Multivariate Exploration of Metric Dilation

Aritra Banik[1], Fedor V. Fomin[2], Petr A. Golovach[2], Tanmay Inamdar[3], Satyabrata Jana[4], Saket Saurabh[2,5]

[1]NISER Bhubaneswar, India

[2]UiB, Bergen, Norway [3]IIT Jodhpur, India

[4]University of Warwick, UK

[5]IMSc, Chennai, India

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Outline

Basic Definitions

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- G **has dialation** t

  for each pair $u, v \in V(G)$,

  $$d_G(u, v) \leq t \cdot d_M(u, v)$$

Example

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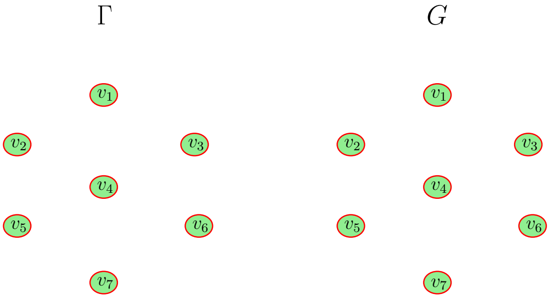

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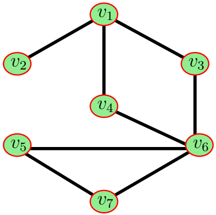

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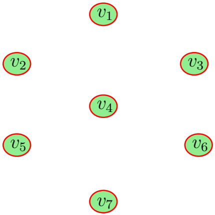

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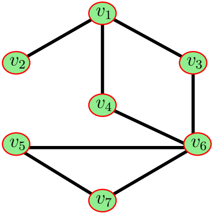

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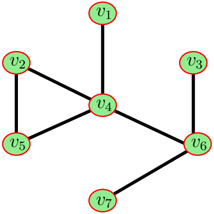

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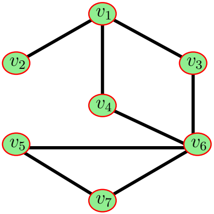

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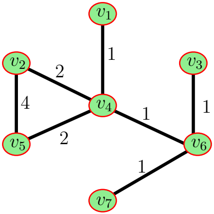

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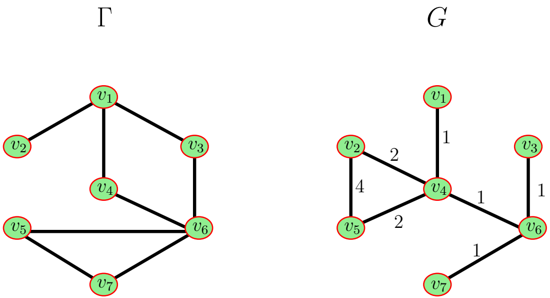

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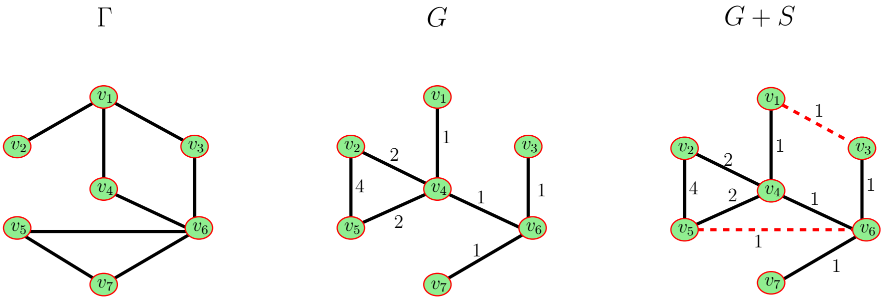

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- ☞ For which pairs, $(\mathcal{G}, \mathcal{M})$, Dilation t-Augmentation admit an FPT algorithm?

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☞ Dilation t-Augmentation **can not** be solved in time $g(k) \cdot n^{\mathcal{O}(t)}$?

