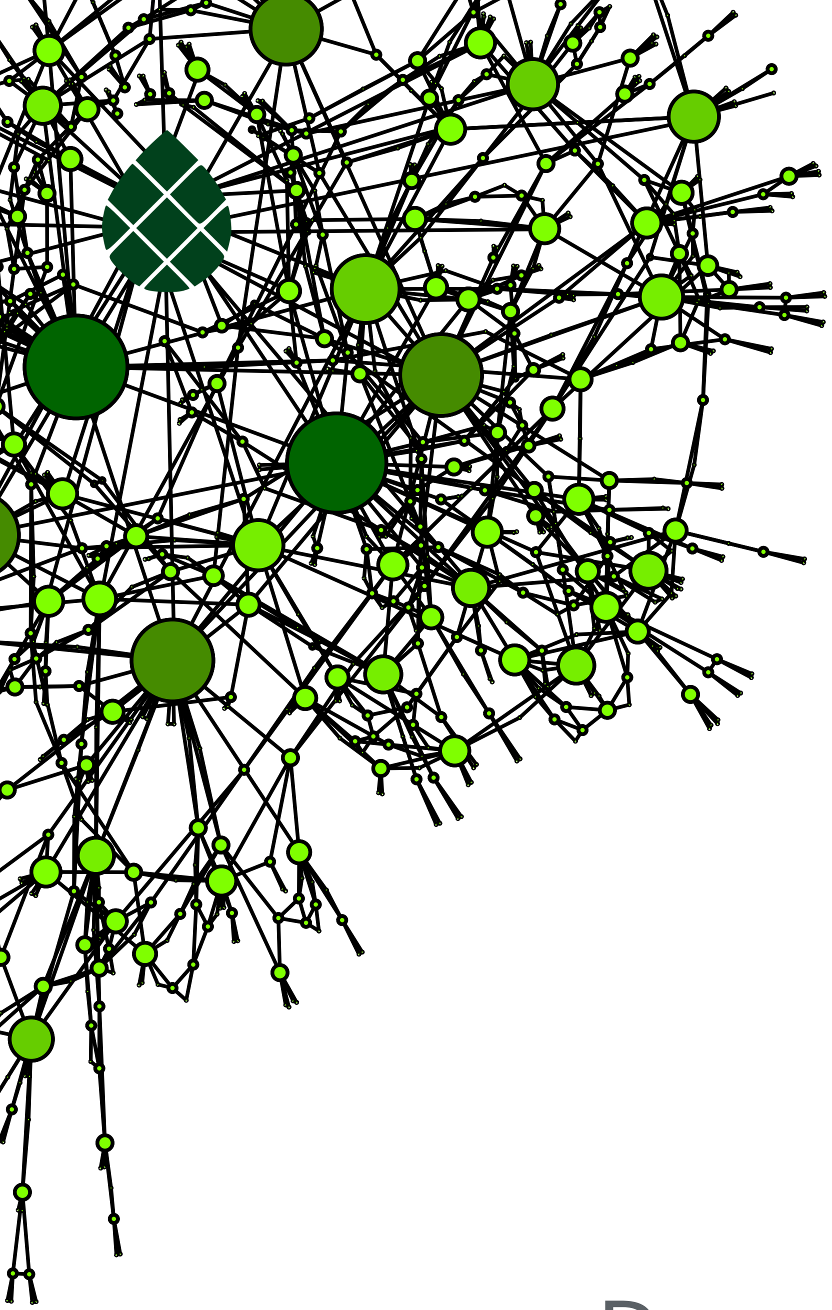
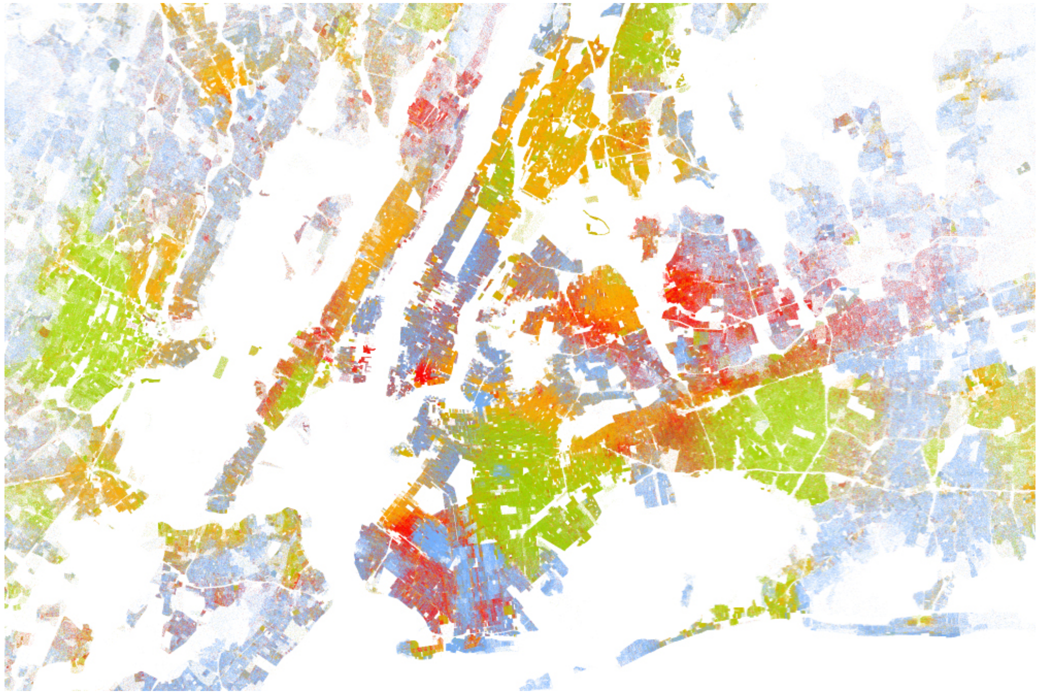
