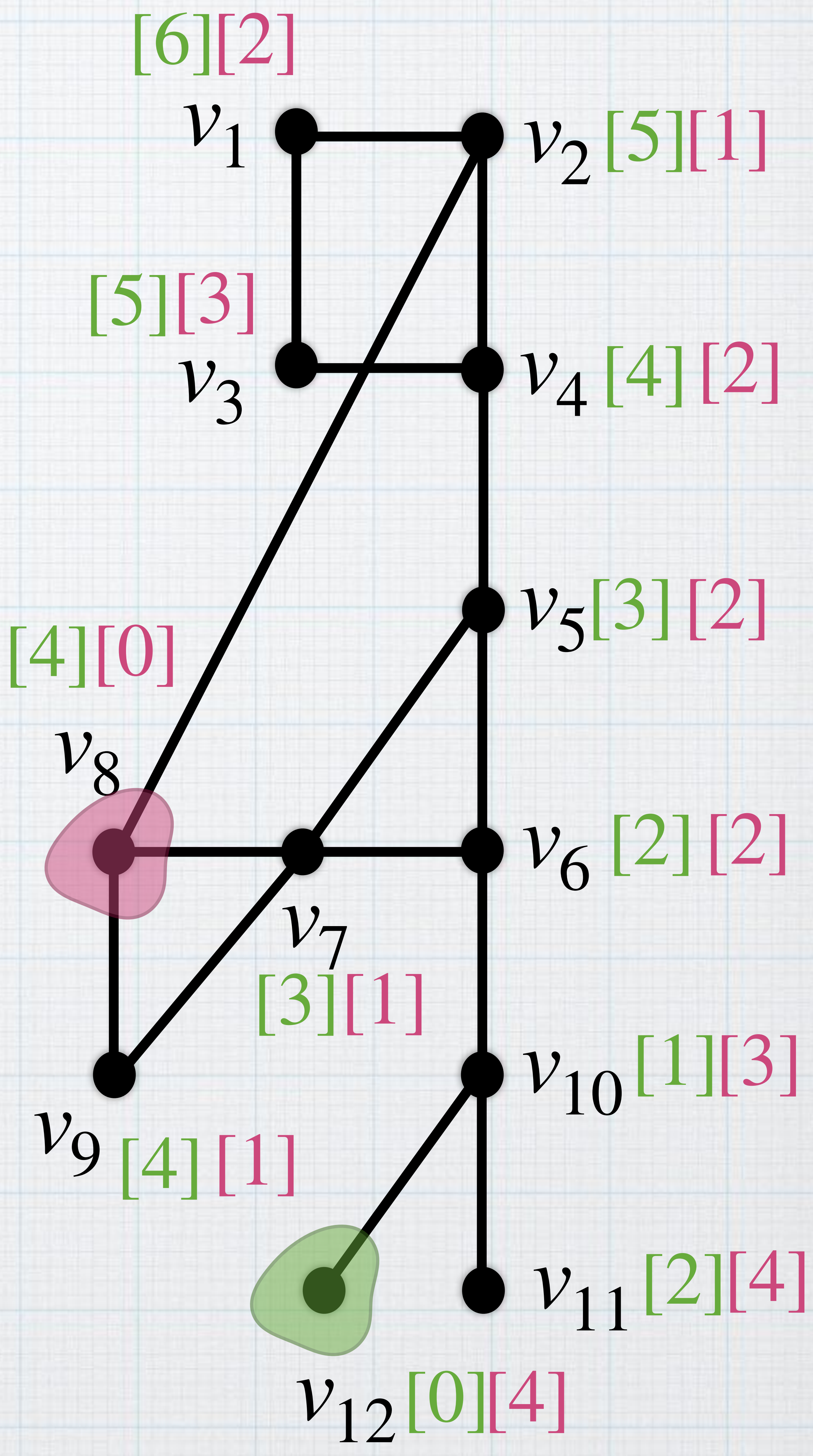

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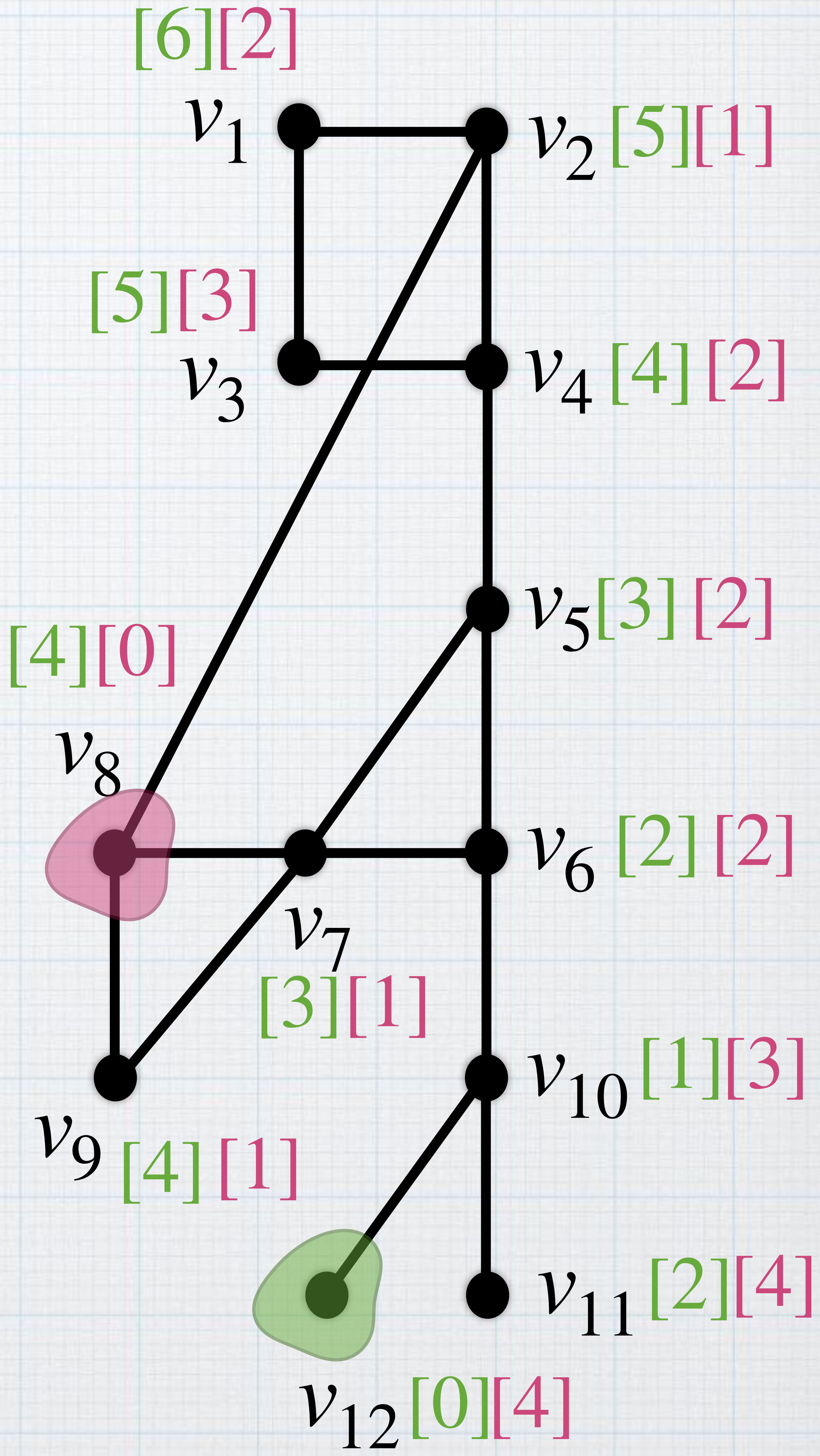

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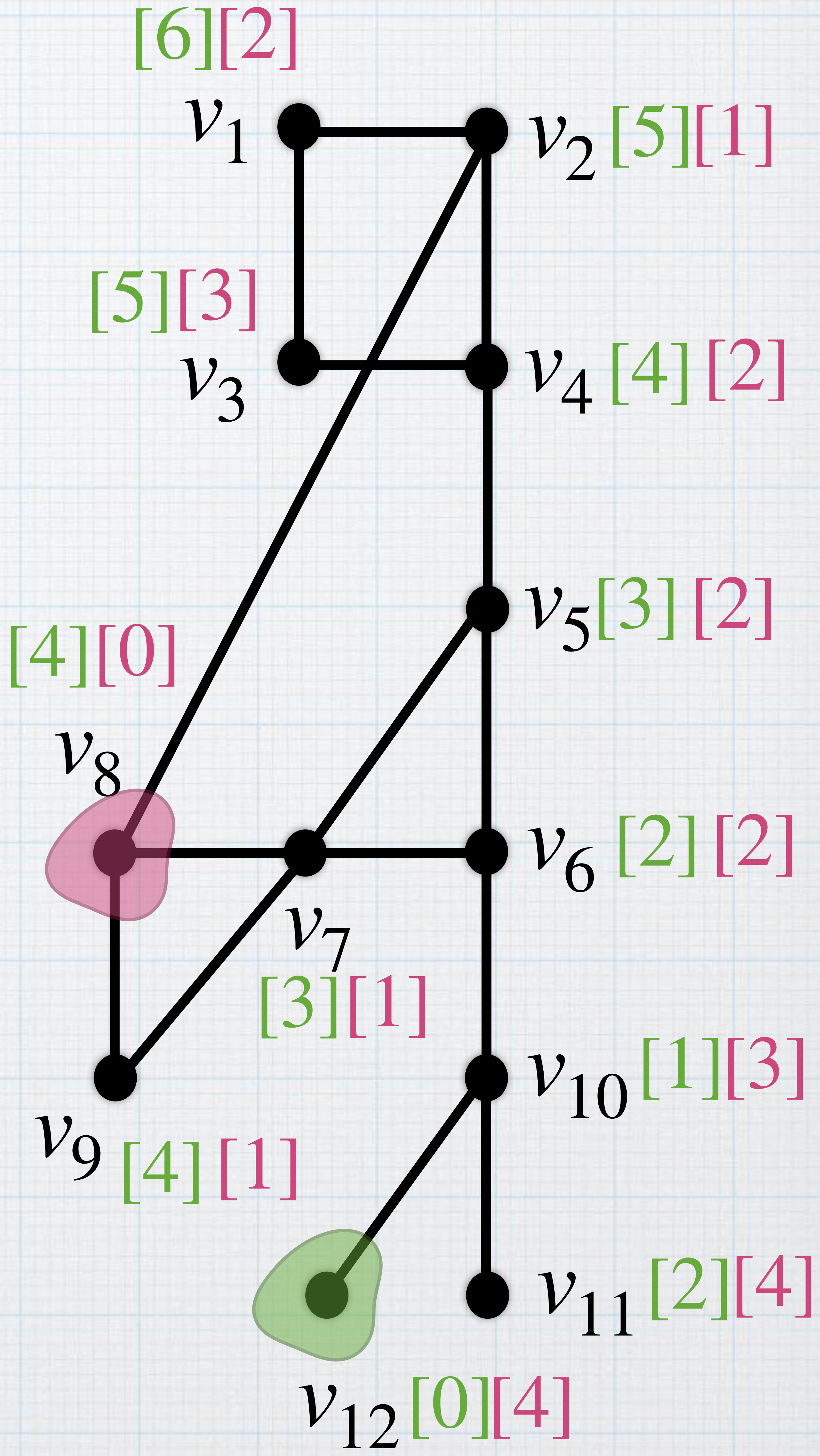

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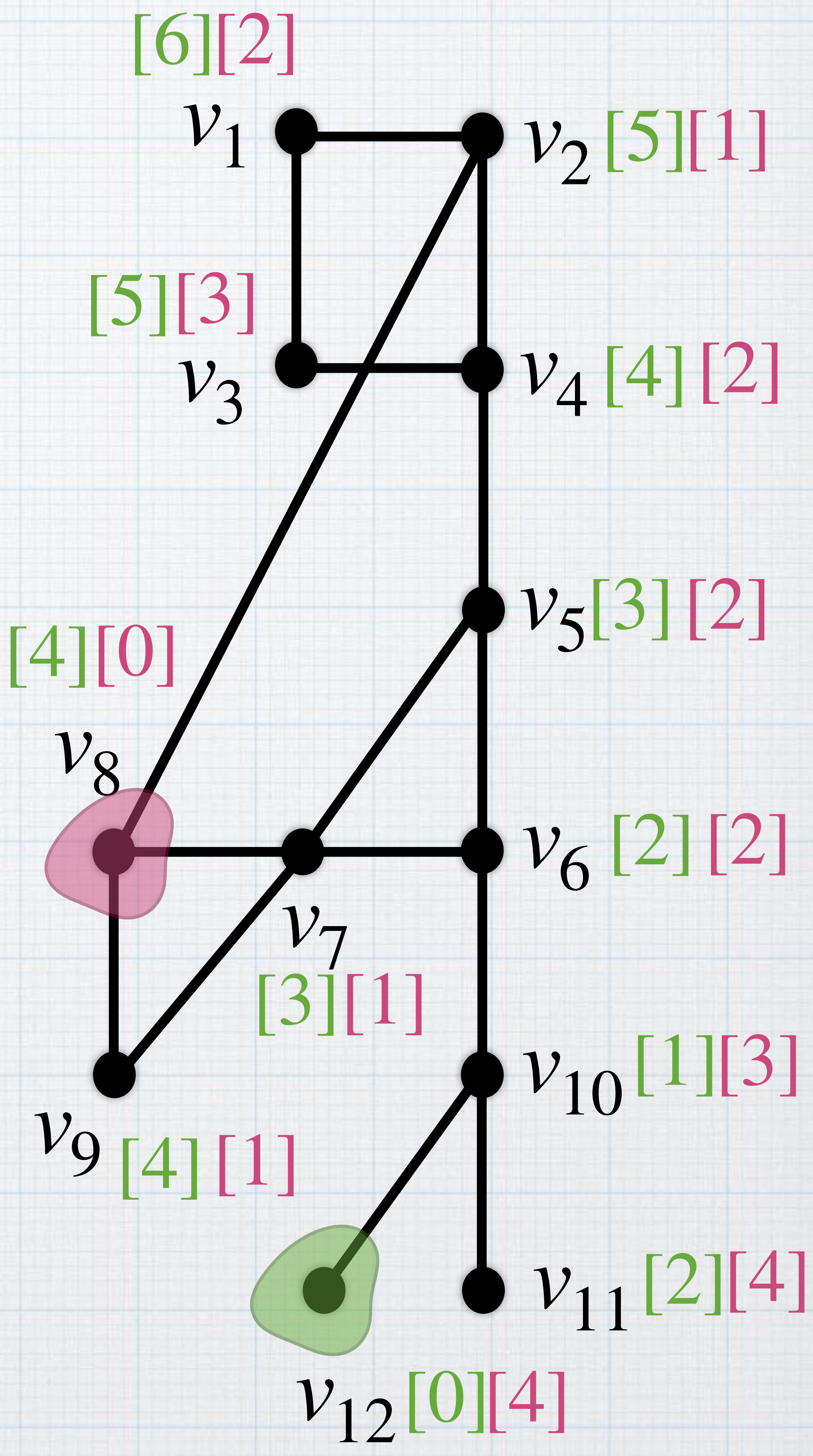
– but also makes it ideal to obtain exotic lower bounds.



Thm [FGKLIST (ICALP'24)]. The Metric Dimension problem on bounded diameter graphs

- admits $2^{2^{\mathcal{O}(\mathrm{tw})}} \cdot n^{\mathcal{O}(1)}$–time algo, but
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How about vertex cover?

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(Edge Clique Cover [CPP, SODA'13], Biclique Cover [CIK, IPEC'16], Strong Met–Dim [FGKLIST, ICALP'24])

Geodetic Set

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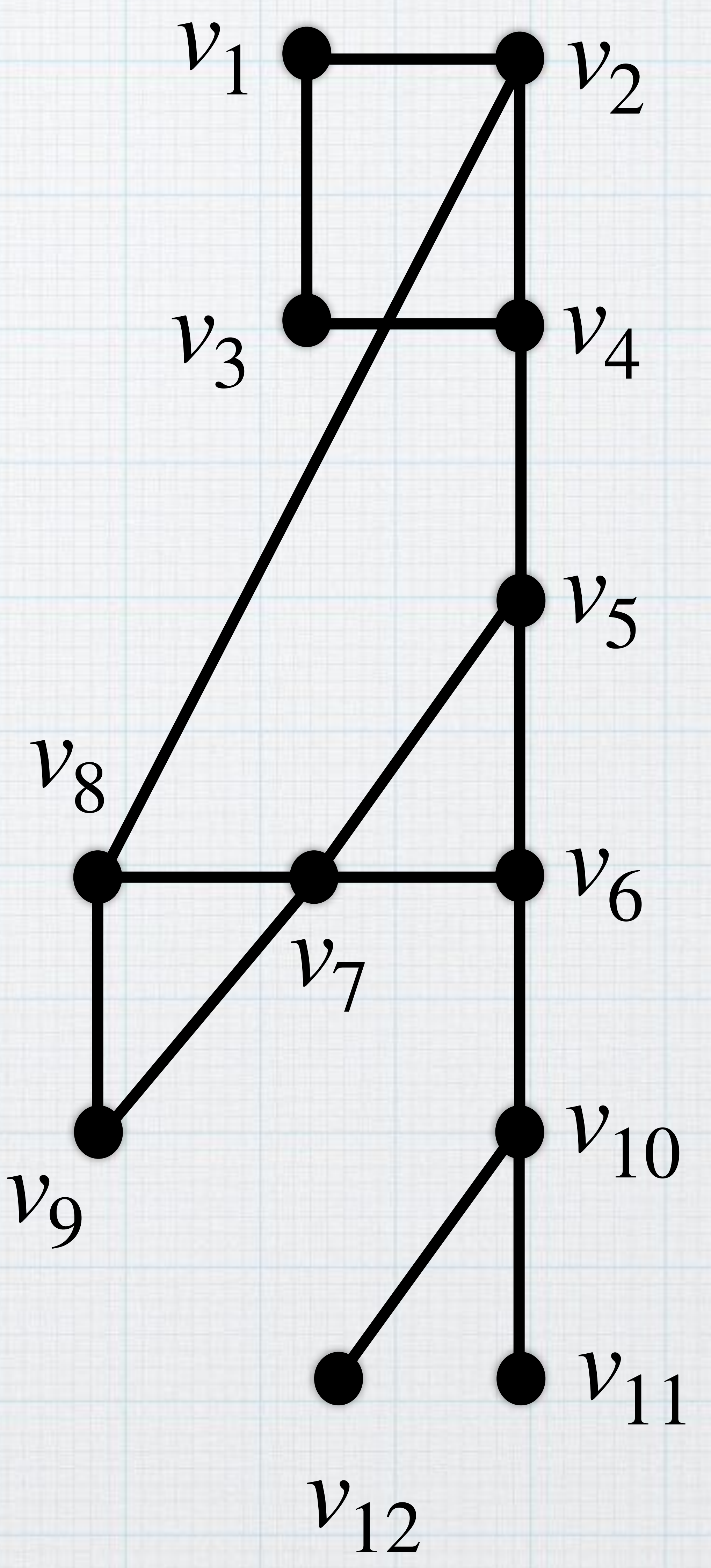
Output: $\exists ?$ a subset S of k vertices s.t. for any vertex u is covered by a shortest path between two vertices in S