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- ☞ Any connected weighted graph with n vertices contains a t-spanner with $\mathcal{O}(tn^{1+\frac{2}{t+1}})$ edges [Baswana and Sen.; ICALP '03]
- ☞ Deciding whether a geometric graph contains a t-spanner with $\leq k$ edges is NP-hard? [Gudmundsson and Michiel Smid .; SWAT '06]

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Our Results

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G	Metric (Γ)	Parameter(s)	t	Complexity
$\mathcal{K}_{d,d}$-free	General	$k + d$	2	FPT
Star forest	General	k	3	W[1]-hard
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General	Tree	k	2	P
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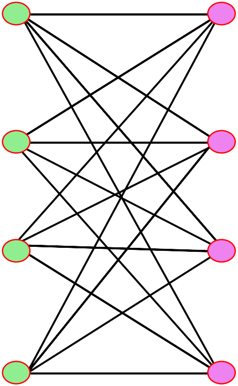

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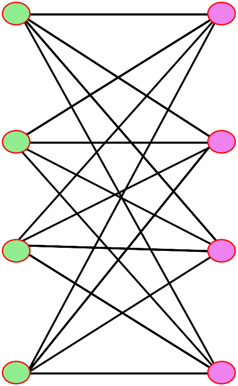

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It includes trees, planar graphs, H minor-free graphs, graphs of bounded expansion, nowhere dense graphs, and graphs of bounded degeneracy

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☞ **We only need to focus on adjacent conflicts**

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Bound the number of conflict pairs.

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Definition (Conflict Graph $\mathbb{C}$)

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- Towards Bound $|E(\mathbb{C})|$,

  We bound the degree of each vertex in R in $\mathbb{C}$.

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- So in each step, the budget k reduces by at least one
- Stop when $k = 0$ or degree of each vertex of R gets bounded.

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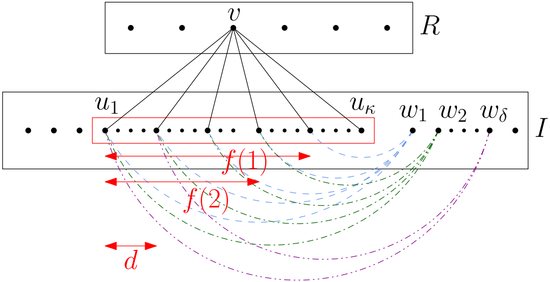

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- Guess the edges in S which is in b/w W' and R (due to invariant).
- Add W' to the vertex cover R

How is f?

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$$f(i) = \begin{cases} d & \text{if } i = d \\ d \cdot k + k^2 + k & \text{if } i = d - 1 \\ d \cdot k^{d-i} + k^{d-i+1} + \{2 \cdot \sum_{j=2}^{d-i} k^j\} + k & \text{if } 0 \leq i \leq d - 2 \end{cases}$$

- For any $1 \leq i \leq d$, $f(i-1) = (f(i) + k) \cdot k + k$.

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Annotated Dilation 2-Augmentation

Input: G, Γ, k, R, V_c. Here $|R| \leq 5k$ and $|V_c| = 5k \cdot f(0)$
Task: Find a dilation 2-augmentation set S such that every edge in S has at least one end-point outside R.

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Definition For each $A, B \subseteq V_c$ with $A \cap B = \emptyset$, let $O(A, B)$ denote the set of vertices $v \in O$ satisfying

1. $A = \{u \ : \ u \in V_c,\ d_G(u, v) = 1\}$, and
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- Now we **guess** end-points of the solution edges.

Final Result

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Theorem

DILATION 2-AUGMENTATION can be solved in time $f(k,d) \cdot n^{\mathcal{O}(1)}$ when G is a $\mathcal{K}_{d,d}$-free graph for any $d \in \mathbb{N}$.

Outline

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- Other considerable special graph classes include intersection graphs such as interval graphs, unit-disk graphs, disk-graphs, string graphs.
- Exploring these problems from the perspective of FPT-approximation

Thank you!