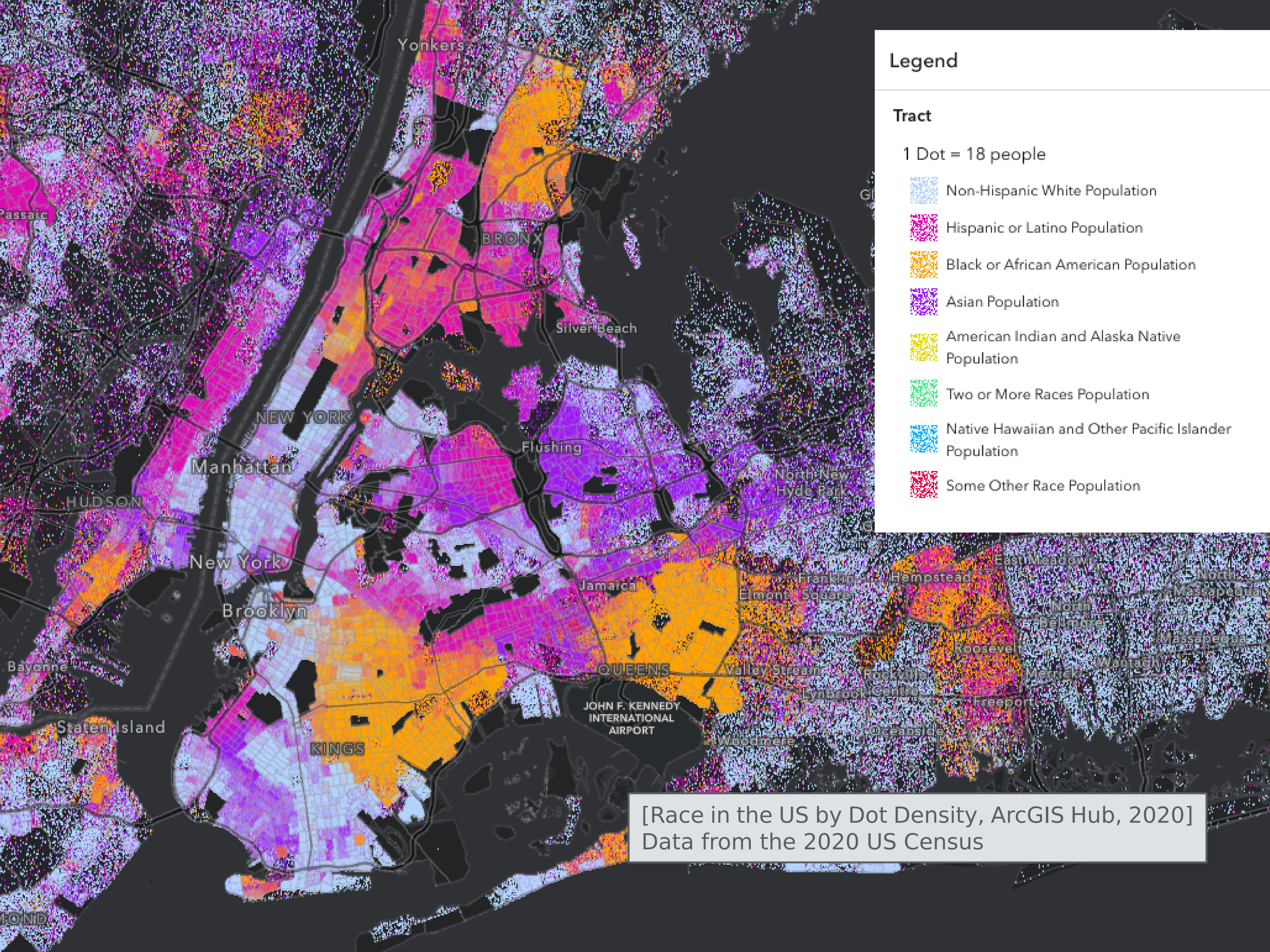