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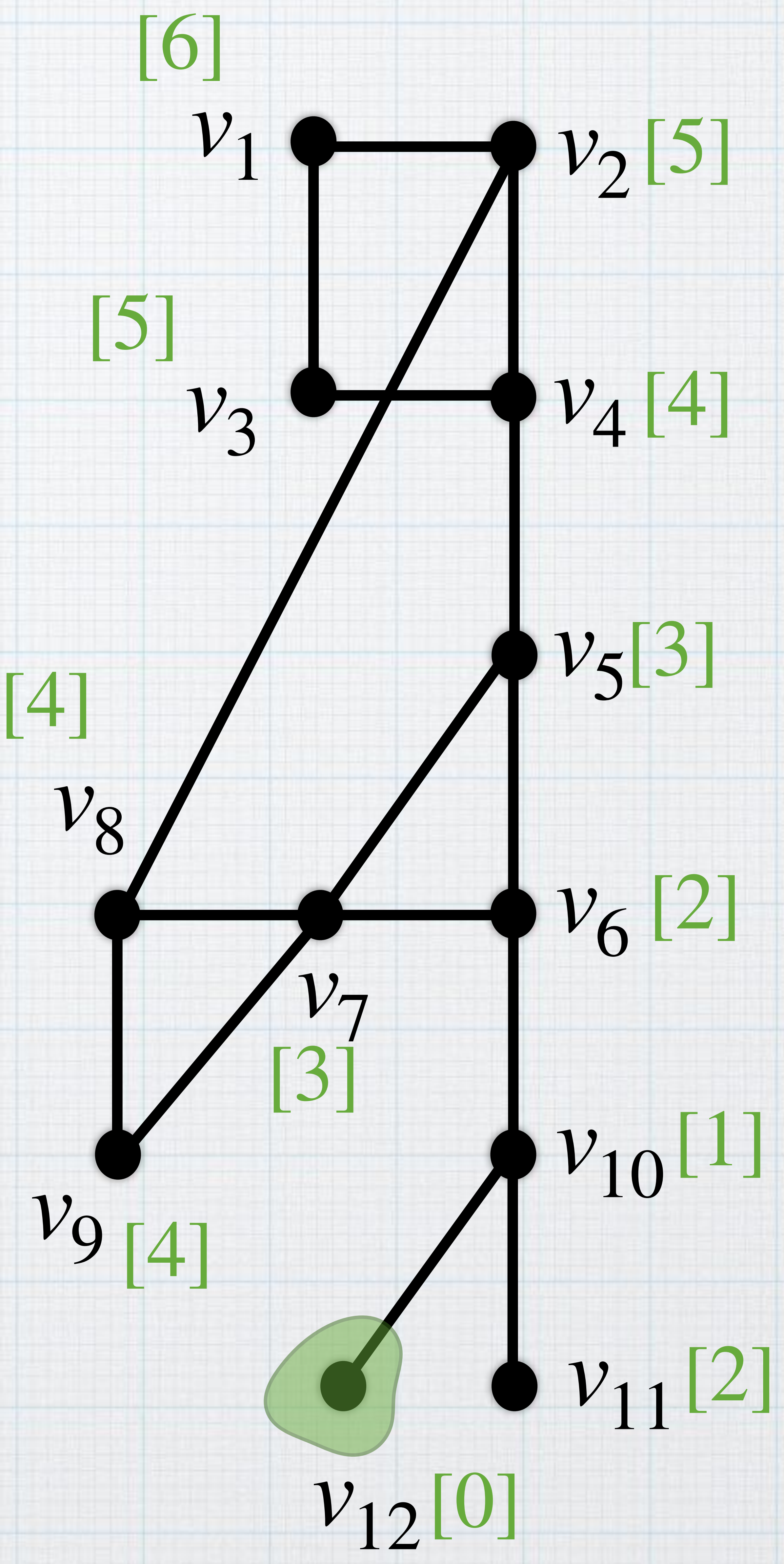

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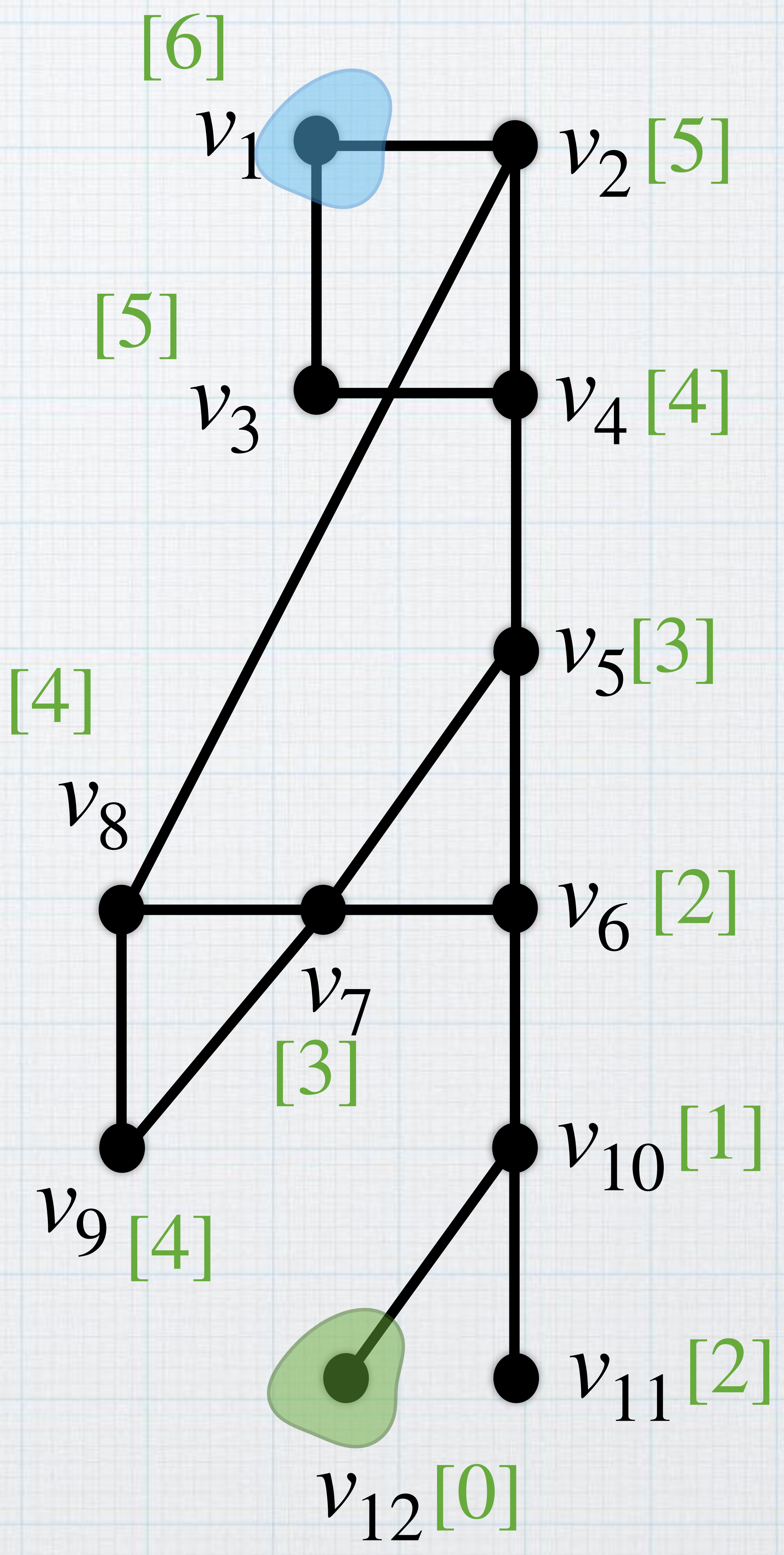

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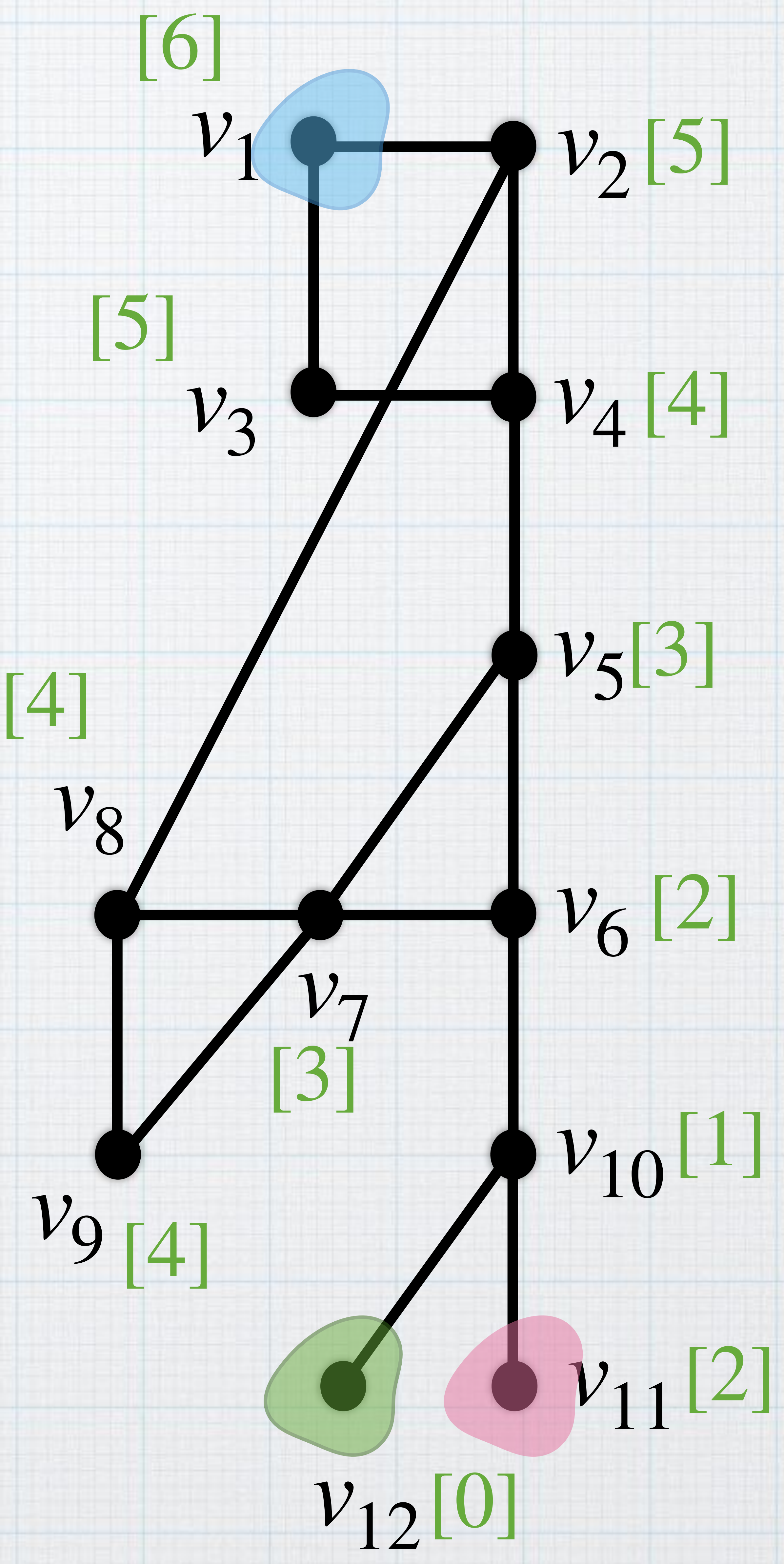

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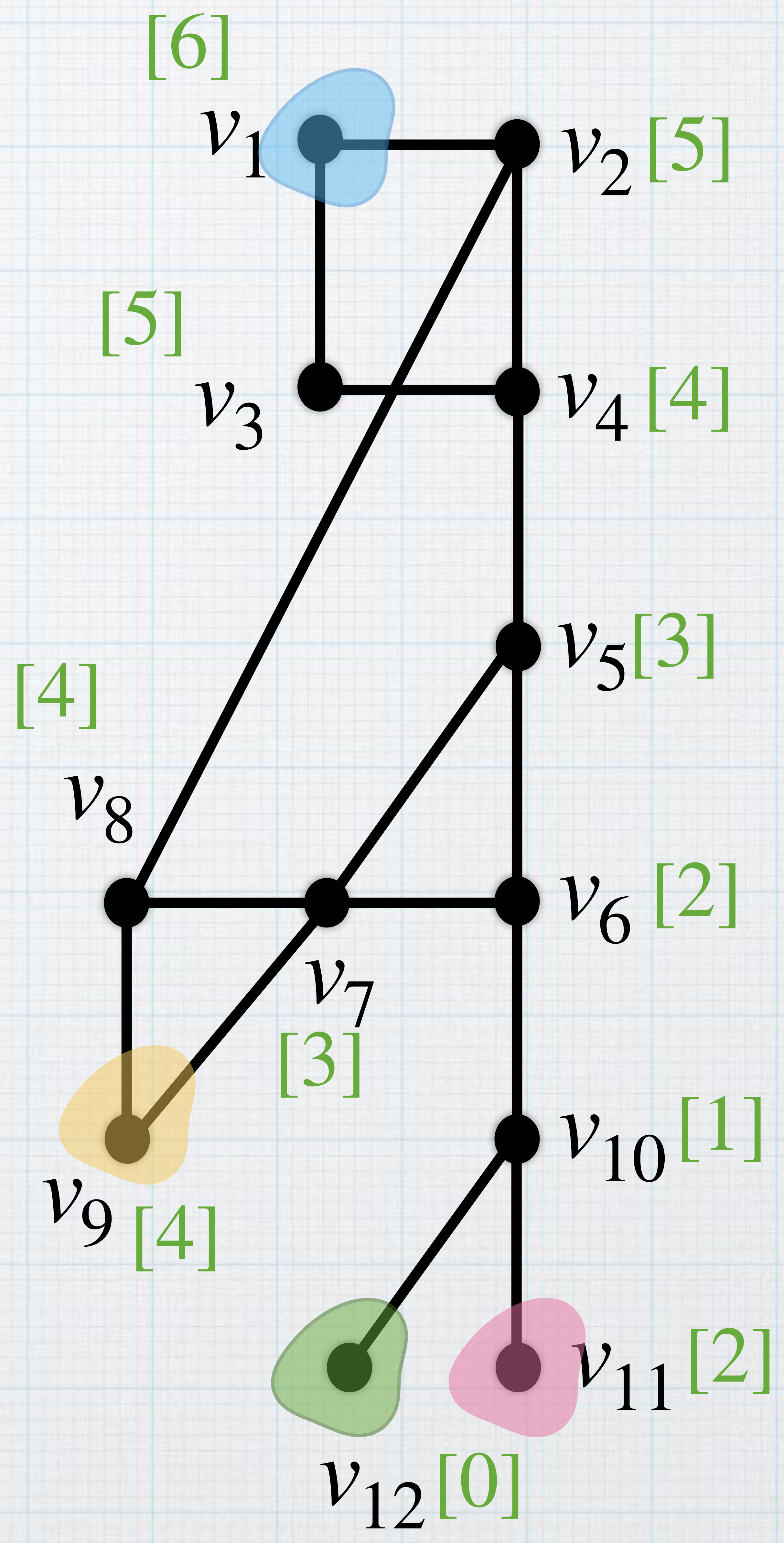

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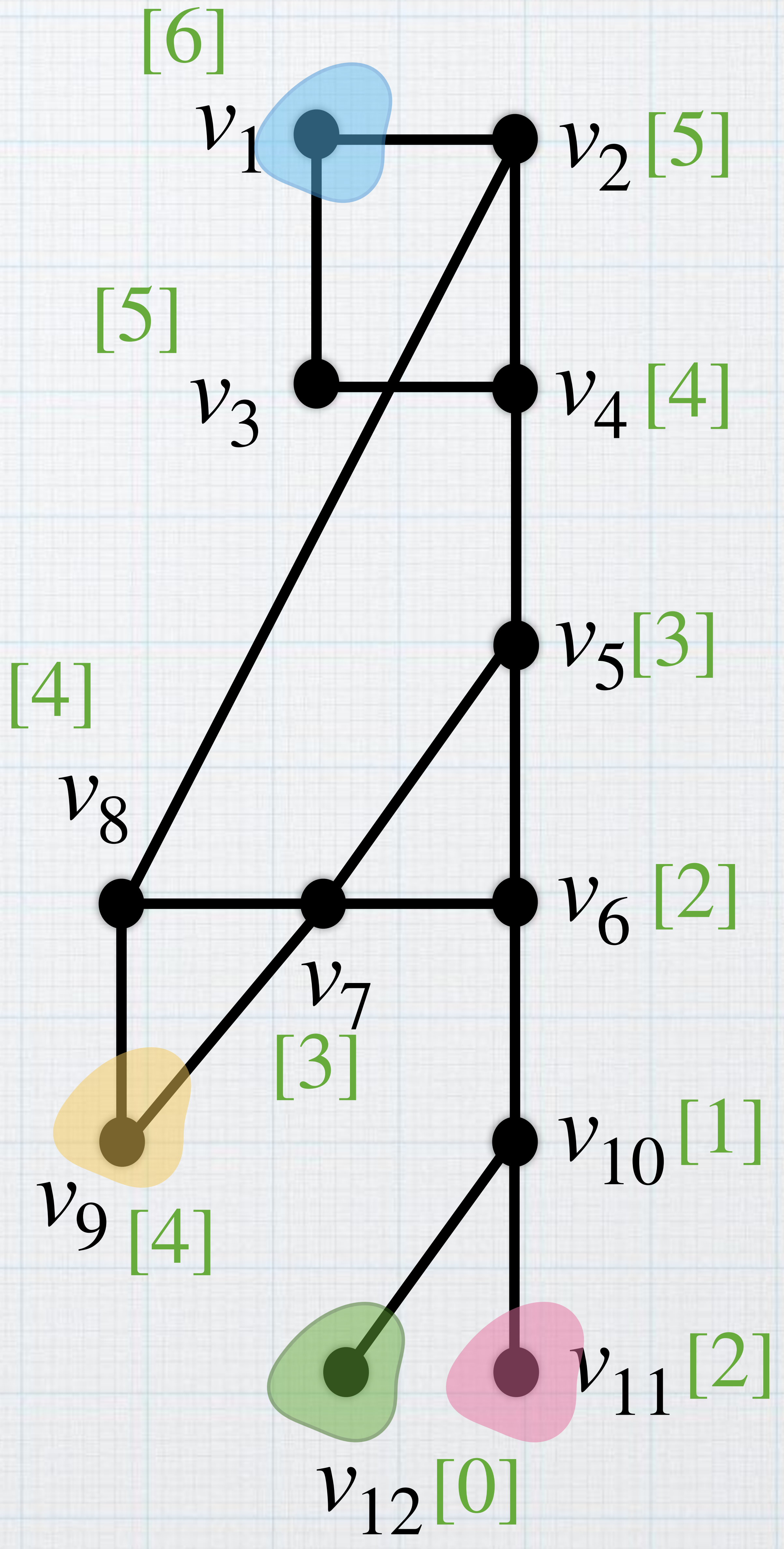

Solution at one part of the graph, plays important role in other part of the graph.

