Minimizing the Number of Tardy Jobs with Uniform Processing Times on Parallel Machines

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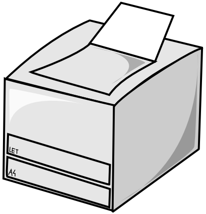
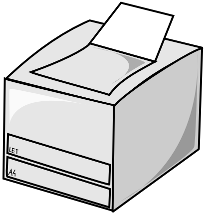
Machines



Introduction to Scheduling

Machines



Jobs

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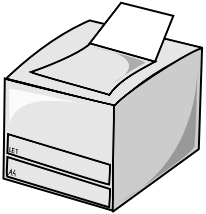

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Provide a schedule that
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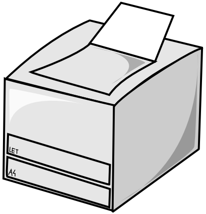

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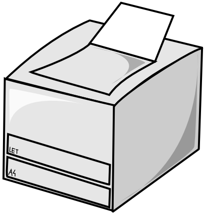

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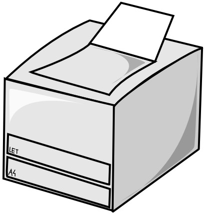

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- Jobs typically have processing times and sometimes weights, release dates, and due dates.
- Objective e.g.: print as many documents as possible before their respective deadlines.

Scheduling with Due Dates on a Single Machine

Machines

Single machine

Scheduling with Due Dates on a Single Machine

Machines	**Jobs**
Single machine	Processing time p, due date d

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Minimize number of jobs that finish after due date

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Jobs: $\boxed{p = 2, d = 4}$ $\boxed{p = 4, d = 6}$

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Background: Most Important Known Results

Single Machine Scheduling with Due Dates

Solvable in polynomial time. [Moore '68]

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  [Lenstra et al. ’77]

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Processing time p,
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“Our Scheduling Problem”

Our Scheduling Problem: Motivation

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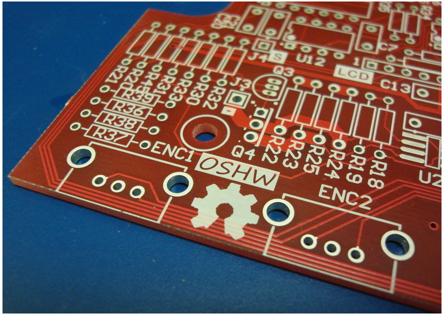

Etching of PCBs.

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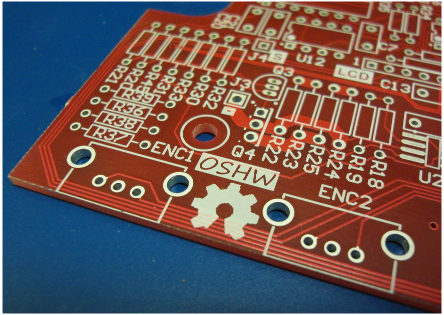

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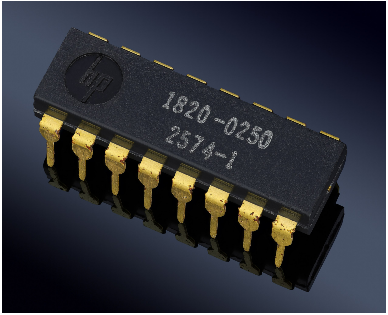

Burn-in of ICs.

Known Results

Theorem [Babtiste et al. ’00, ’04]

Our Scheduling Problem can be solved in $n^{O(m)}$ time, where n is the number of jobs and m is the number of machines.

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J Sched (2011) 14:435–444
DOI 10.1007/s10951-011-0231-3

Parallel machine problems with equal processing times: a survey

Svetlana A. Kravchenko · Frank Werner

by Kravchenko and Werner [2011]

2.1.19 Note that currently the most interesting open problems are $P \mid r_j, p_j = p \mid \sum U_j$ and $P \mid r_j, p_j = p, D_j \mid \sum w_j C_j$.