Universität Augsburg
Fakultät für Angewandte Informatik

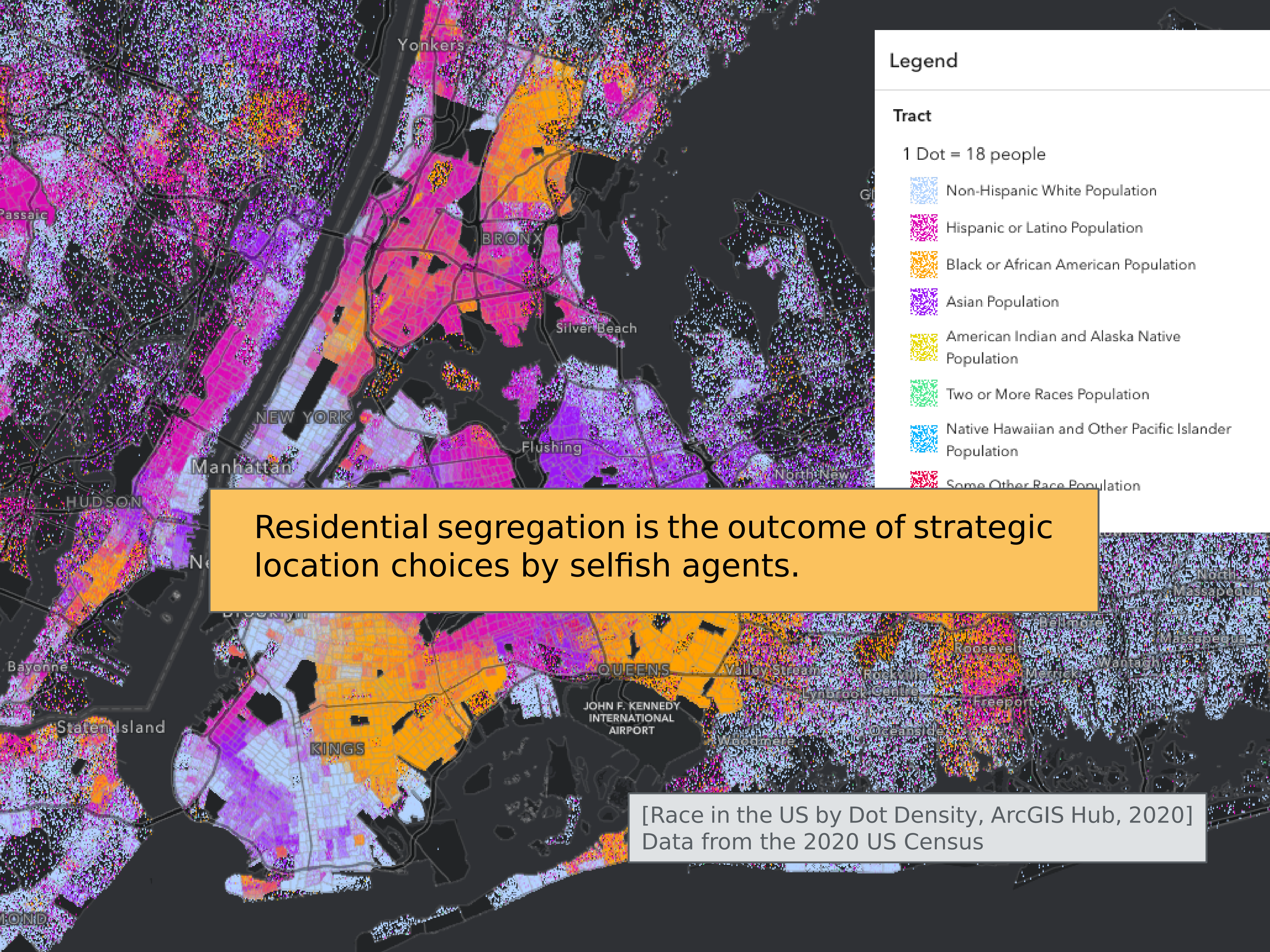


Dynamics of Schelling Games

Pascal Lenzner



[Race in the US by Dot Density, ArcGIS Hub, 2020]
Data from the 2020 US Census

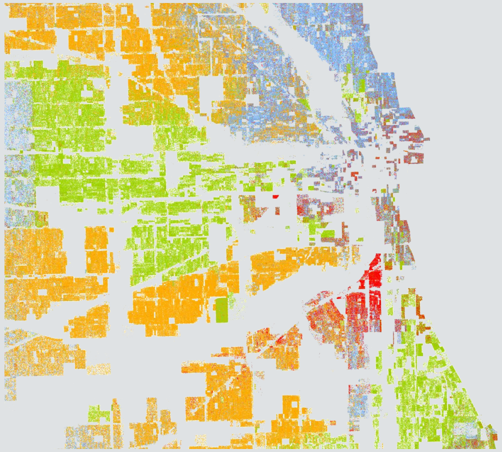


Residential segregation is the outcome of strategic location choices by selfish agents.

[Race in the US by Dot Density, ArcGIS Hub, 2020]
Data from the 2020 US Census

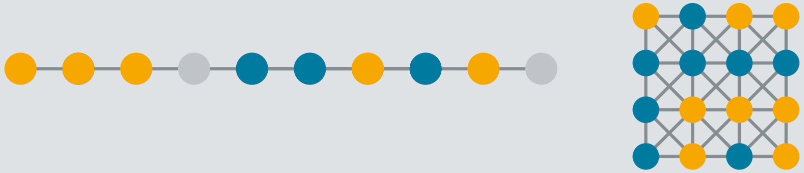
Schelling's Segregation Model

Real-World Segregation:



Residential Segregation in Chicago

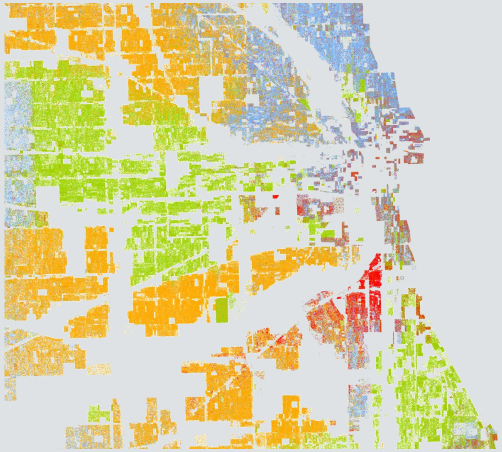
Schelling's Model (1971):



- two types of players on a line or grid.