Ex. v_{12} and v_1 covers a lot of vertices.

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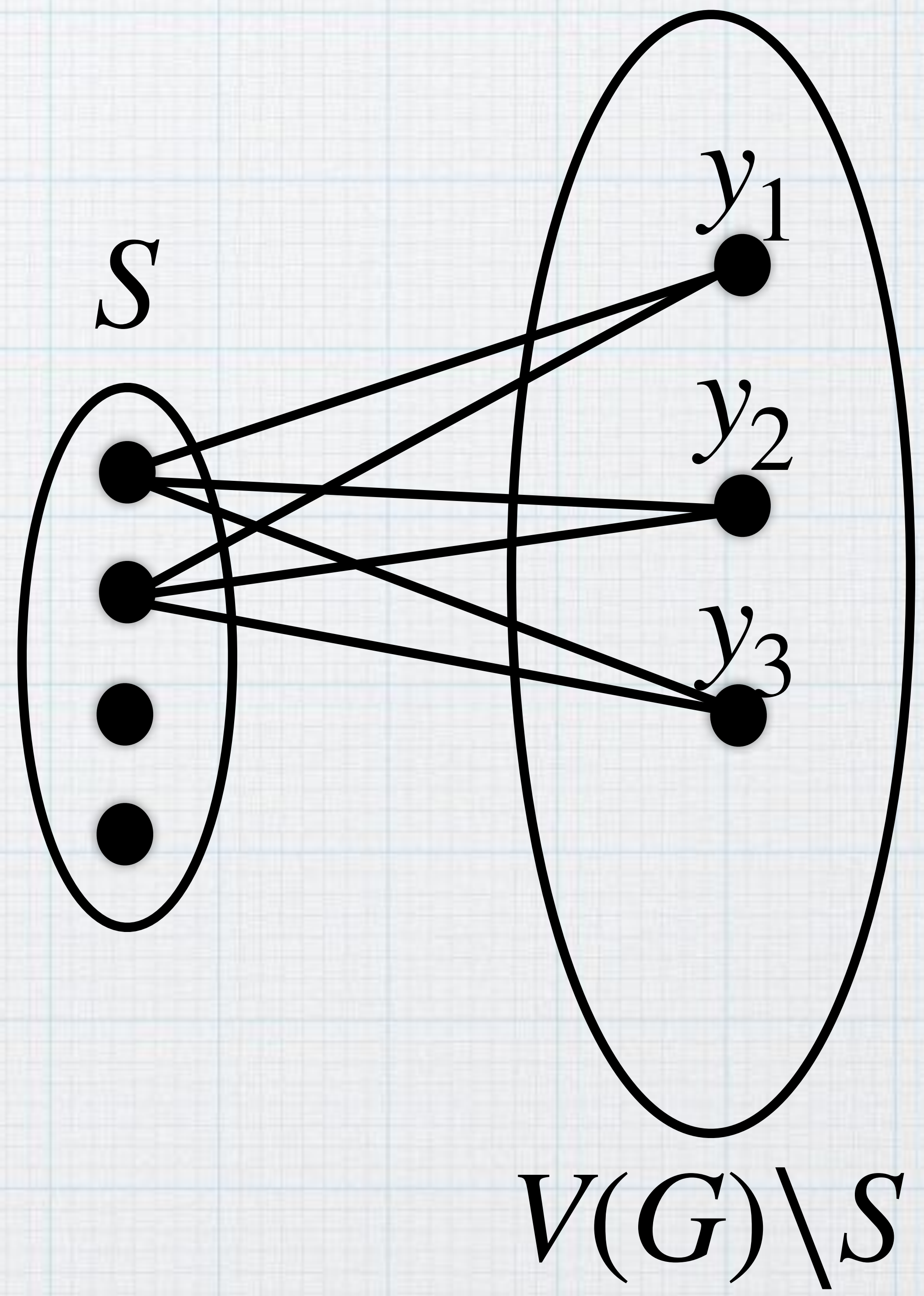

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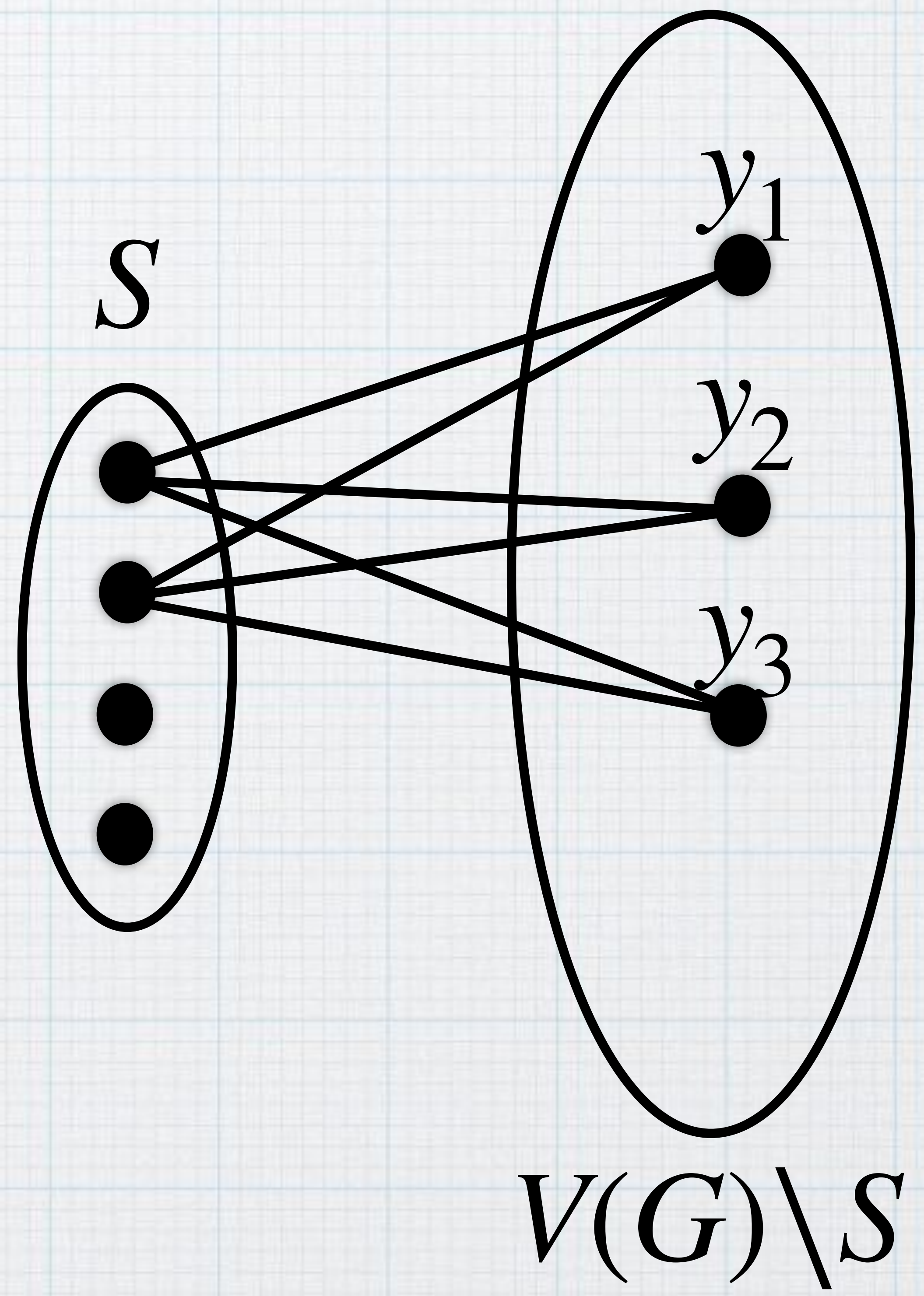
Obs: If $N(y_1) = N(y_2) = N(y_3)$ for y_1, y_2, y_3 in ind–set, then any resolving set contains at least two of them.



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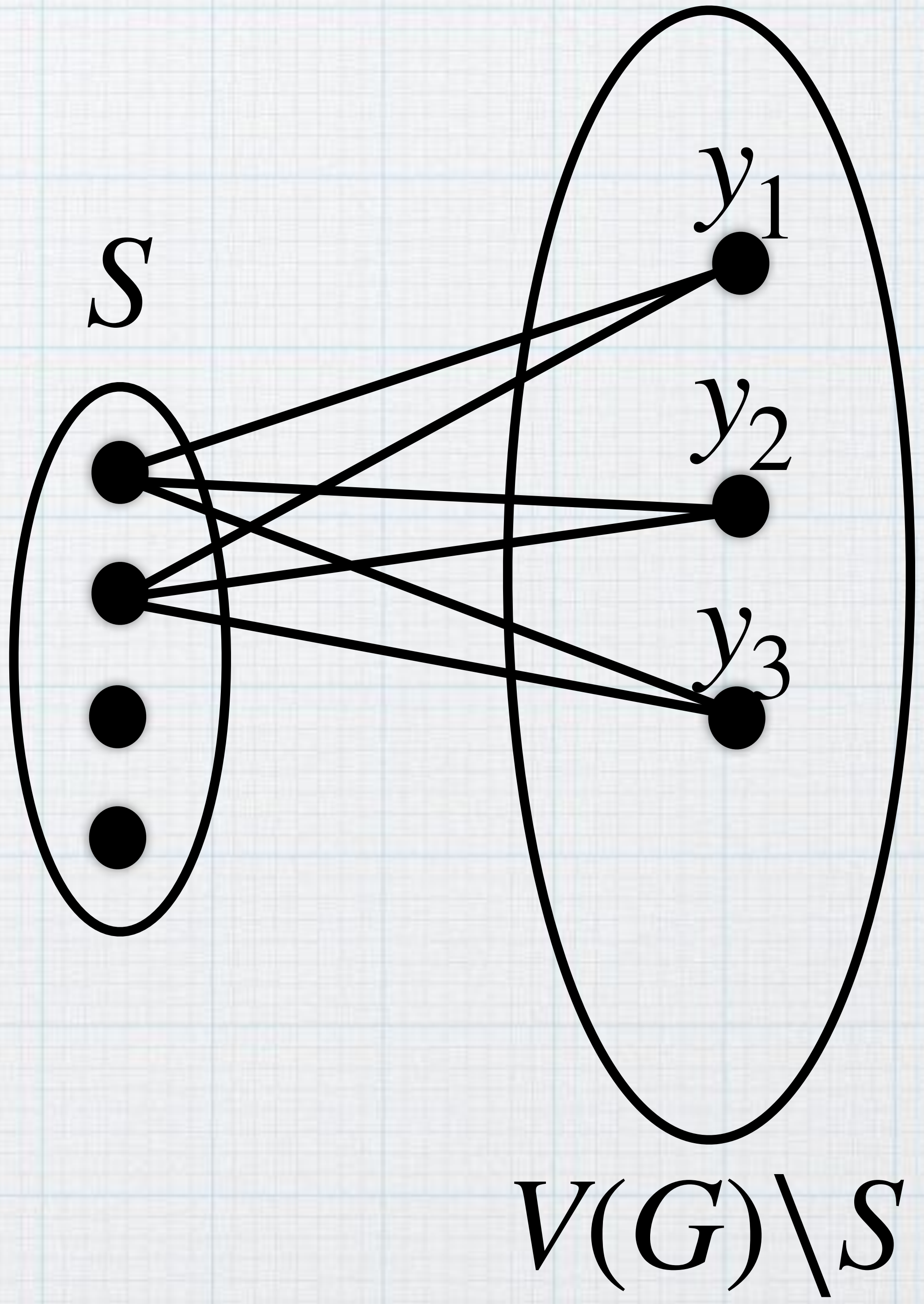

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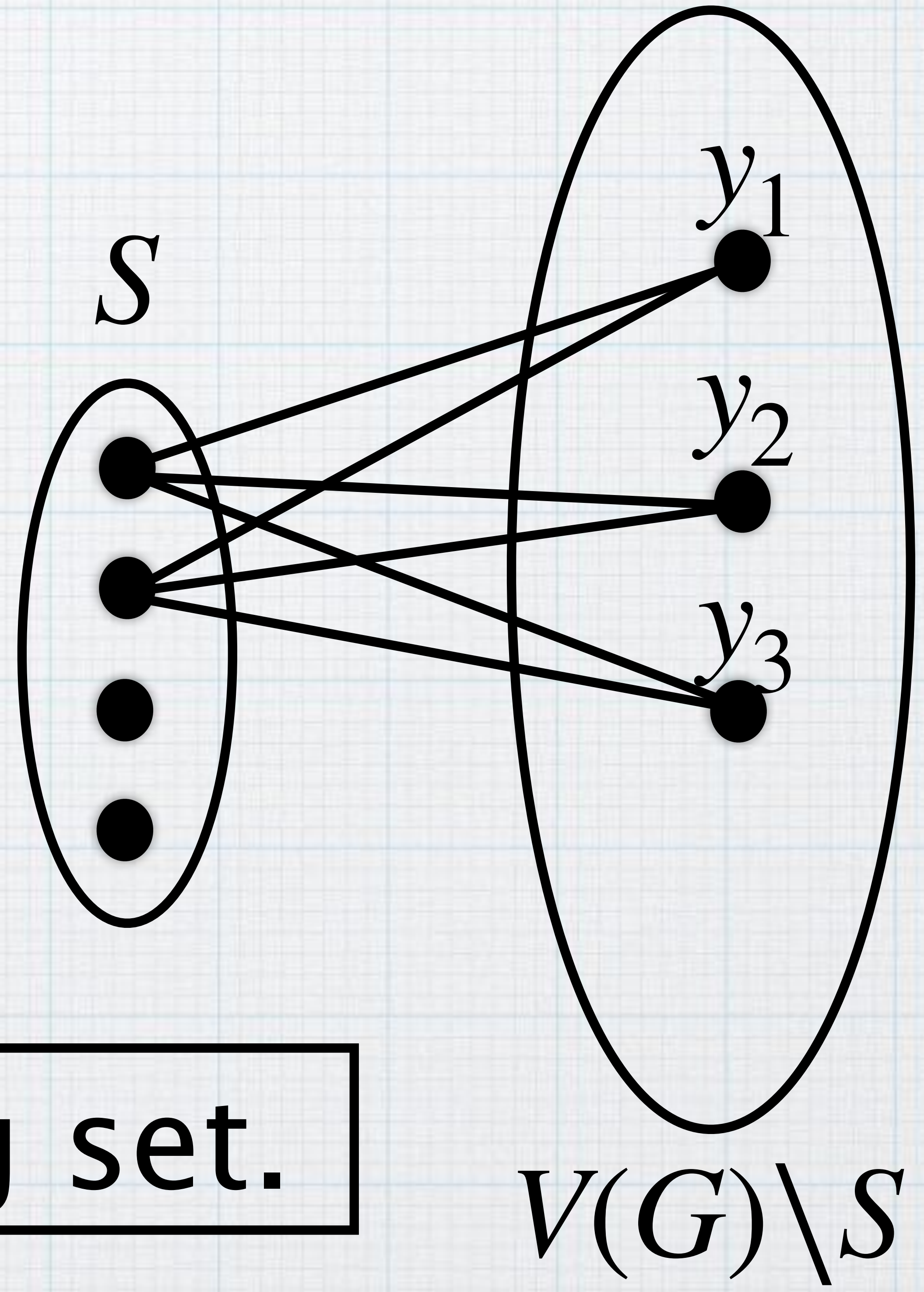

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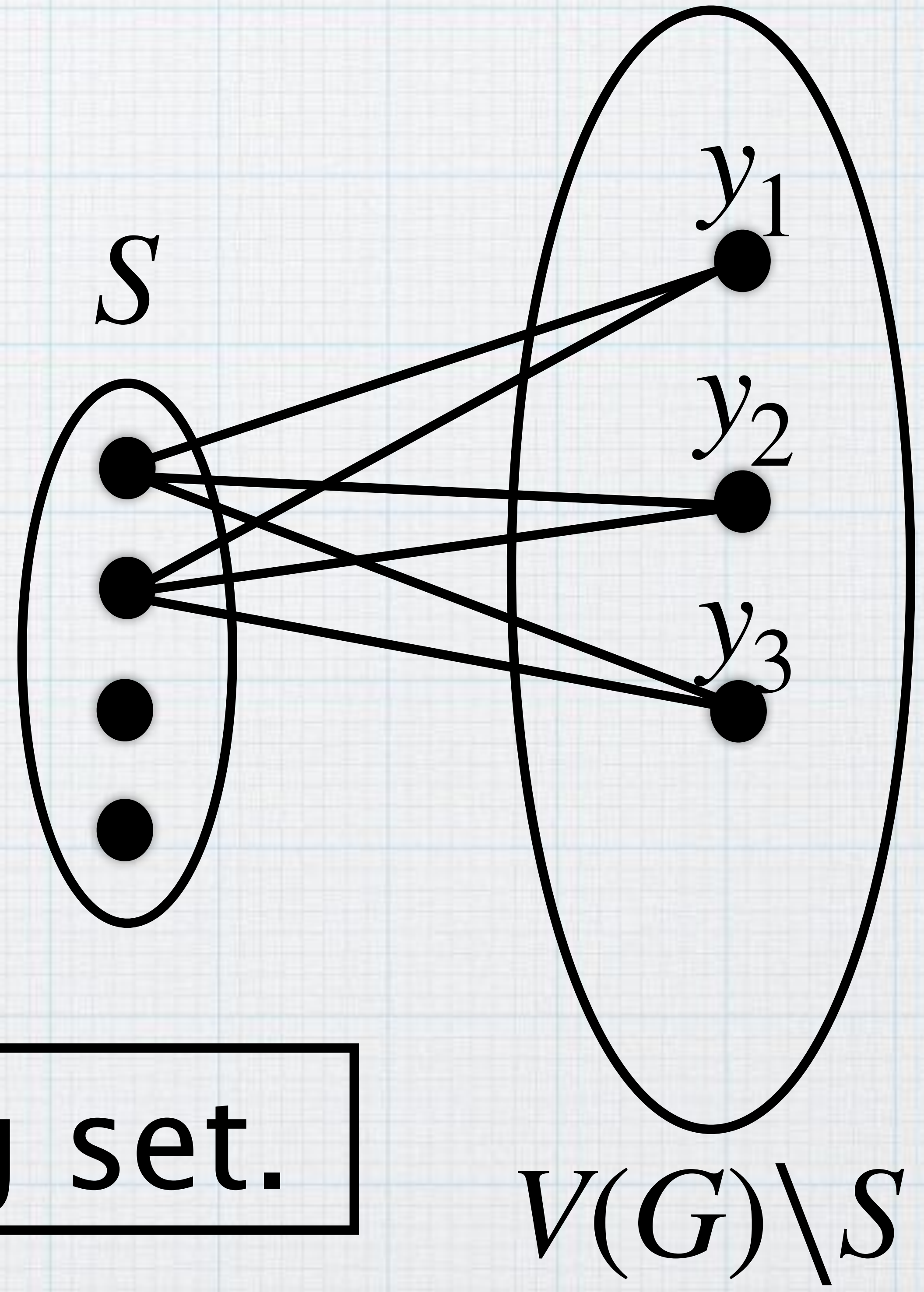

