The Complexity of Learning LTL, CTL and ATL Formulas

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Context

General setting

- Derive formal specification from observations of system behavior
- Formal specifications: written as **temporal logic** formulas
- Synthesis: in a **passive learning** context
- Goal: find tractable cases for the learning decision problem

Linear Temporal Logic (LTL)

Basic characteristics LTL (Pnueli, 1977)

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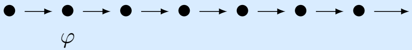

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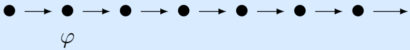

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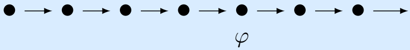

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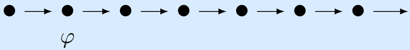

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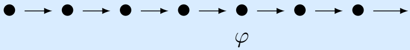
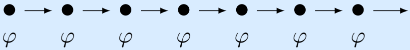


  - (Globally) G φ:


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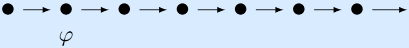

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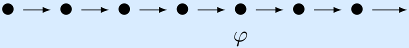

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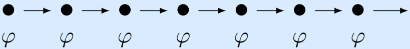
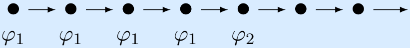


  - (Until) φ_1 U φ_2:



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Example of evaluation

For $\varphi_1 := \mathsf{F}\ (\mathsf{Blck} \wedge \mathsf{X}\ \mathsf{Blue})$:

$$\pi \models \varphi_1$$

For $\varphi_2 := \mathsf{Red}\ \mathsf{U}\ \mathsf{Prpl}$:

$$\pi \models \varphi_2$$

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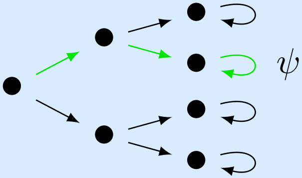

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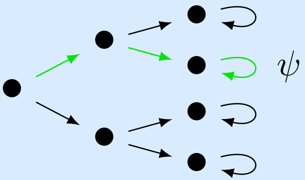
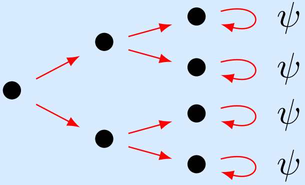


  - (Universal) $\forall\ \psi$:



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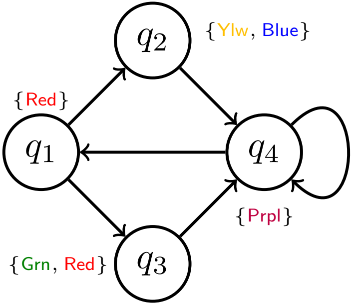

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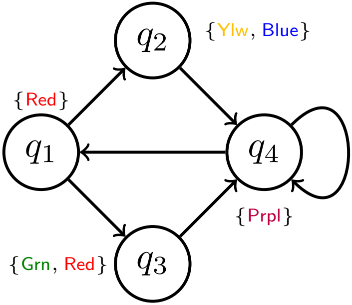

Example

- For $\varphi_1 := \exists\, \mathbf{X}$ Blue:
  - $q_1 \models \varphi_1$
  - $q_2, q_3, q_4 \not\models \varphi_1$
- For $\varphi_2 := \forall\, \mathbf{F}$ Prpl:
  - $q_1, q_2, q_3, q_4 \models \varphi_2$

Alternating-time Temporal Logic (ATL)

Basic characteristics ATL_k (Alur, Henzinger, Kupferman, 2002)

- Generalizes CTL
- Evaluated on labeled k-player games
- Propositional logic $+$ (Strategic quantification) temporal operators

Passive learning problem

Logic $\mathcal{L} \in \{\mathsf{LTL}, \mathsf{CTL}, \mathsf{ATL}_k \mid k \in \mathbb{N}_1\}$, subset of operators $\mathsf{O} \subseteq \mathsf{Op}$

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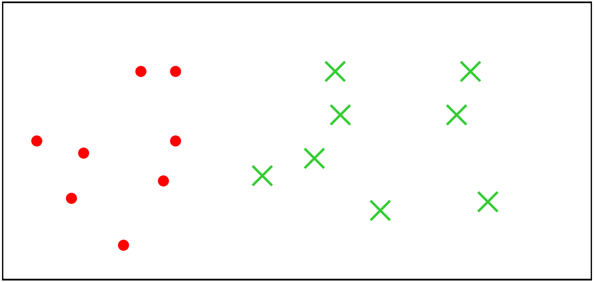

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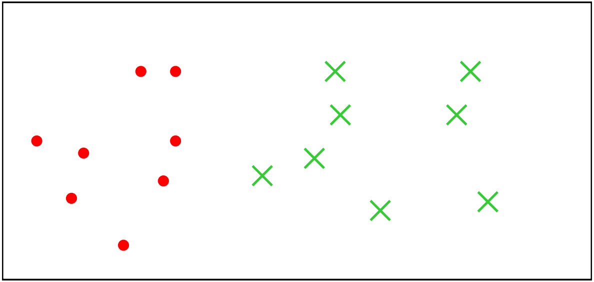