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NP-hardness vs. polynomial-time solvability: **Open!**

Open Problems in Throughput Scheduling

Jiří Sgall*

Computer Science Institute of Charles University, Faculty of Mathematics and Physics, Malostranské nám. 25, CZ-11800 Praha 1, Czech Republic
sgall@iuuk.mff.cuni.cz

Abstract. In this talk we survey the area of scheduling with the objective to maximize the throughput, i.e., the (weighted) number of completed jobs. We focus on several open problems.

by Sgall [2012]

In case of equal length jobs, the problem of maximizing the number of scheduled jobs is still far from trivial. On a single machine it is polynomial [3,7], but on parallel machines, it is known to be polynomial only if the number of machines m is a constant [1]. The following is still open:

Open problem 2. *Consider scheduling of equal-length jobs on m parallel machines with m a part of the input. Is there a polynomial-time algorithm to compute a schedule maximizing the number of completed jobs? Is it NP-hard to compute the maximal weight of scheduled jobs for some?*

Known Results

Theorem [Babtiste et al. ’00, ’04]

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ELSEVIER

Computers & Operations Research
Volume 100, December 2018, Pages 254-261

Survey in Operations Research and Management Science

Parameterized complexity of machine scheduling: 15 open problems

Matthias Mnich [a, b], René van Bevern [c, d]

by Mnich and van Bevern [2018]

Baptiste et al. (2004) showed that $Pm|r_j, p_j{=}p|\Sigma w_j U_j$ is polynomial-time solvable. According to Sgall (2012), this is open when the number of machines is not a constant. A first step to resolving this question is solving the following problem.

Open Problem 7. *Are $P|r_j, p_j{=}p|\Sigma w_j U_j$ and $P|r_j, p_j{=}p|\Sigma U_j$ fixed-parameter tractable parameterized by the number of machines?*

A negative answer to this question would also be interesting since it is open whether these problems are even NP-hard if the number m of machines is part of the input. Notably, the special

Reminder: Parameterized Algorithms and Complexity

Parameterized Problem

Each instance I is associated with a (small) parameter k.

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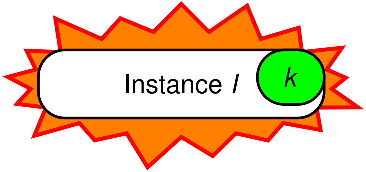
Classical worst-case running time:



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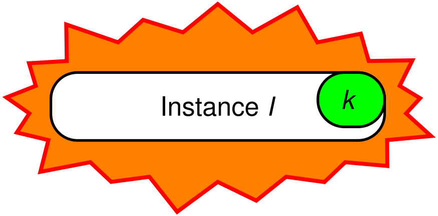

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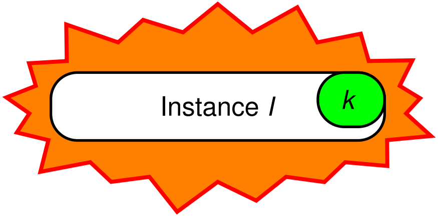

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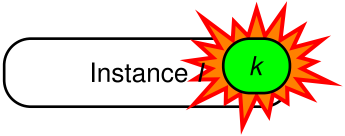

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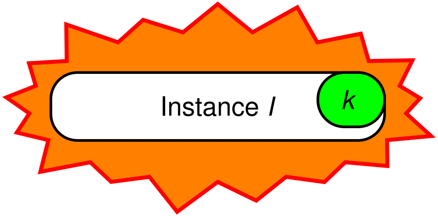

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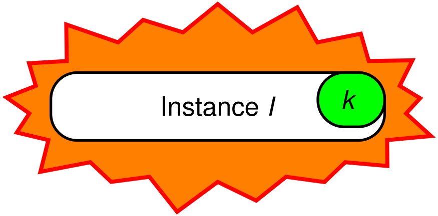

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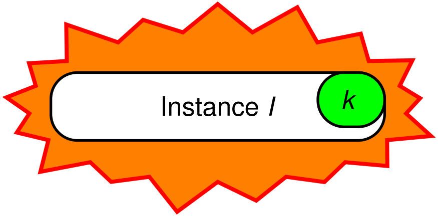

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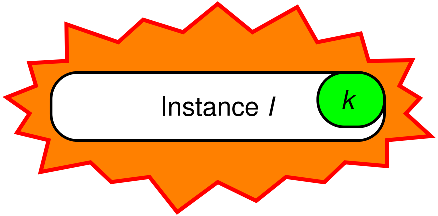

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Parameterized Hardness: W[1]-hard or W[2]-hard $\Rightarrow$ presumably not in FPT.

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Our Scheduling Problem is (strongly) **NP-hard** and **W[2]-hard** when parameterized by the number m of machines, even if all jobs have the same weight.

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Theorem 3

Our Scheduling Problem in **FPT** when parameterized by number of *different* release dates or the number of *different* due dates.

We obtain this by giving an appropriate MILP formulation of the problem.

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Reduction from **Hitting Set**.