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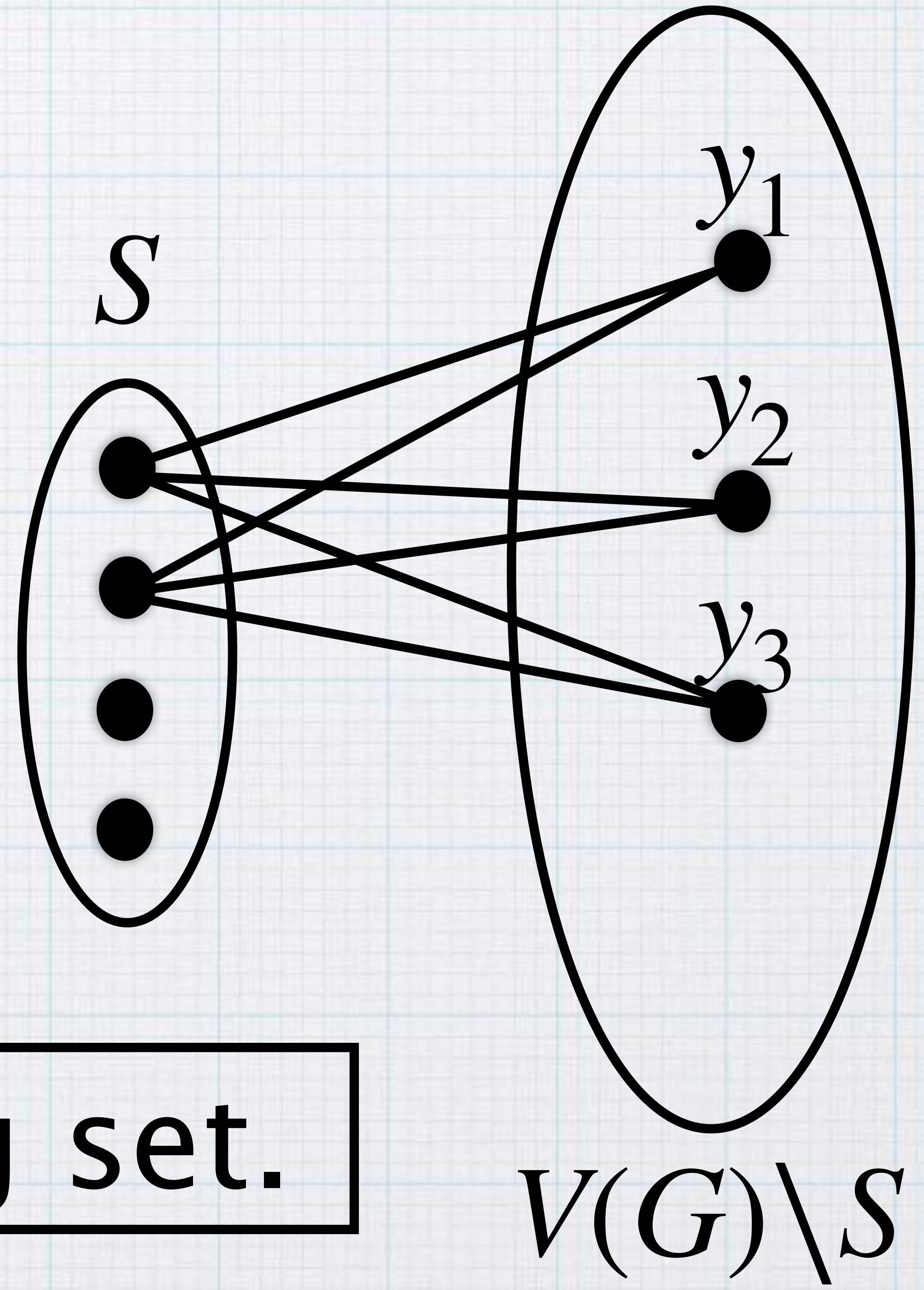

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3-SAT (with n var) to Metric Dimension with vc $= \mathcal{O}(\sqrt{n})$

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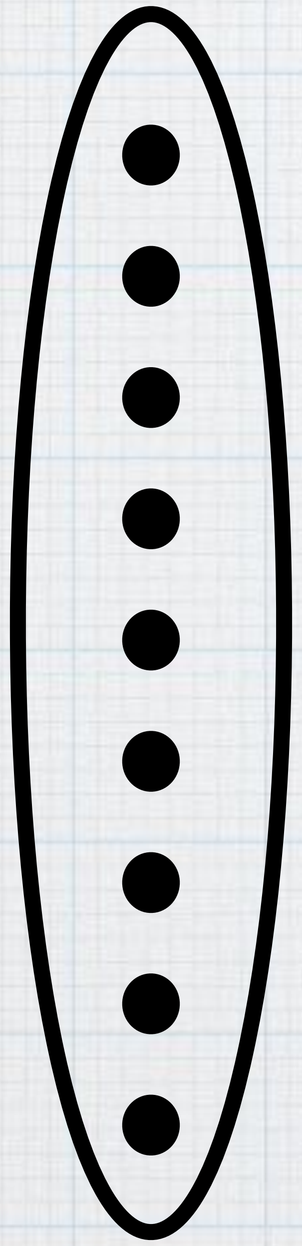

n var

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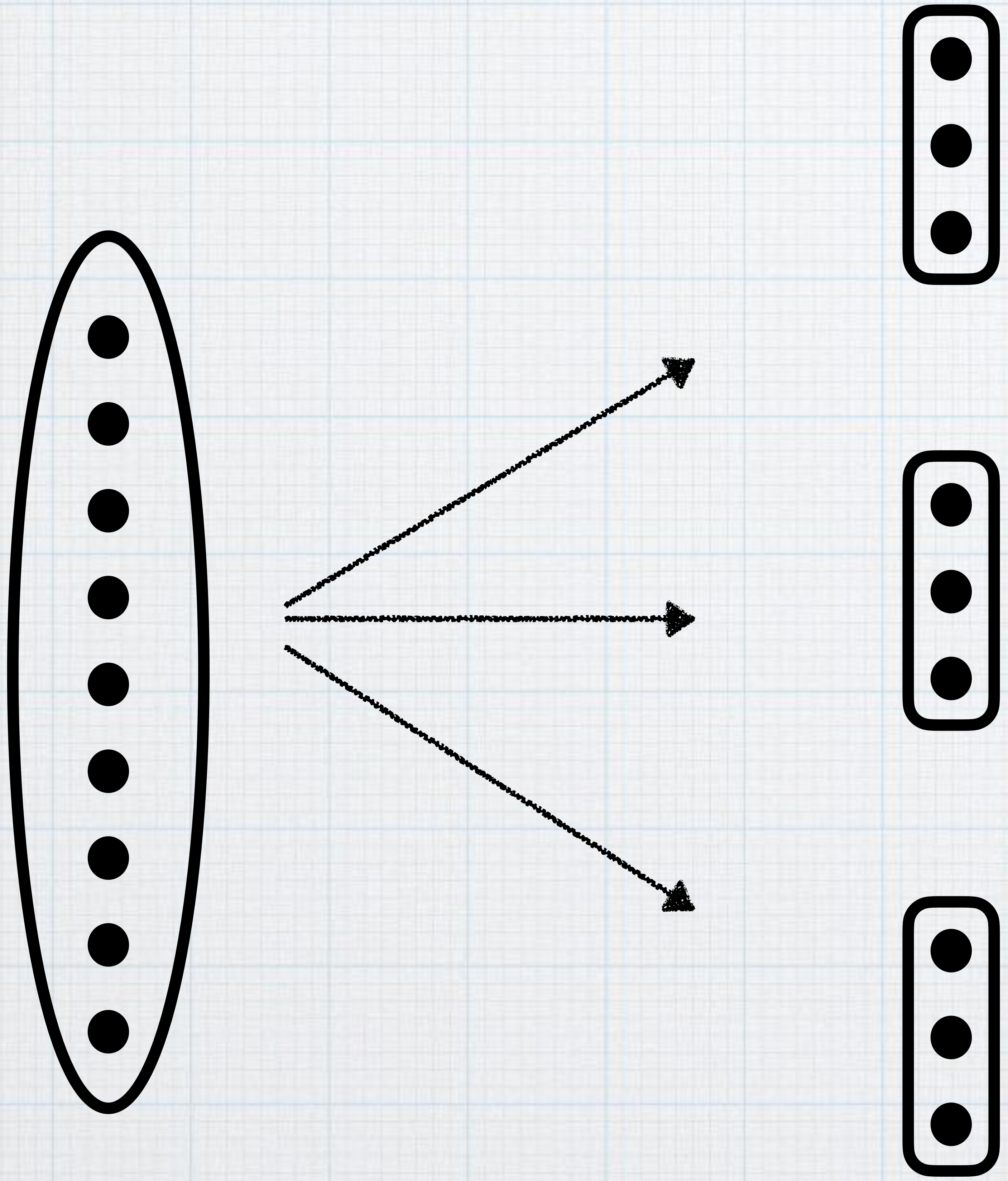

n var

$\sqrt{n}$ buckets with
$\sqrt{n}$ var each

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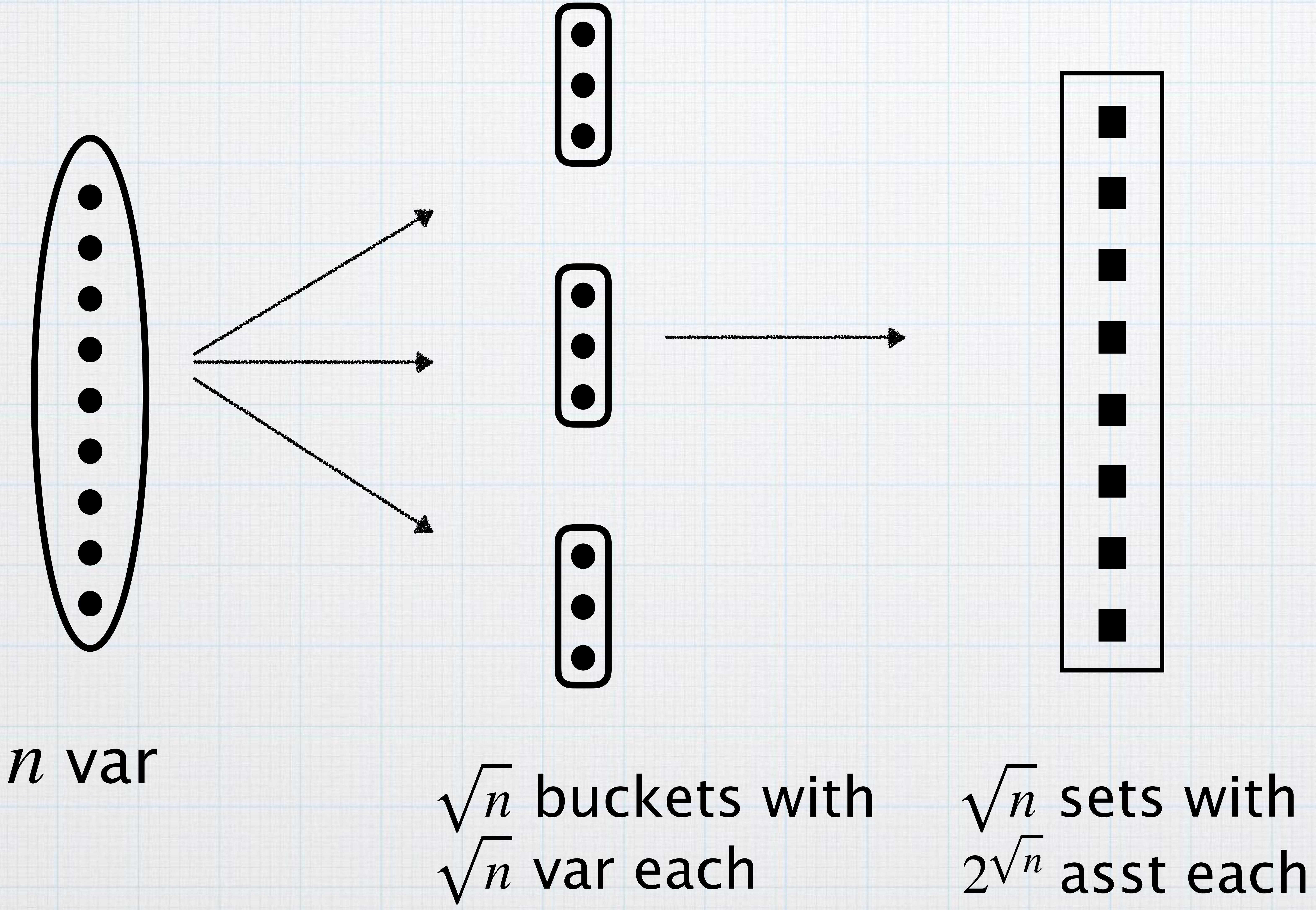

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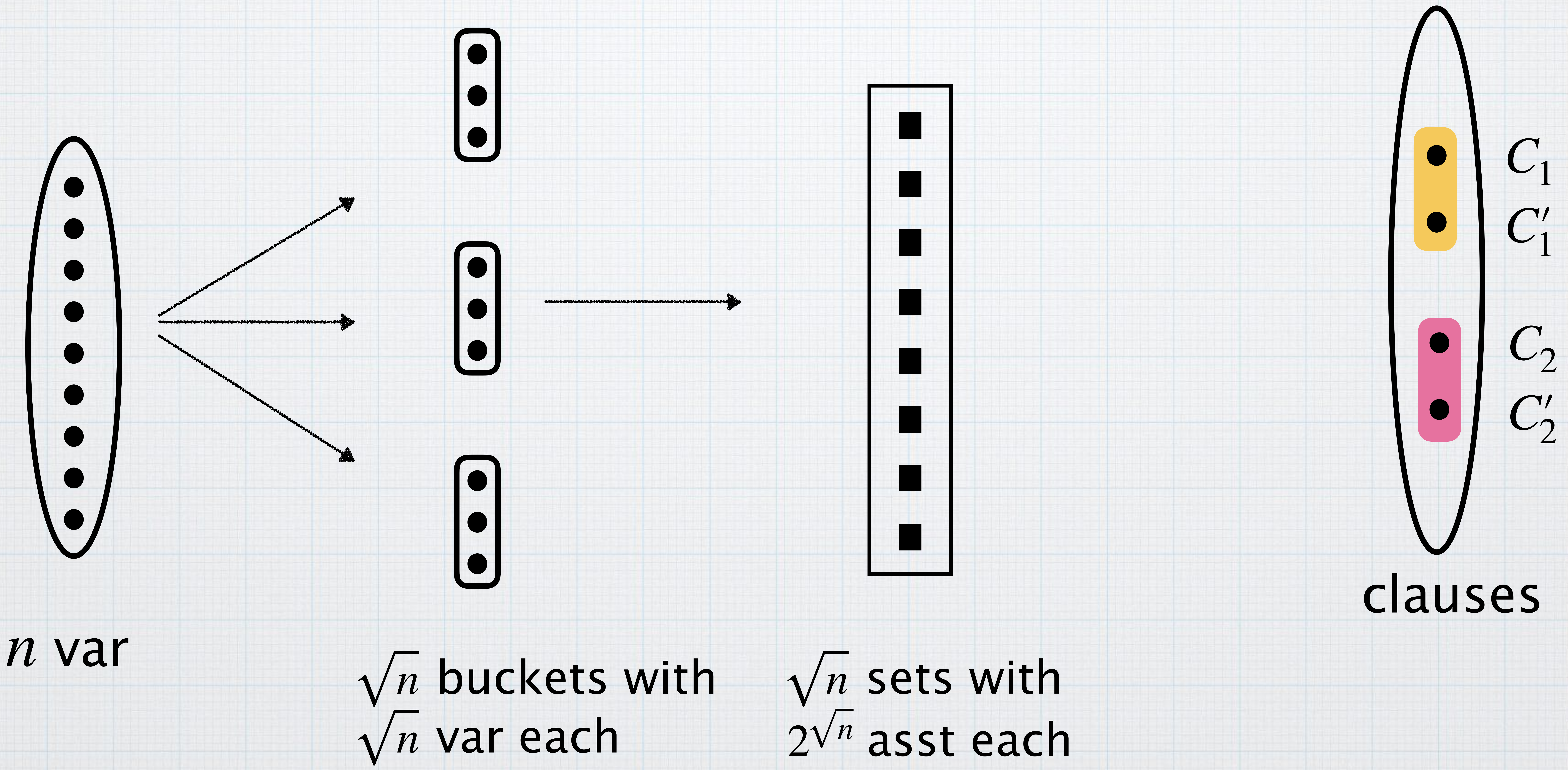