- players have tolerance parameter $\tau \in [0, 1]$
- player is content if at least τ-fraction of neighbors is of own type
- discontent players swap or jump randomly

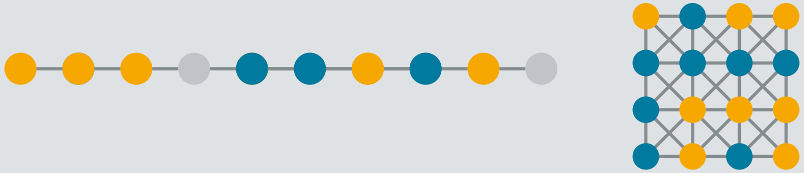
Schelling’s Segregation Model

Real-World Segregation:



Residential Segregation in Chicago

Schelling’s Model (1971):

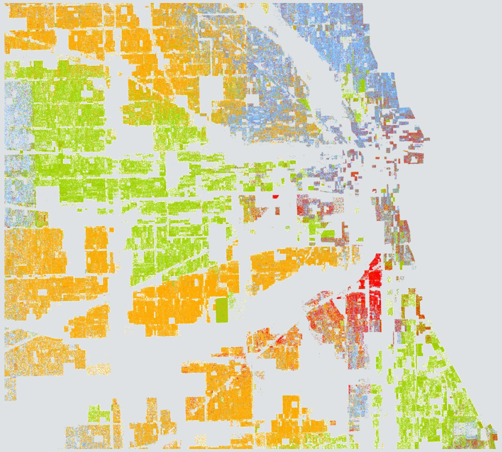


- two types of players on a line or grid.
- players have tolerance parameter $\tau \in [0, 1]$
- player is content if at least τ-fraction of neighbors is of own type
- discontent players swap or jump randomly

Phenomenon: From initial random placement the process reaches a severely segregated state even if $\tau < \frac{1}{2}$.