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Input: A universe $U = \{u_1, u_2, \ldots, u_n\}$, a family $\mathscr{S} = \{S_1, S_2, \ldots, S_m\}$ of subsets of U, and an integer k.

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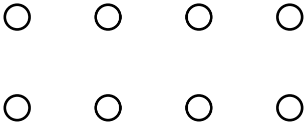

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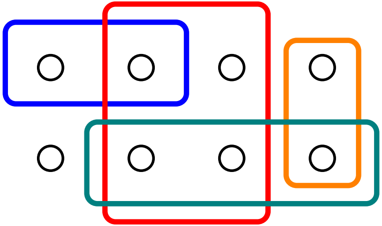
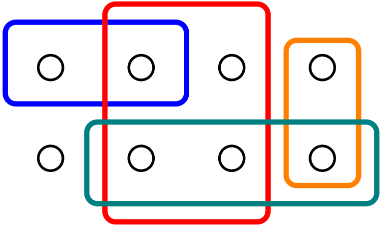


Hardness of Our Scheduling Problem I

Reduction from **Hitting Set**.



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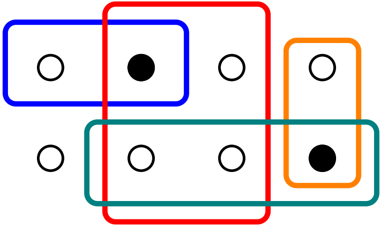

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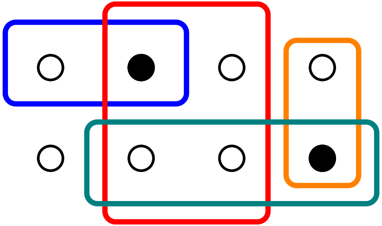

Main Idea:

Hardness of Our Scheduling Problem I

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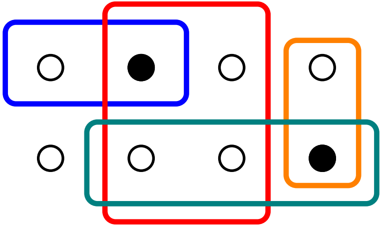

Main Idea:

- Use k machines, set $p = 2n$.

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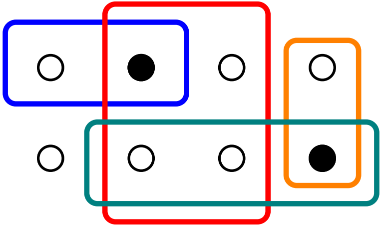

Main Idea:

- Use k machines, set $p = 2n$.
- We want to enforce that each machine only has idle time at the beginning.
  The idle time encodes an element of U that is selected for the set cover.

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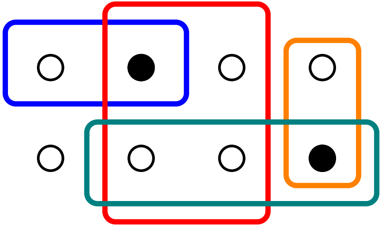

Main Idea:

- Use k machines, set $p = 2n$.
- We want to enforce that each machine only has idle time at the beginning. The idle time encodes an element of U that is selected for the set cover.
- For each set S_i we create $|S_i|$ "element jobs" and $k-1$ "dummy jobs" jobs. We want to schedule k jobs for each set S_i.

Hardness of Our Scheduling Problem II

$$U = \{u_1, u_2, u_3, u_4, u_5, u_6\},\ S_1 = \{u_1, u_2, u_3\},\ S_2 = \{u_3, u_4, u_5\},\ S_3 = \{u_1, u_5, u_6\},\ k = 2$$

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- For each set S_i, there is a “region” in the processing time.

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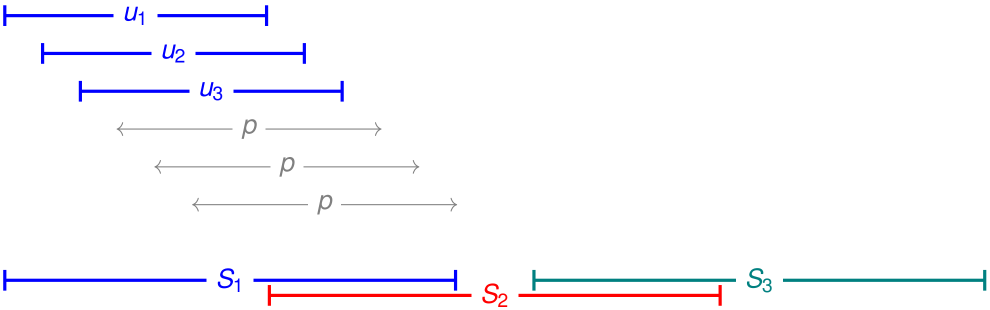