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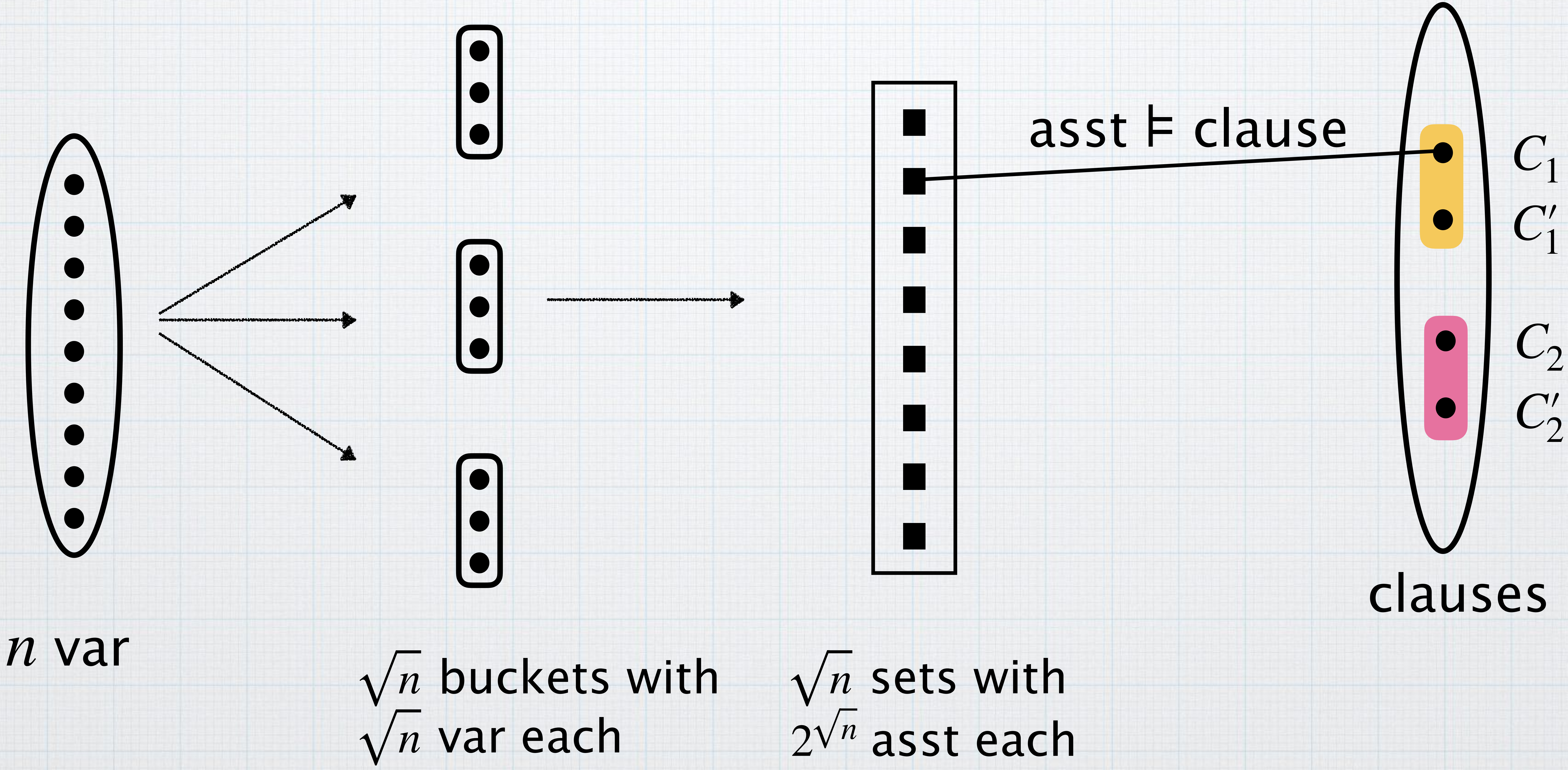

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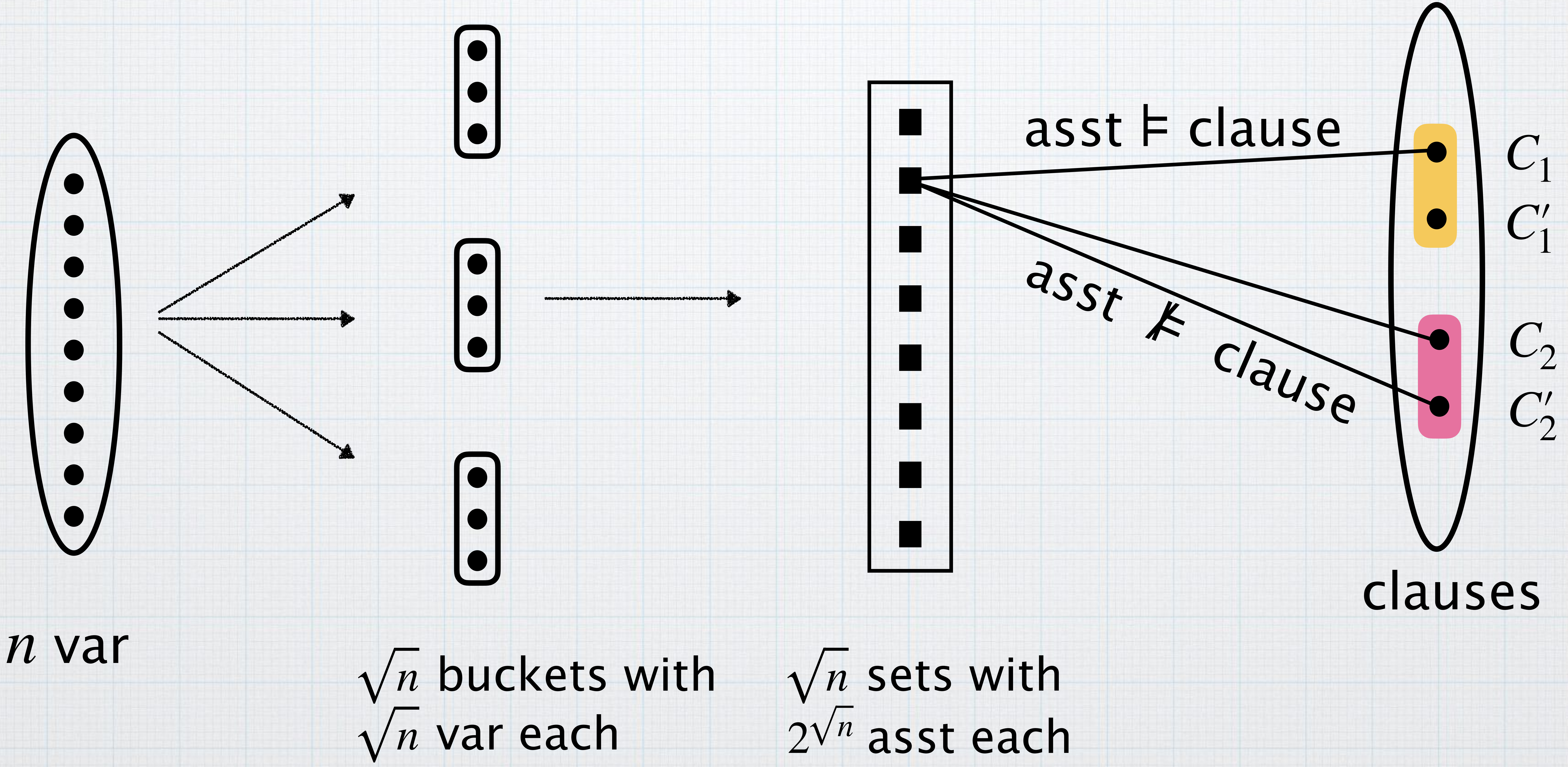

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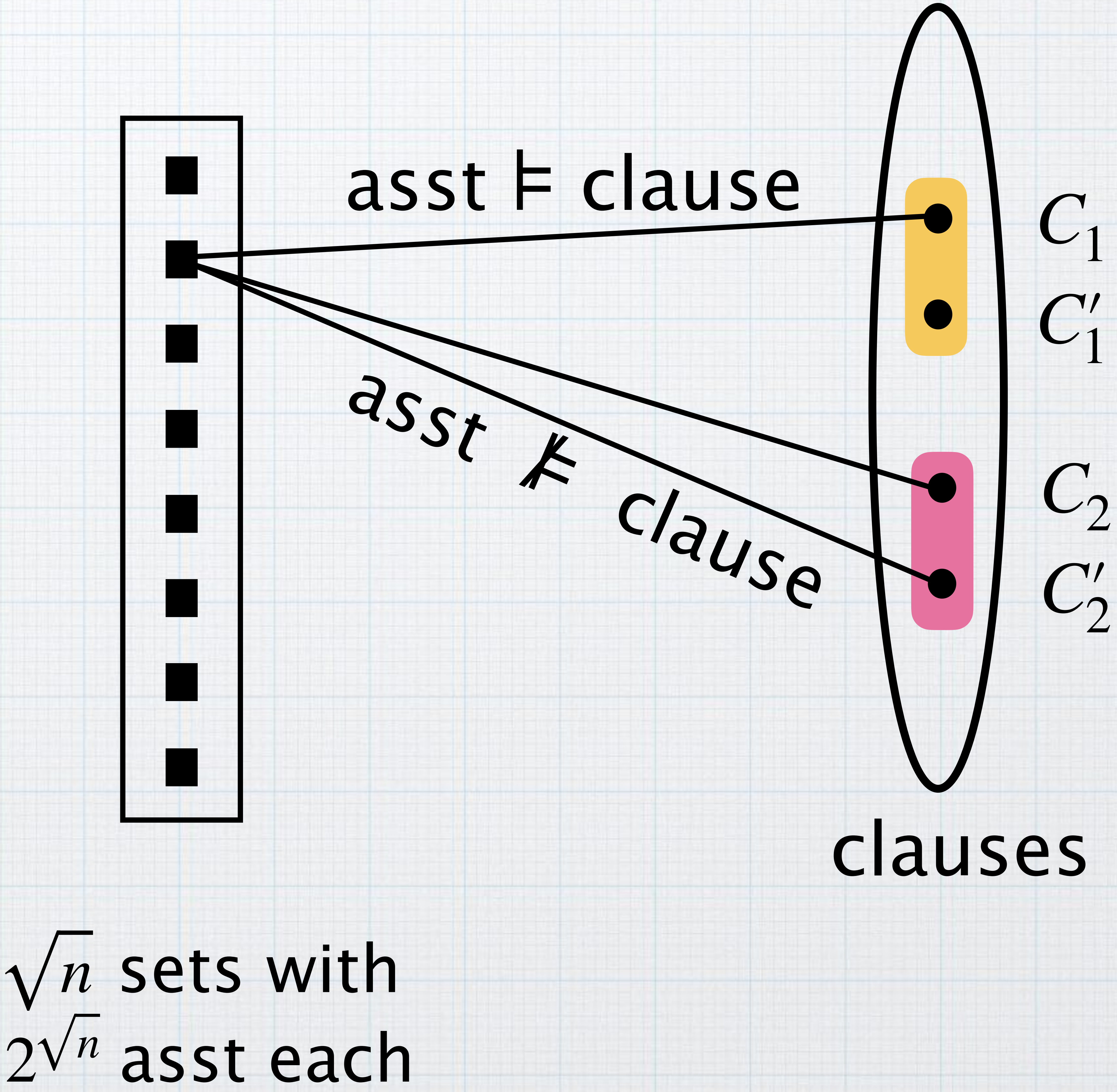
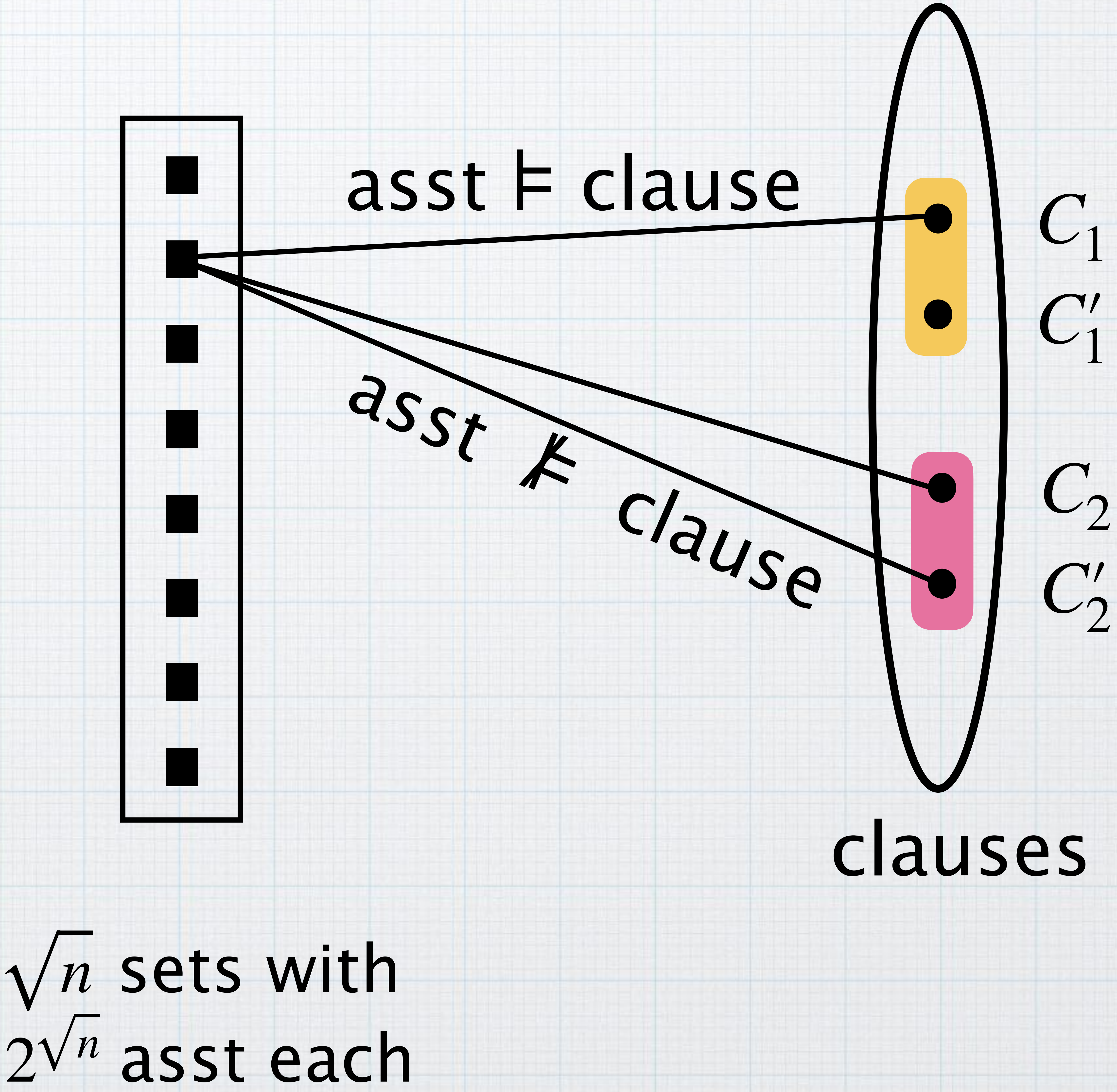


3-SAT (with n var) to Metric Dimension with vc = $\mathcal{O}(\sqrt{n})$

Q: How to identify vertices corresponding to assignments?



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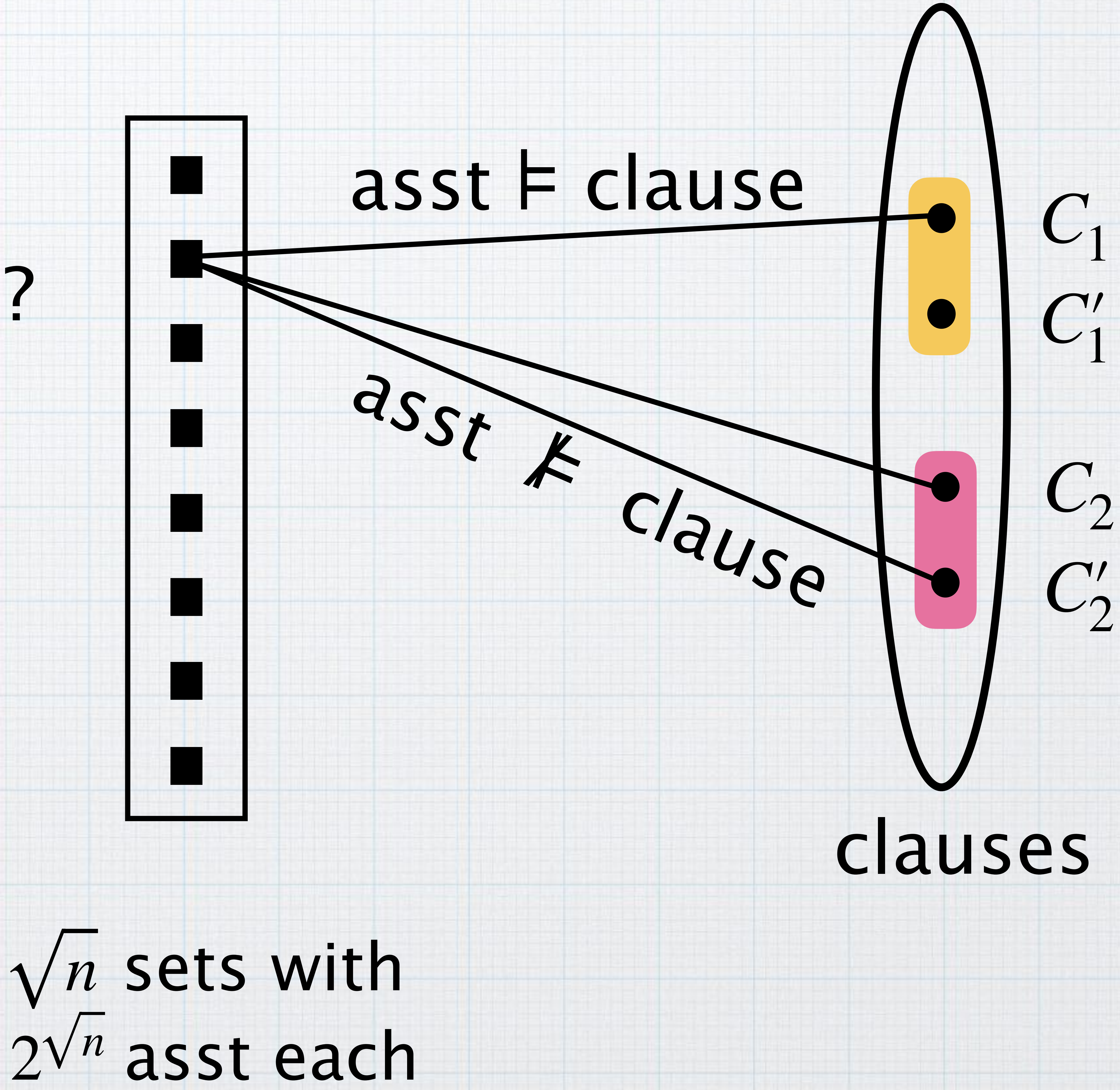
$\{C_1, C_2\}$ $\{C'_1, C_2\}$ ...