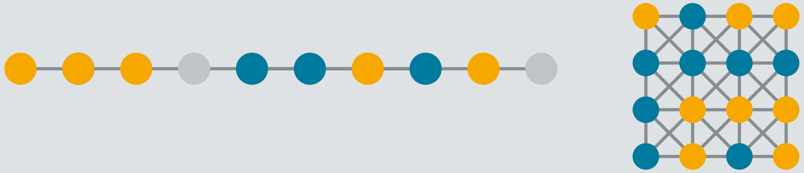
Schelling's Segregation Model

Real-World Segregation:



Residential Segregation in Chicago

Schelling's Model (1971):



- two types of players on a line or grid.
- players have tolerance parameter $\tau \in [0, 1]$
- player is content if at least τ-fraction of neighbors is of own type
- discontent players swap or jump randomly

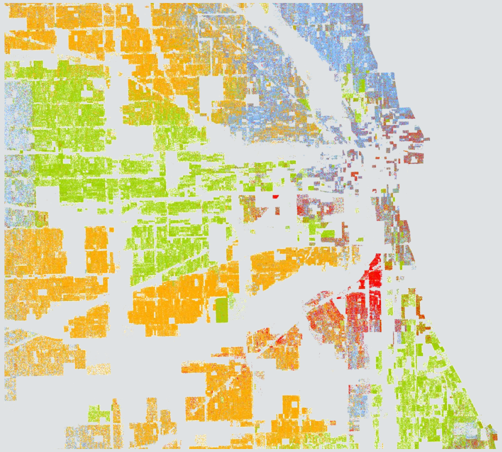
Phenomenon: From initial random placement the process reaches a severely segregated state even if $\tau < \frac{1}{2}$.

- explains why even a population of tolerant players can end up in a segregated state

"Micromotives vs. Macrobehavior"

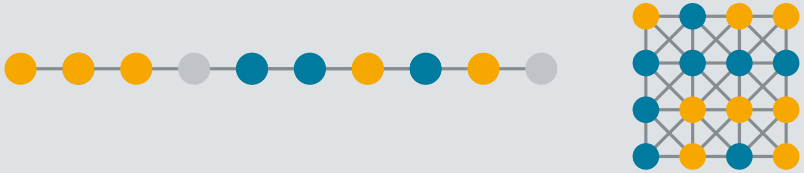
Schelling's Segregation Model

Real-World Segregation:



Residential Segregation in Chicago

Schelling's Model (1971):



- two types of players on a line or grid.
- players have tolerance parameter $\tau \in [0, 1]$
- player is content if at least τ-fraction of neighbors is of own type
- discontent players swap or jump randomly

Phenomenon: From initial random placement the process reaches a severely segregated state even if $\tau < \frac{1}{2}$.

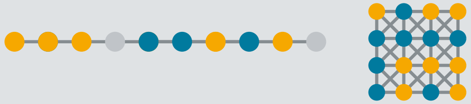
- explains why even a population of tolerant players can end up in a segregated state

"Micromotives vs. Macrobehavior"

- 2005: "Economics Nobel Prize" for Thomas Schelling

Game-Theoretic Schelling Segregation

Schelling's Model: [J. Math. Soc. 1(2), '71]



- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0, 1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-Theoretic Schelling Segregation

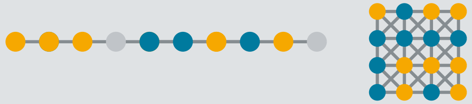
Related Work:

- analysis of random process on the ring and on grids

[STOC’12, FOCS’14, SODA’14, J. Stat. Physics’16, SODA’17]

- no strategic location choice by agents

Schelling’s Model: [J. Math. Soc. 1(2), ’71]



- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0, 1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-Theoretic Schelling Segregation

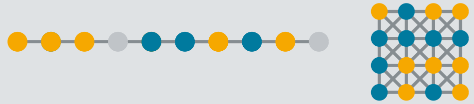
Related Work:

- analysis of random process on the ring and on grids

[STOC’12, FOCS’14, SODA’14, J. Stat. Physics’16, SODA’17]