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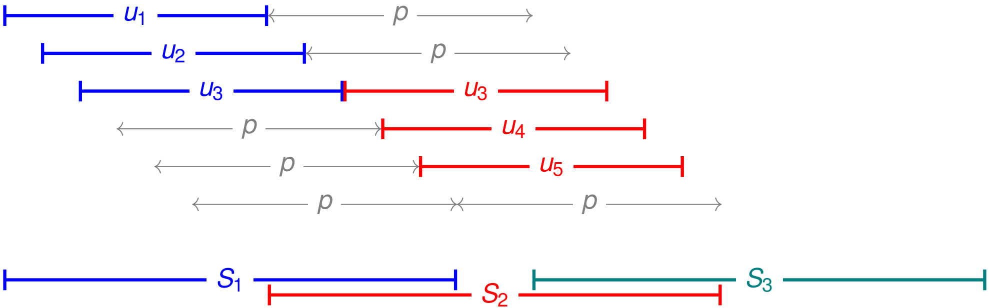

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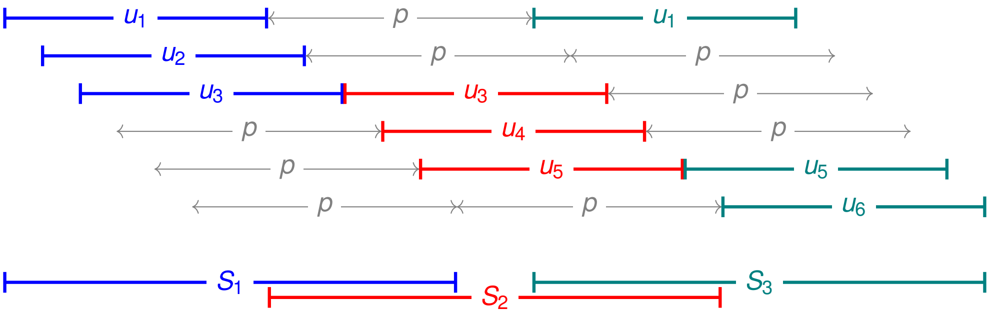

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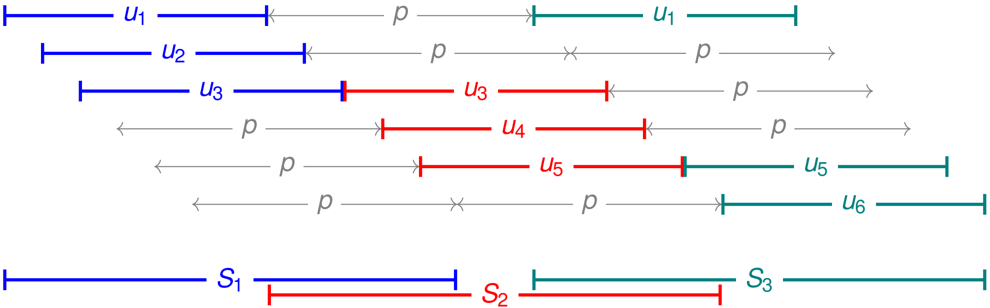
- For each set S_i, there is a “region” in the processing time.
- The element job can only be placed in a specific interval. It creates no idle time if beginning idle time encodes element.
- Dummy jobs can be placed anywhere in the region.



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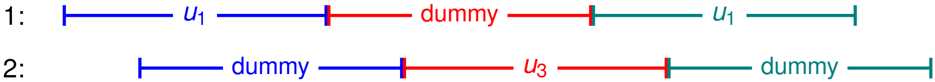

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Problem: Index of initially selected element on a machine can be increased.

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Observation:

Whenever “cheating” happens, the index of at least one encoded element is increased. This can only happen $k \cdot (n - 1)$ times.

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Whenever “cheating” happens, the index of at least one encoded element is increased. This can only happen $k \cdot (n-1)$ times. $\Rightarrow$ Repeat sketched construction $k \cdot (n-1)+1$ times.

Conclusion

Summary:

We give an almost complete picture of the parameterized complexity of **Our Scheduling Problem** wrt.

- the processing time p,
- the nr. $w_\#$ of weights,
- the nr. $d_\#$ of due dates, and
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Thank you!