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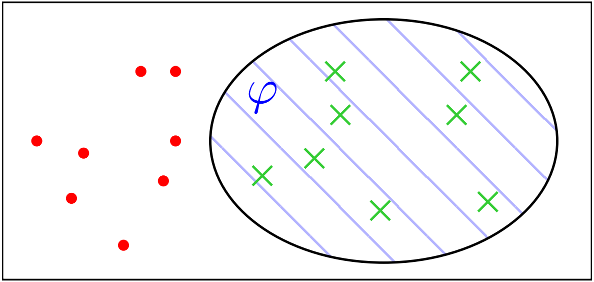

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Bound in unary

- Why a bound? Prevents overfitting
- Why in unary? Generating a formula requires explicitly writing it

Goal

Study the complexity and look for tractable cases

The decision problem $\mathsf{PvLn}_{\mathcal{L}}(\mathsf{O})$ where:

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Central related work[1]

The complexity of LTL learning with finite words

[1]Learning temporal formulas from examples is hard, C. Mascle, N. Fijalkow, G. Lagarde, arXiv 2023.

Two straightforward observations

Problems in NP

For all $\mathcal{L} \in \{\mathsf{LTL}, \mathsf{CTL}, \mathsf{ATL}\}$ and subsets of operators $\mathsf{O} \subseteq \mathsf{Op}$:

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First summary

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CTL		
ATL_2		
ATL		

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ATL		NP-c		

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- Input: $l \in \mathbb{N}_1$, subsets $C = C_1, \ldots, C_n \subseteq [1, \ldots, l]$, $k \leq l$
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3. Define a $-$instance that $\phi^l(H)$ accepts iff $|H| \geq k + 1$

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2. Define the bound $B \in \mathbb{N}$ and $(+, -)$ instances that a formula separates iff it is equal to some $\phi^l(H)$, for $H \subseteq [1, \ldots, l]$
3. Define a $-$instance that $\phi^l(H)$ accepts iff $|H| \geq k + 1$
4. For $i \leq n$, define a $+$instance that $\phi^l(H)$ accepts iff $H \cap C_i \neq \emptyset$

Abstract recipe

Hitting set problem

- Input: $l \in \mathbb{N}_1$, subsets $C = C_1, \ldots, C_n \subseteq [1, \ldots, l]$, $k \leq l$
- Output: Yes iff there is $H \subseteq [1, \ldots, l]$ such that:
  - $|H| \leq k$
  - for all $1 \leq i \leq n$, we have $H \cap C_i \neq \emptyset$

Pick a logic $\mathcal{L}$ and set of operators $\mathsf{O} \subseteq \mathsf{Op}$

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Application: when $\mathcal{L} = \mathsf{CTL}$, $\mathsf{X} \in \mathsf{O}$

Step 1: type of formulas

For $l \in \mathbb{N}_1$ and $H \subseteq [1, \ldots, l]$:

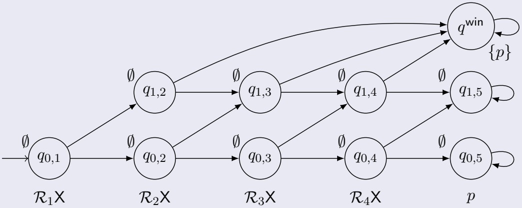
$$\phi^l(H) := \mathcal{R}_1\mathsf{X}\, \mathcal{R}_2\mathsf{X}\ \ldots\ \mathcal{R}_l\mathsf{X}\, p$$

with $\mathcal{R}_i \in \{\exists, \forall\}$ and $\mathcal{R}_i = \exists$ iff $i \in H$.

Application: when $\mathcal{L} = \mathsf{CTL}$, $\mathsf{X} \in \mathsf{O}$

Step 3: bounding the size of H = bounding the number of $\exists$

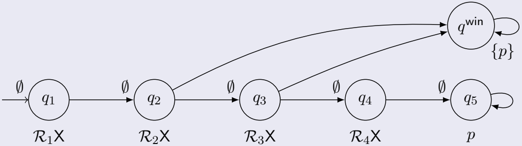
- When $l = 4$ and $k = 1$, define a $-$instance
- Kripke structure $K_{4,2\leq}$:



- We have $K_{4,2\leq} \models \phi^4(H)$ iff $|H| \geq 2$

Application: when $\mathcal{L} = \mathsf{CTL}$, $\mathsf{X} \in \mathsf{O}$

Step 4: H intersects the set C_i = an $\exists$ index is in C_i

- When $l = 4$ and $C_i = \{2, 3\} \subseteq [1, \ldots, 4]$, define a $+$instance
- Kripke structure K_{4,C_i}: 

- We have $K_{4,C_i} \models \phi^4(H)$ iff $H \cap C_i = \emptyset$

Summary of our results

Summary

	Unlimited use of binary operators	Limited use of binary operators		
		With $\mathbf{X}$	Without $\mathbf{X}$	
			With $\mathbf{F}, \mathbf{G}$	Only $\mathbf{F}$ or only $\mathbf{G}$
LTL	NP-c	L		
CTL		NP-c	NL-c	
ATL_2		NP-c		P-c
ATL		NP-c		

Take home message and future work

The passive learning of temporal logic formulas

- NP-hard as soon as there is an unlimited use of binary operators
- By limiting the use of binary operators, we recover some tractable cases

Take home message and future work

The passive learning of temporal logic formulas

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Various questions could be explored

- Allow a restricted use of binary temporal operators
- Look for tractable approximation algorithms
- What if the bound is written in binary?
- ...