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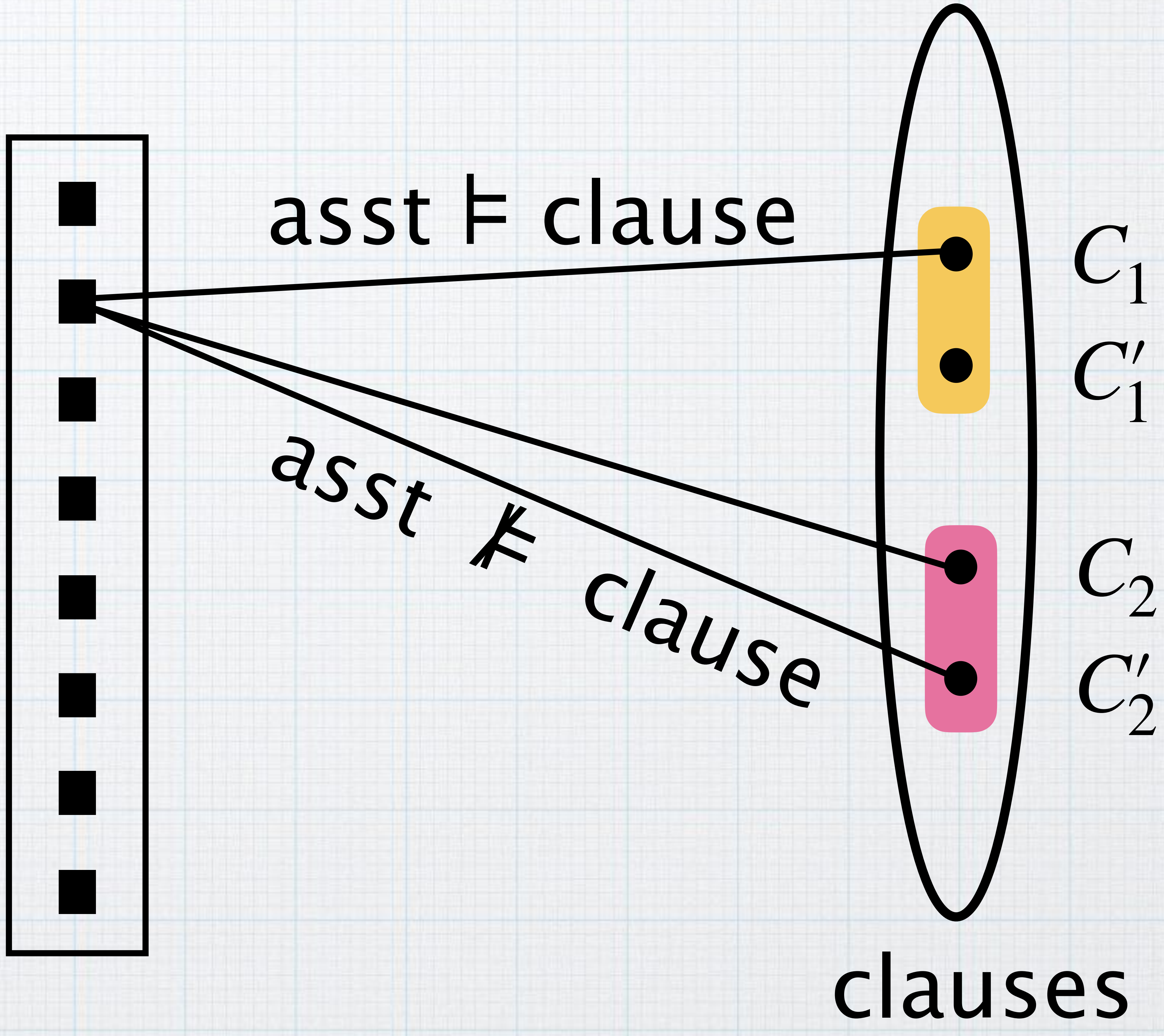
Q: How to control the vertex cover? i.e. limit the interactions



$\sqrt{n}$ sets with
$2^{\sqrt{n}}$ asst each

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Using bit encoding which cost log(set-size) extra vertices

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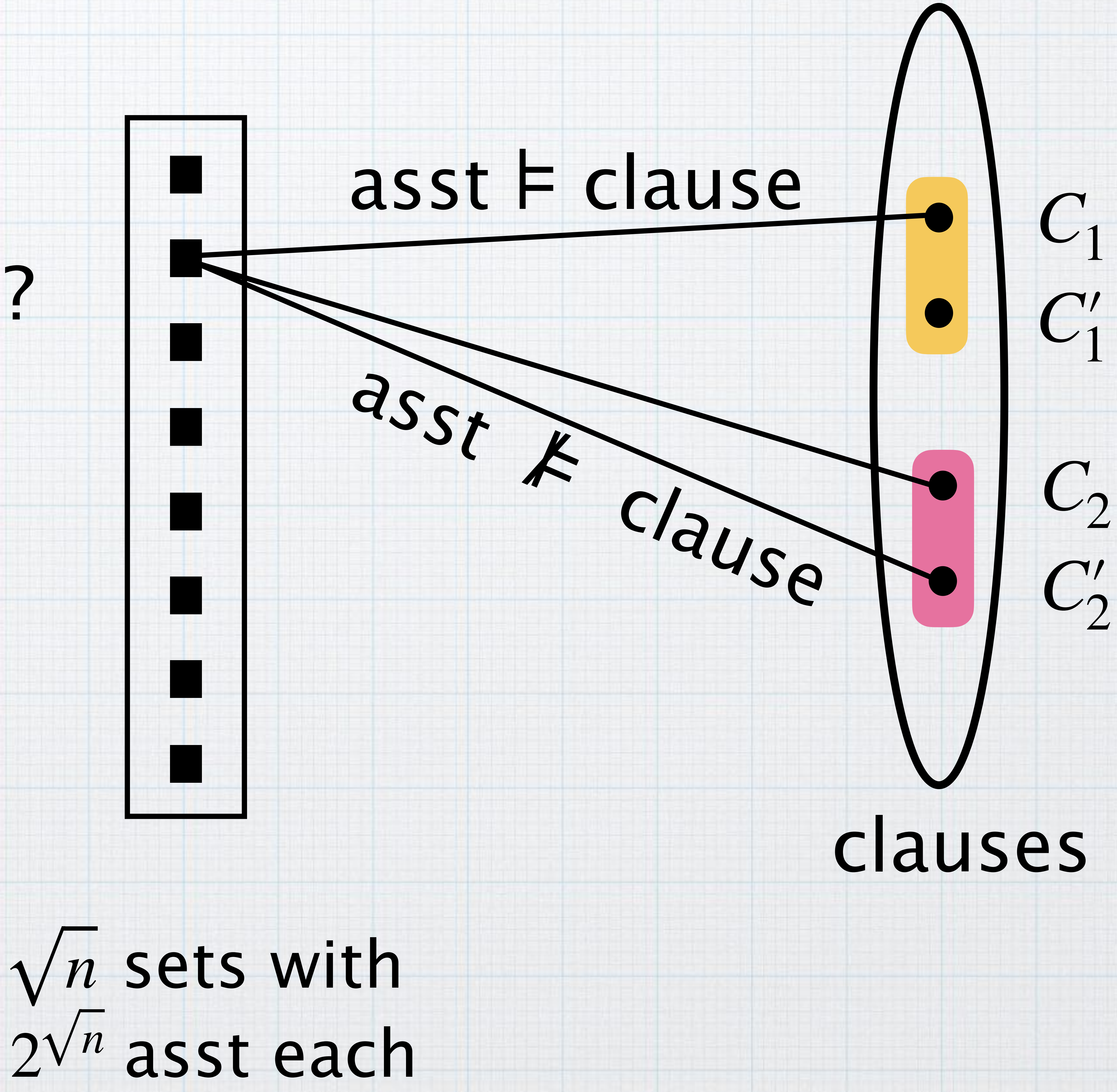

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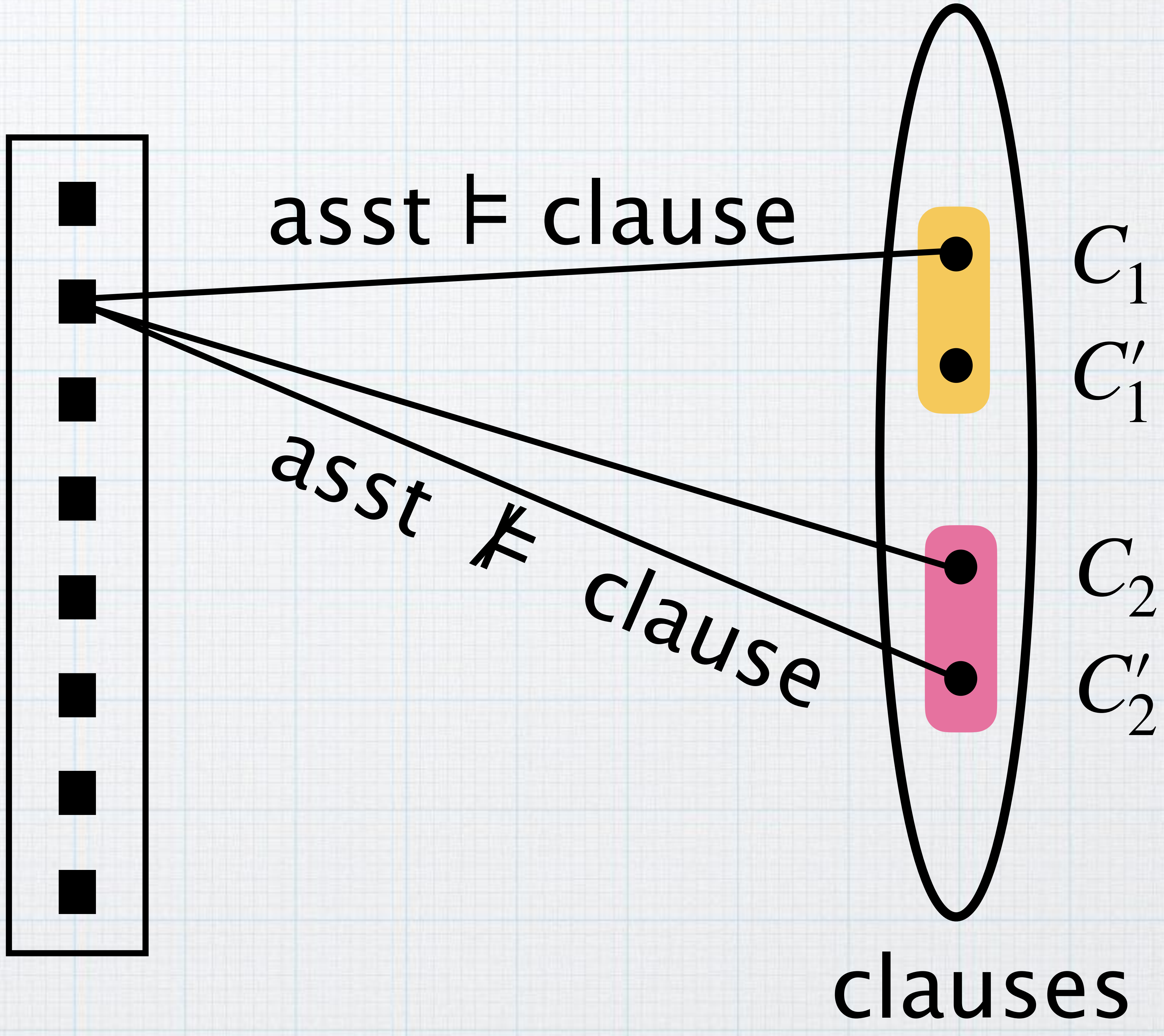
Sperner Family



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Sperner Family: Collection $\mathcal{F}$ of subsets of a universe such that for any two sets A_1, A_2 in $\mathcal{F}$ neither $A_1 \subseteq A_2$ nor $A_2 \subseteq A_1$.

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For $p = \mathcal{O}(\sqrt{n})$, $\mathcal{F}$ is of size $2^{\mathcal{O}(\sqrt{n})}$, i.e. unique set for each asst.

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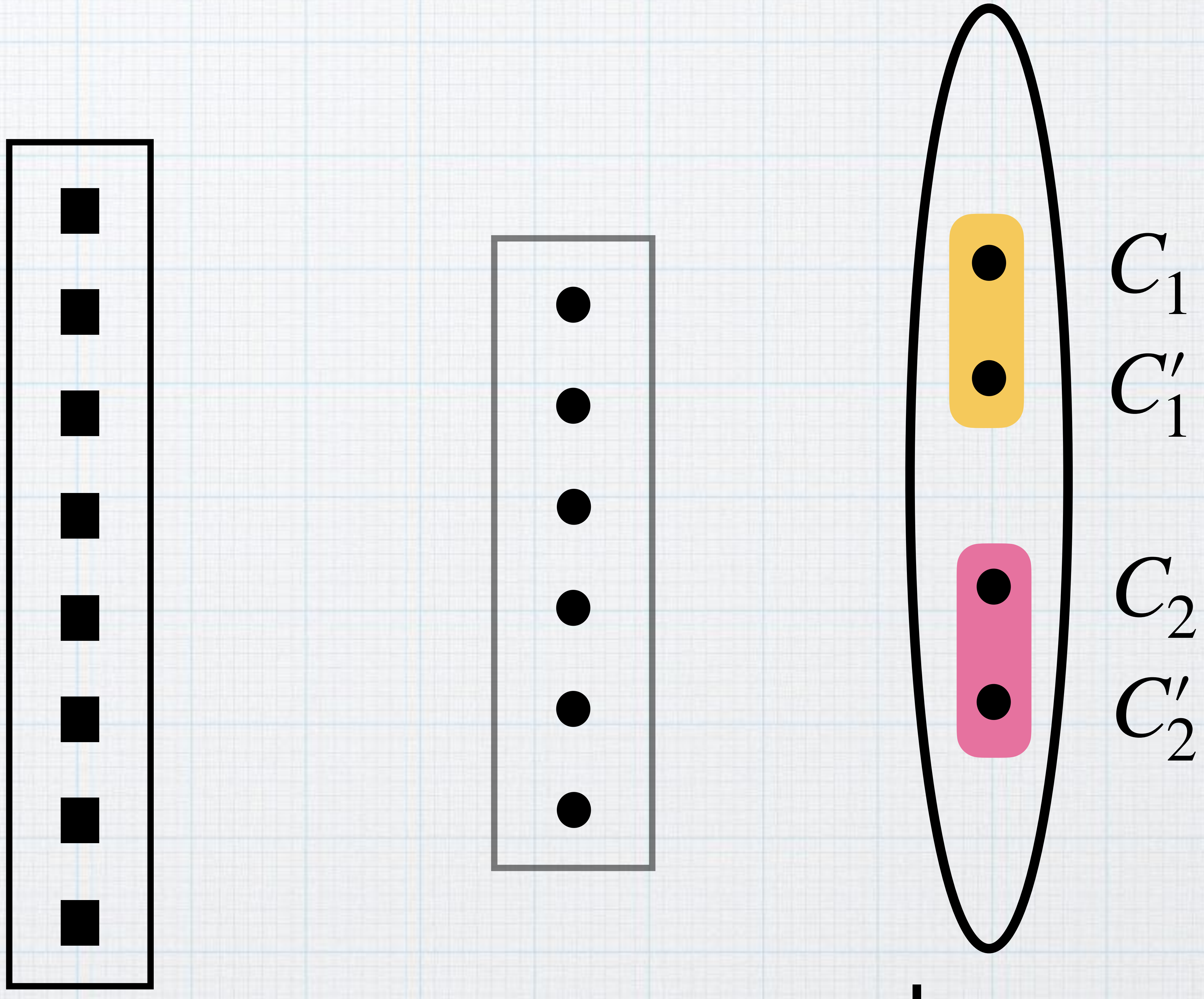

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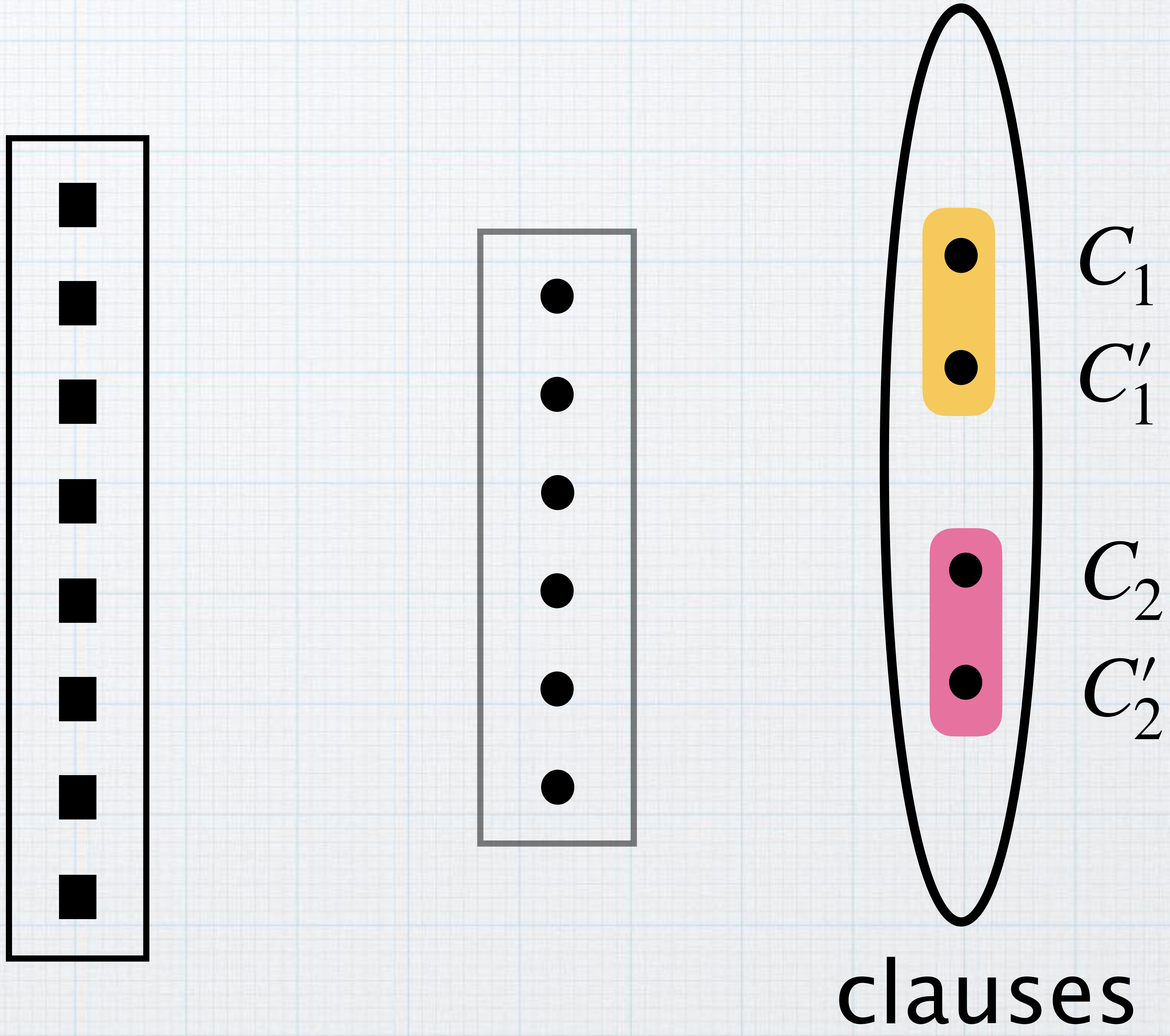
Replace direct connection via universe of Sperner Family