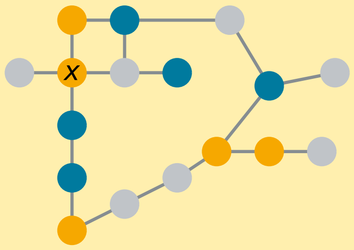
- no strategic location choice by agents

Schelling’s Model: [J. Math. Soc. 1(2), ’71]



- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0, 1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-theoretic version: [Chauhan, L., Molitor: SAGT’18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if

$$\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Game-Theoretic Schelling Segregation

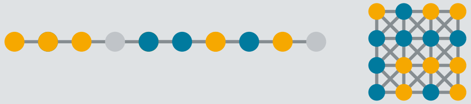
Related Work:

- analysis of random process on the ring and on grids

[STOC’12, FOCS’14, SODA’14, J. Stat. Physics’16, SODA’17]

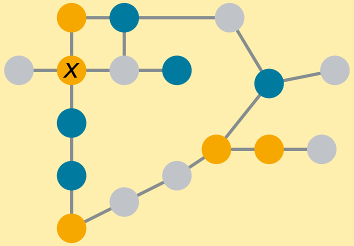
- no strategic location choice by agents

Schelling’s Model: [J. Math. Soc. 1(2), ’71]



- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0, 1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-theoretic version: [Chauhan, L., Molitor: SAGT’18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if

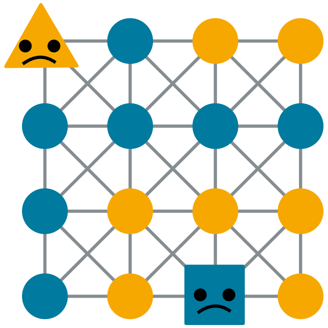
$$\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game

- pairs of discontent agents can swap positions

$\tau = \frac{1}{2}$:



Game-Theoretic Schelling Segregation

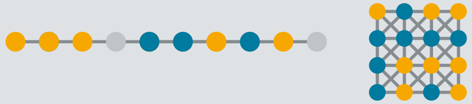
Related Work:

- analysis of random process on the ring and on grids

[STOC’12, FOCS’14, SODA’14, J. Stat. Physics’16, SODA’17]

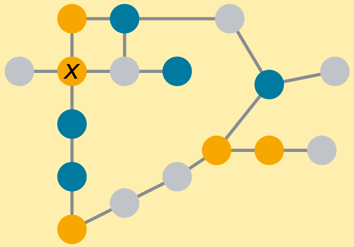
- no strategic location choice by agents

Schelling’s Model: [J. Math. Soc. 1(2), ’71]

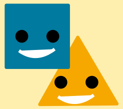


- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0,1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-theoretic version: [Chauhan, L., Molitor: SAGT’18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if



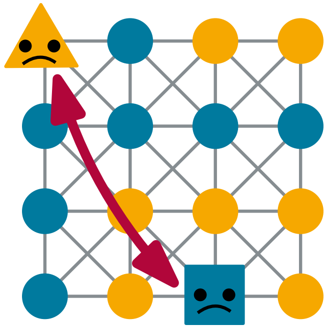
$$\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game

- pairs of discontent agents can swap positions

$\tau = \frac{1}{2}$:



Game-Theoretic Schelling Segregation

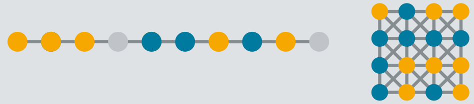
Related Work:

- analysis of random process on the ring and on grids

[STOC'12, FOCS'14, SODA'14, J. Stat. Physics'16, SODA'17]

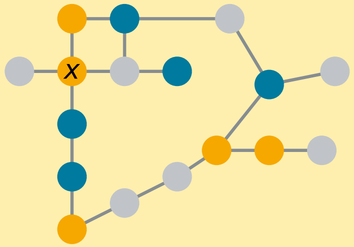
- no strategic location choice by agents

Schelling's Model: [J. Math. Soc. 1(2), '71]

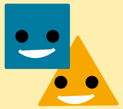


- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0,1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-theoretic version: [Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if



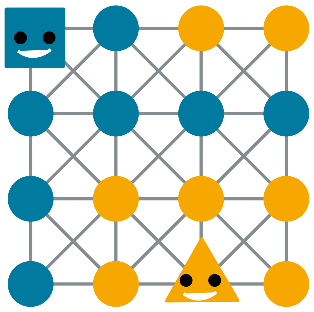
$$\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game

- pairs of discontent agents can swap positions

$\tau = \frac{1}{2}$:



Game-Theoretic Schelling Segregation

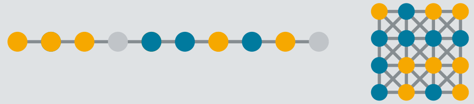
Related Work:

- analysis of random process on the ring and on grids

[STOC'12, FOCS'14, SODA'14, J. Stat. Physics'16, SODA'17]

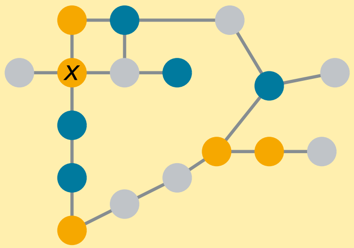
- no strategic location choice by agents

Schelling's Model: [J. Math. Soc. 1(2), '71]

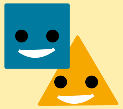


- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0,1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-theoretic version: [Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if



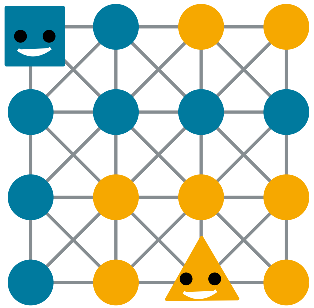
$$\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game

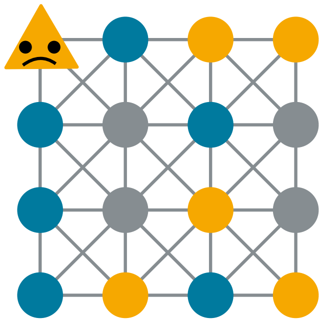
- pairs of discontent agents can swap positions

$\tau = \frac{1}{2}$:



Jump Game

- discontent agents can jump to empty positions



Game-Theoretic Schelling Segregation

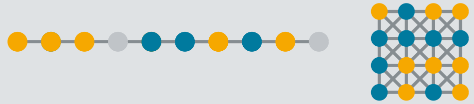
Related Work:

- analysis of random process on the ring and on grids

[STOC’12, FOCS’14, SODA’14, J. Stat. Physics’16, SODA’17]

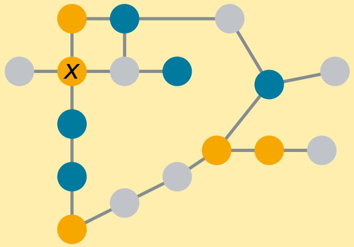
- no strategic location choice by agents

Schelling’s Model: [J. Math. Soc. 1(2), ’71]

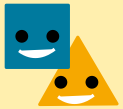


- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0,1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-theoretic version: [Chauhan, L., Molitor: SAGT’18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if



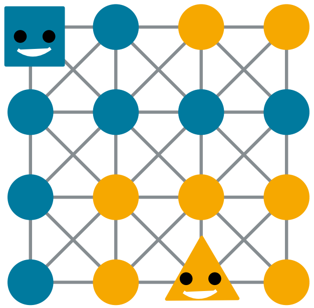
$$\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Swap Game

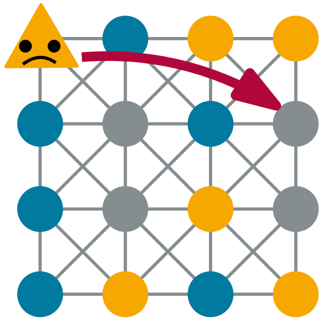
- pairs of discontent agents can swap positions

$\tau = \frac{1}{2}$:



Jump Game

- discontent agents can jump to empty positions



Game-Theoretic Schelling Segregation

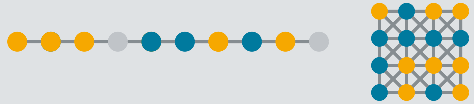
Related Work:

- analysis of random process on the ring and on grids

[STOC'12, FOCS'14, SODA'14, J. Stat. Physics'16, SODA'17]

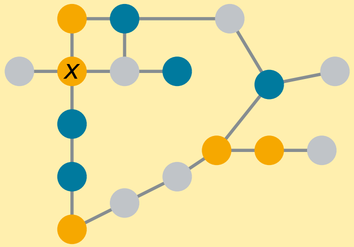
- no strategic location choice by agents

Schelling's Model: [J. Math. Soc. 1(2), '71]



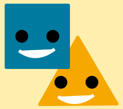
- two types of agents on a line or grid.
- agents have tolerance parameter $\tau \in [0, 1]$
- agent is content if at least τ-fraction of neighbors is of own type
- discontent agents swap or jump randomly