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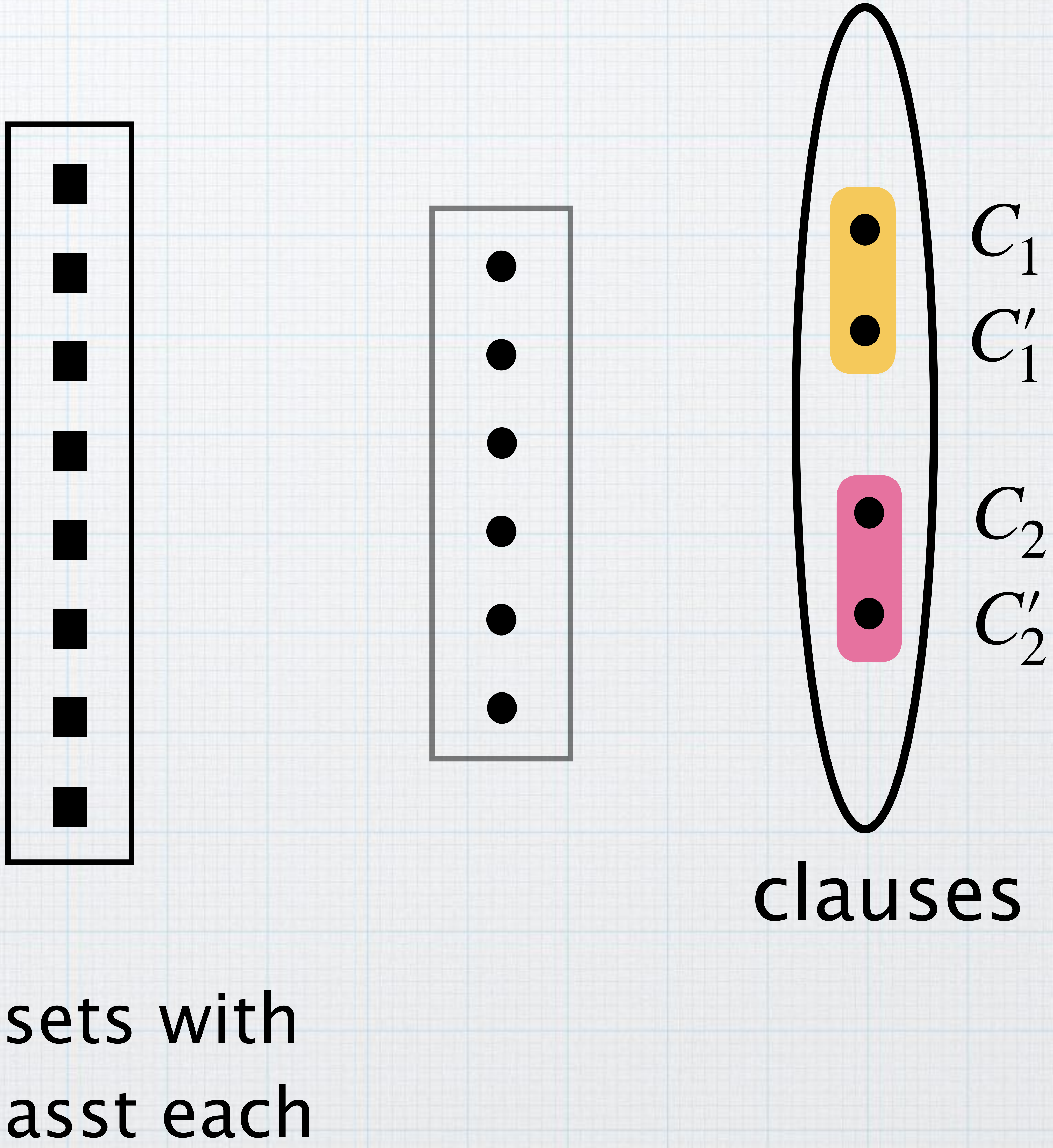

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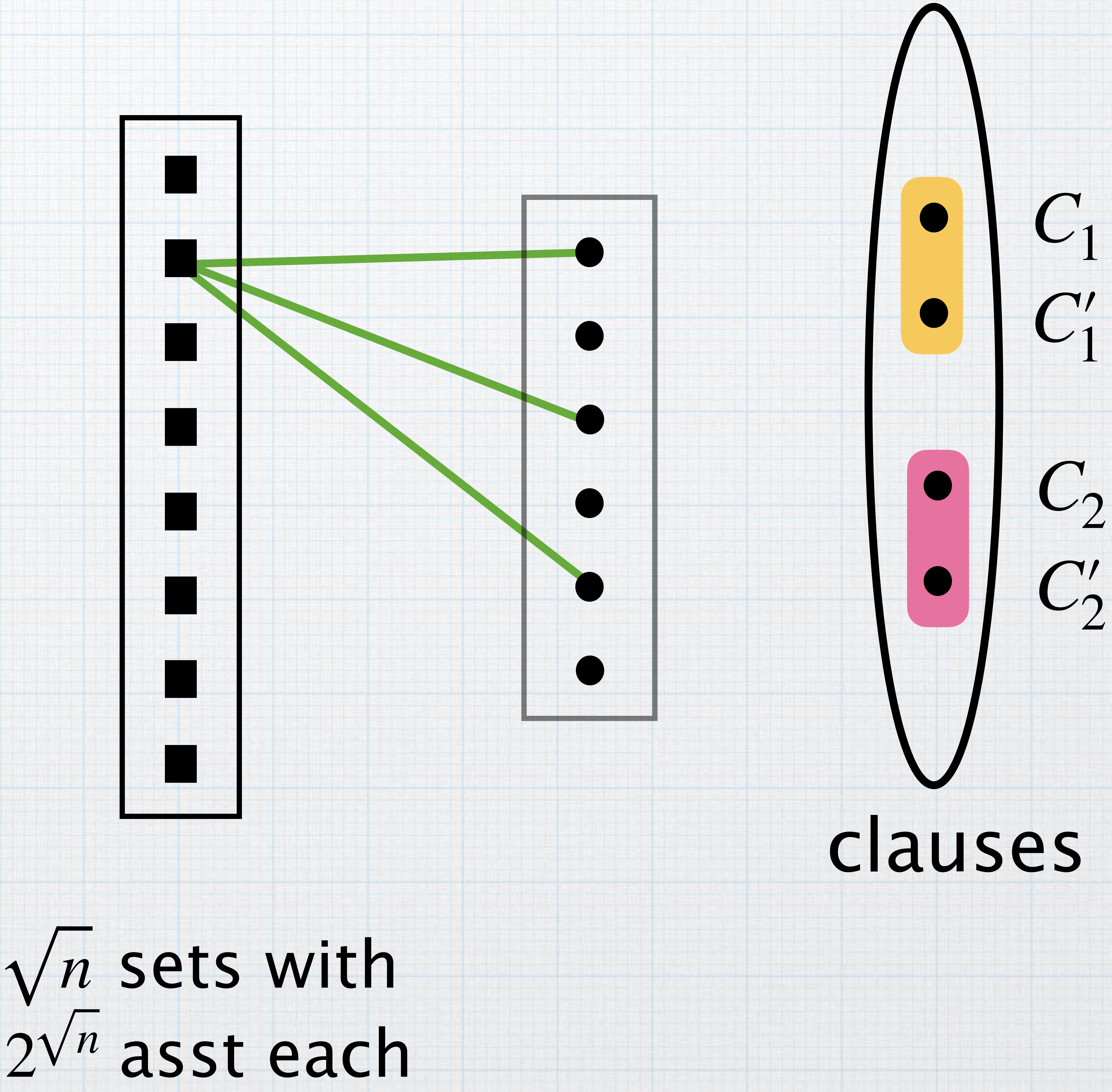

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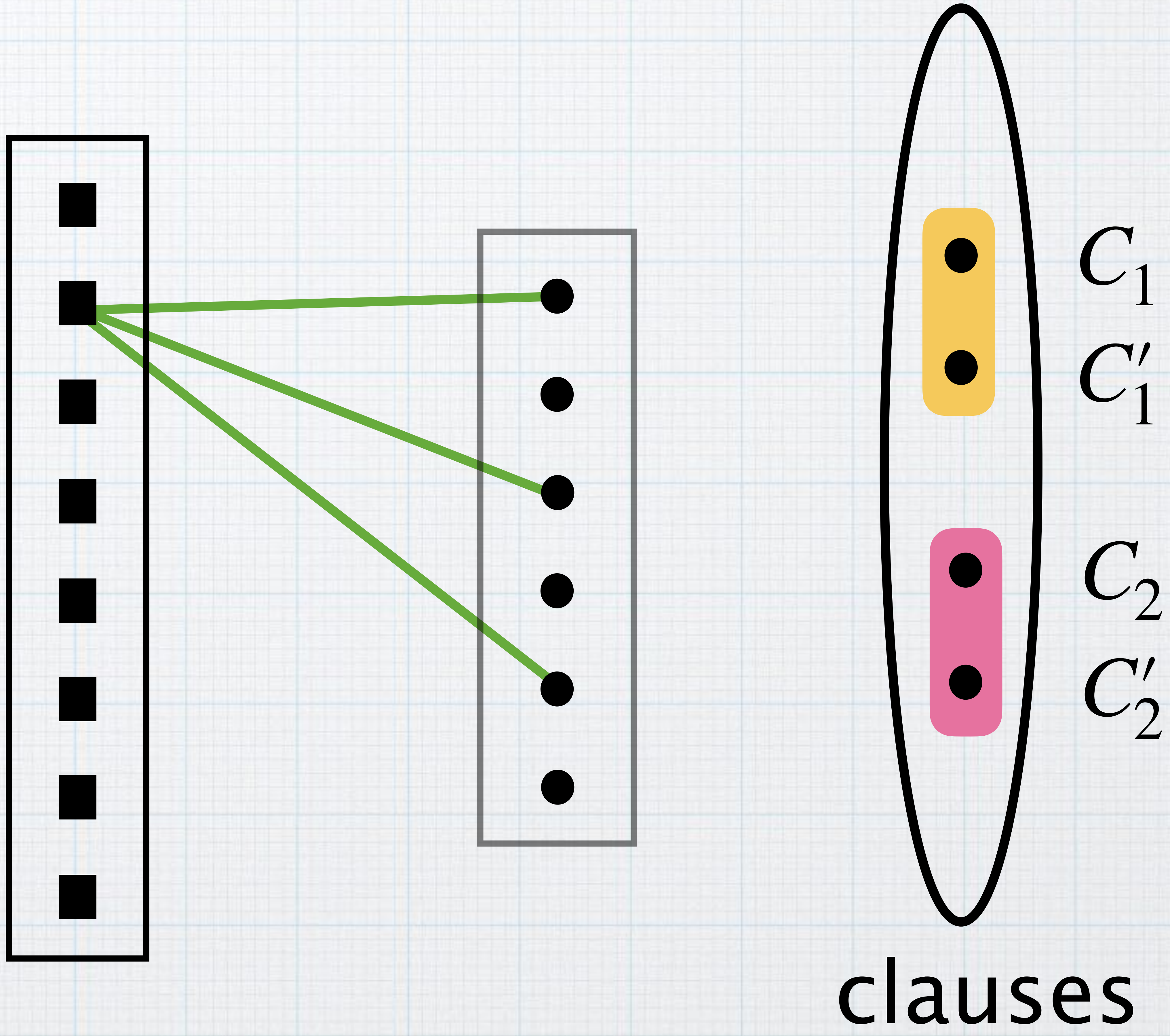

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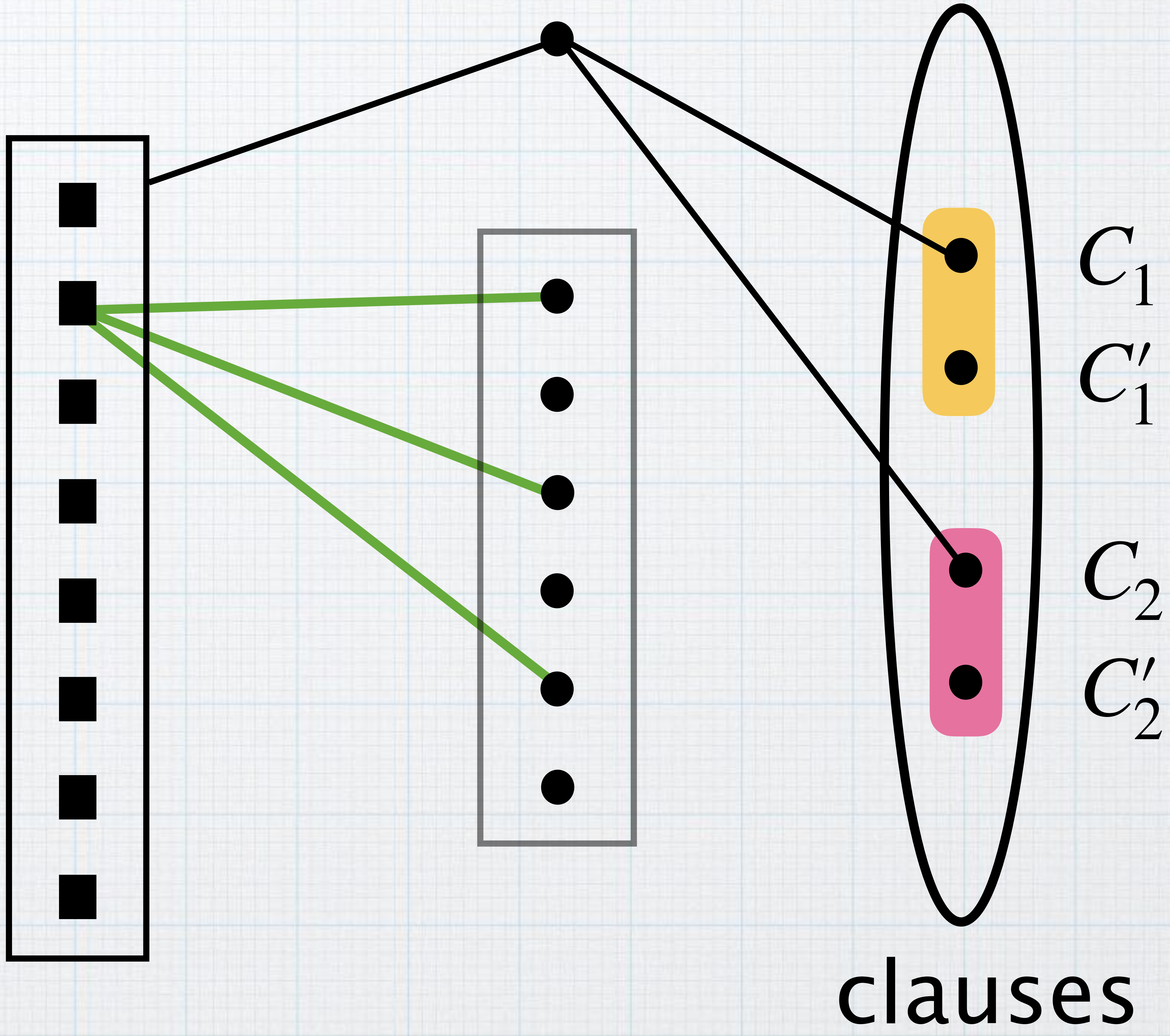

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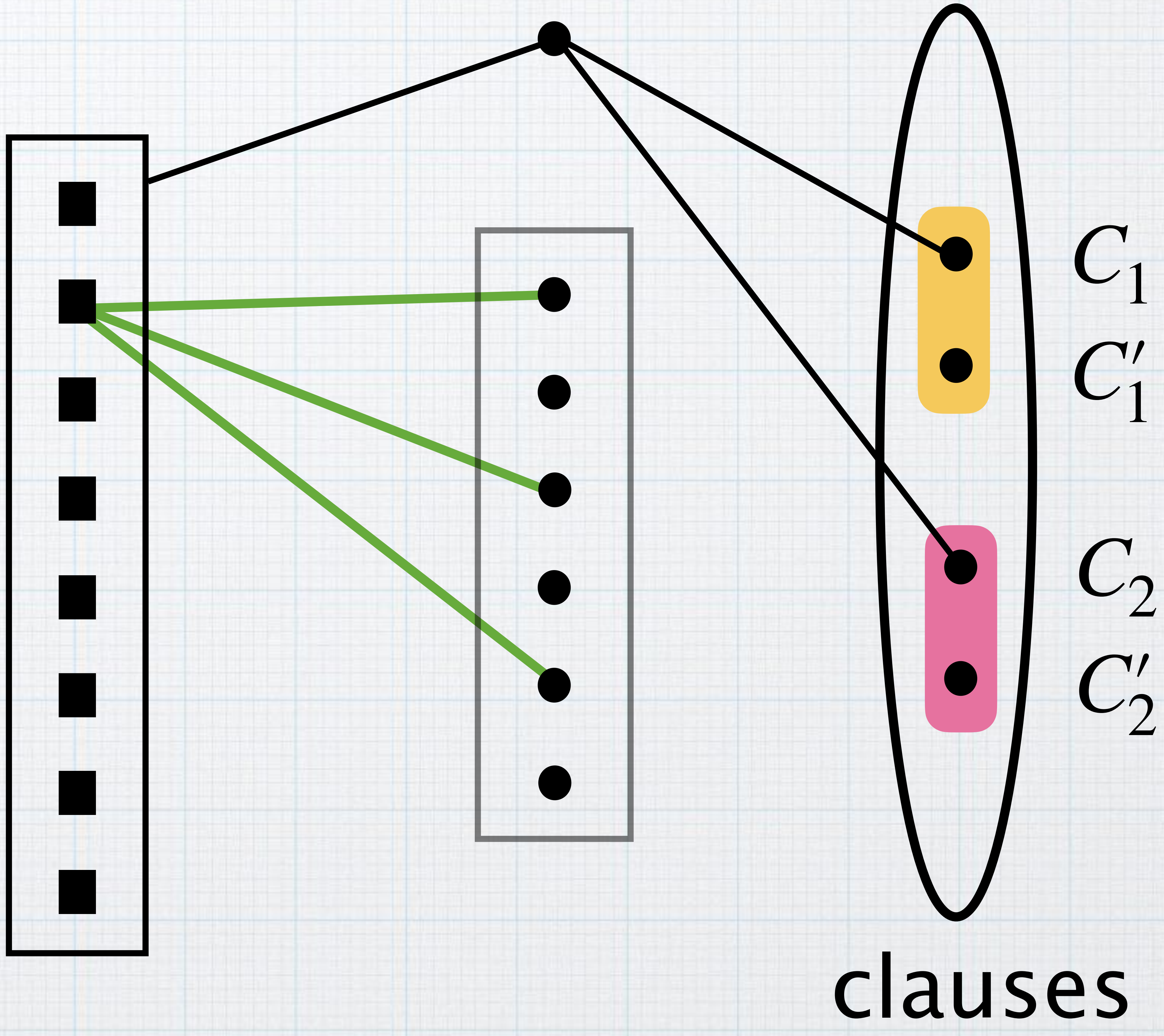

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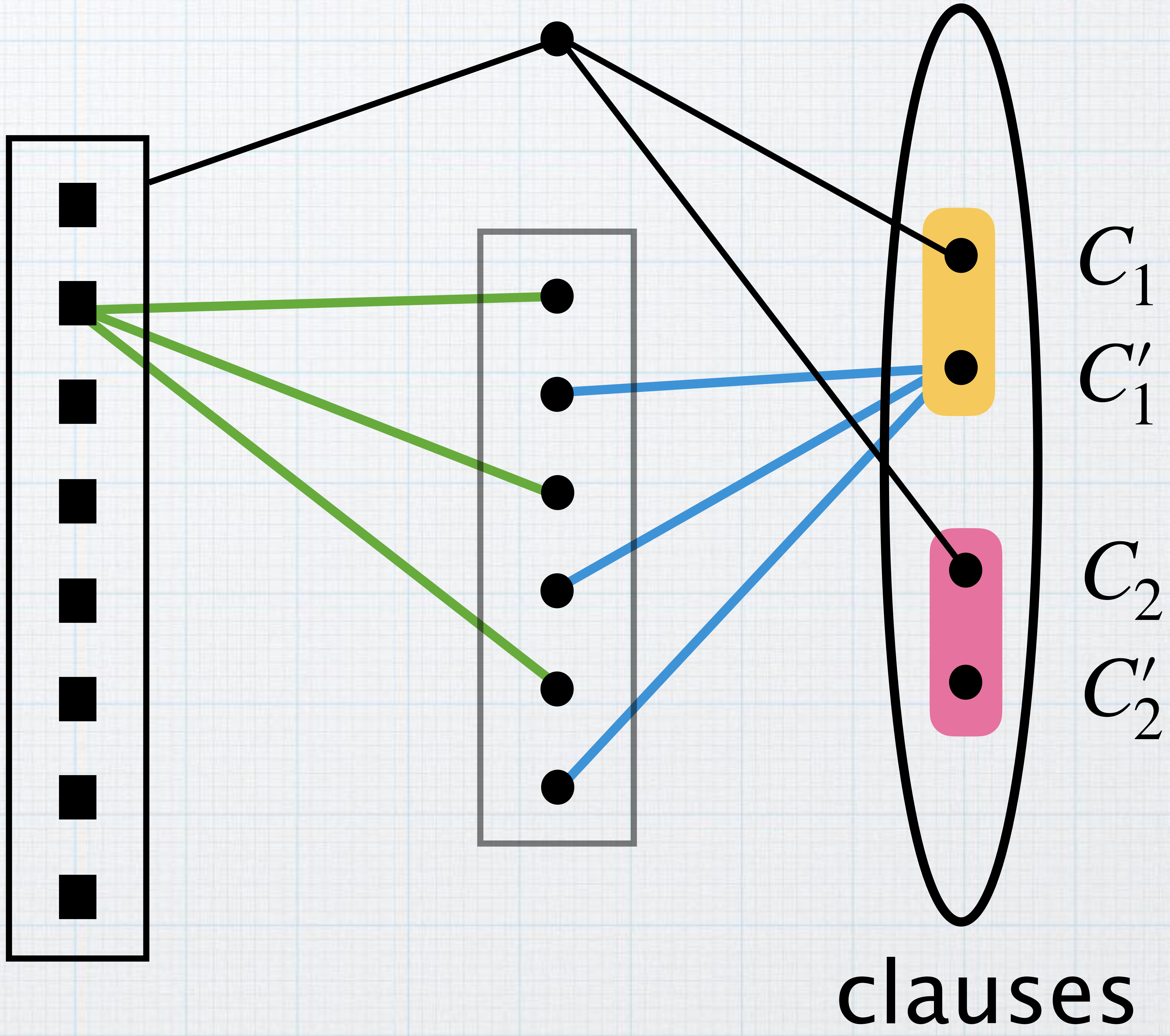

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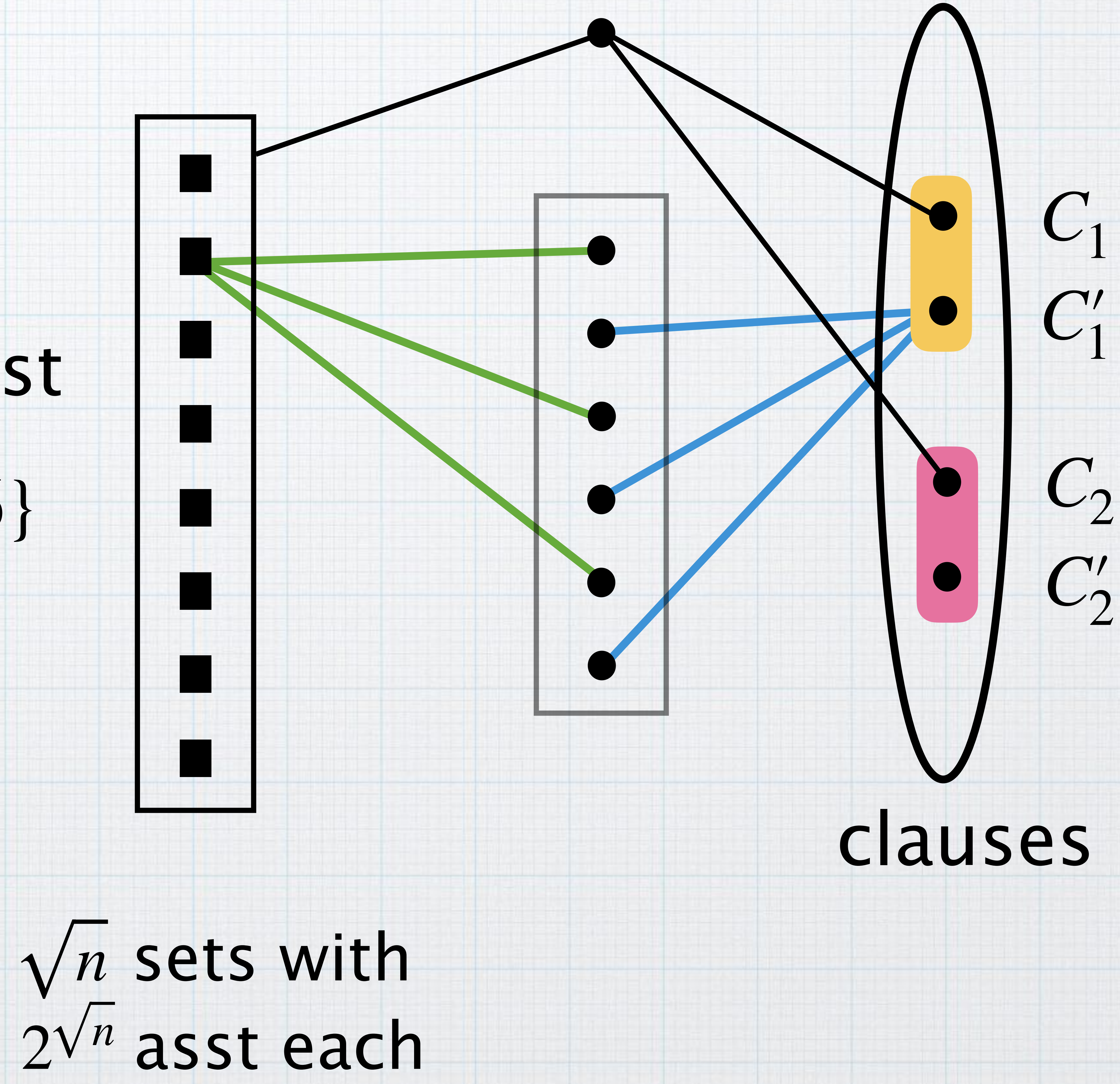

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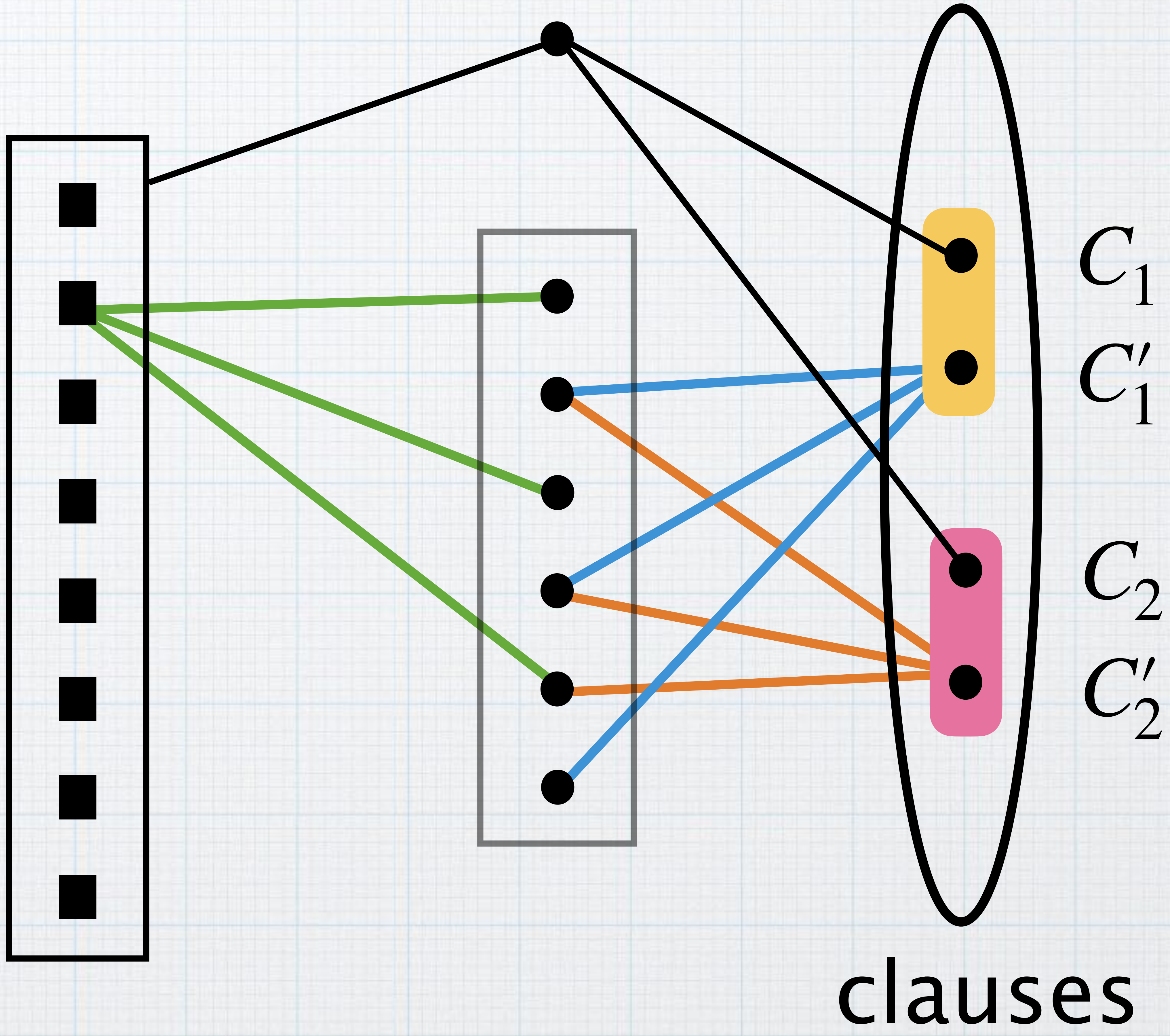

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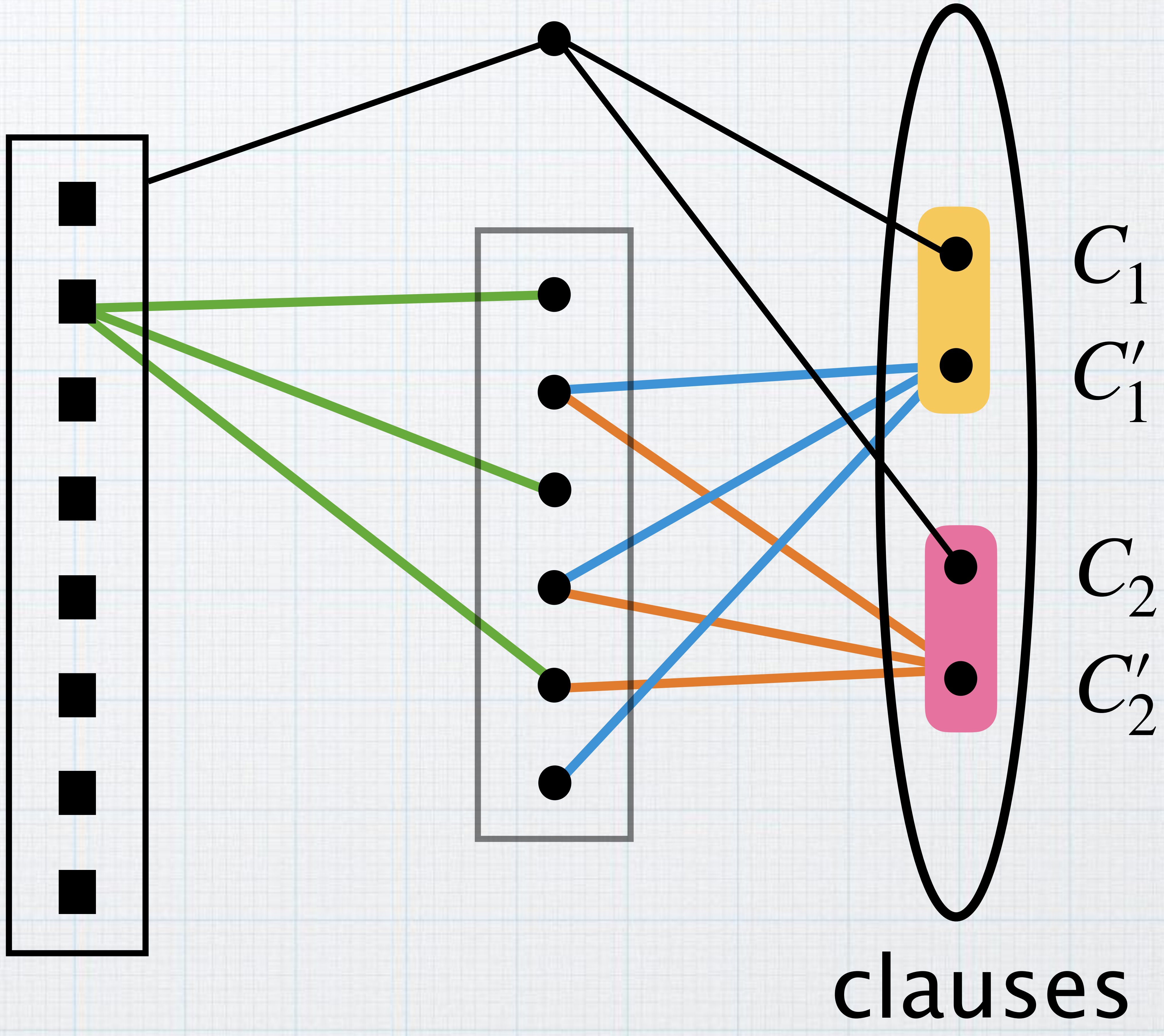

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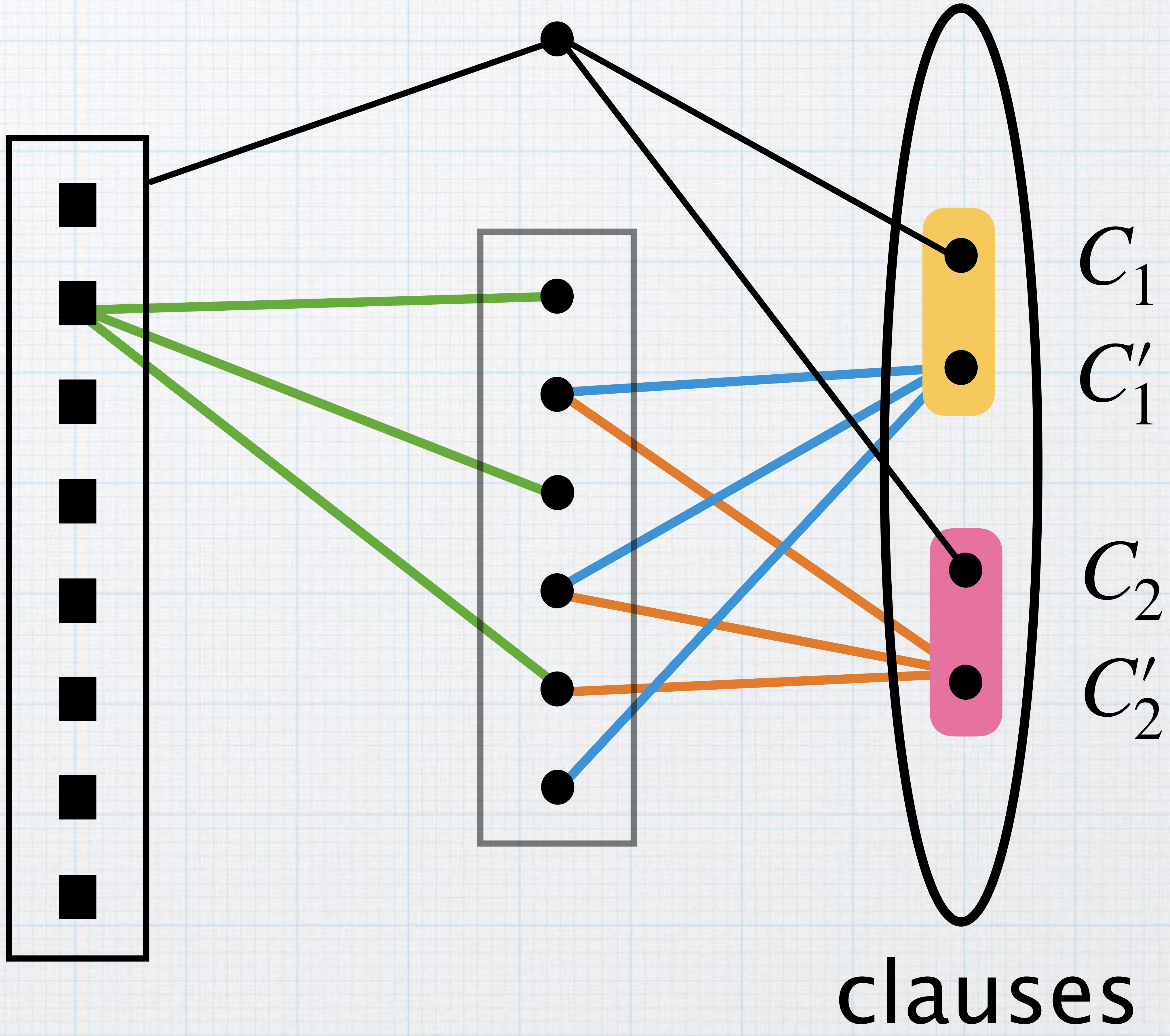

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Any other (metric graph) NP-Complete problems that admit such lower bounds?

Thank you