Game-theoretic version:



[Chauhan, L., Molitor: SAGT'18]

- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if



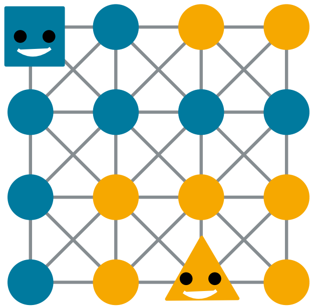
$$\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game

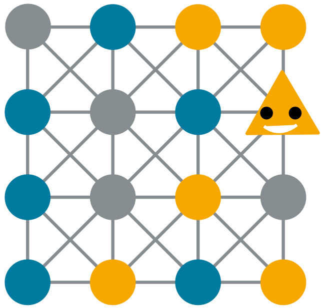
- pairs of discontent agents can swap positions

$\tau = \frac{1}{2}$:



Jump Game

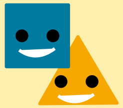
- discontent agents can jump to empty positions



Game-Theoretic Schelling Segregation

Game-theoretic version:

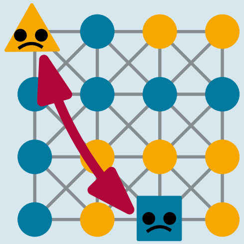
[Chauhan, L., Molitor: SAGT'18]



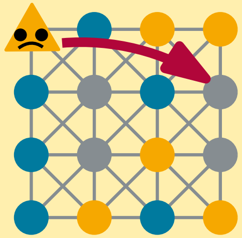
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Swap Game



Jump Game

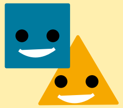


Important concepts:

Game-Theoretic Schelling Segregation

Game-theoretic version:

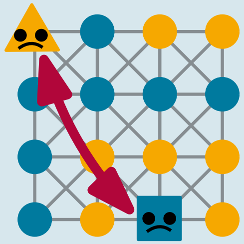
[Chauhan, L., Molitor: SAGT'18]



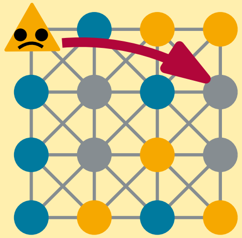
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game



Jump Game



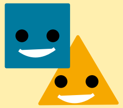
Important concepts:

Jump/Swap Equilibrium (JE/SE):
A strategy profile σ is a JE (SE) if and only if no agent (pair of agents) has an improving jump (swap).

Game-Theoretic Schelling Segregation

Game-theoretic version:

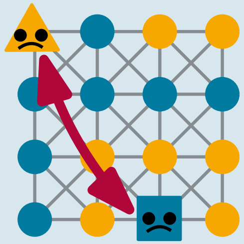
[Chauhan, L., Molitor: SAGT'18]



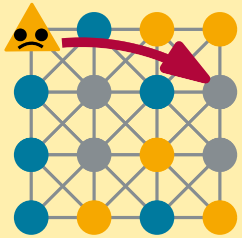
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Swap Game



Jump Game



Important concepts:

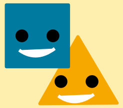
Jump/Swap Equilibrium (JE/SE):
A strategy profile σ is a JE (SE) if and only if no agent (pair of agents) has an improving jump (swap).

Finite Improvement Property (FIP): A Schelling game has the FIP if starting from any strategy profile, a JE/SE is reached after a finite number of improving jumps/swaps.

Game-Theoretic Schelling Segregation

Game-theoretic version:

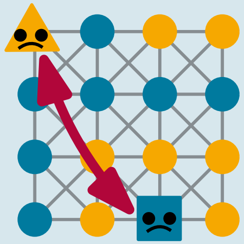
[Chauhan, L., Molitor: SAGT'18]



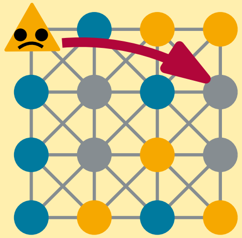
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Swap Game



Jump Game



Important concepts:

Jump/Swap Equilibrium (JE/SE):
A strategy profile σ is a JE (SE) if and only if no agent (pair of agents) has an improving jump (swap).

Finite Improvement Property (FIP): A Schelling game has the FIP if starting from any strategy profile, a JE/SE is reached after a finite number of improving jumps/swaps.

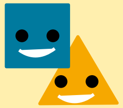
Social Cost/Welfare:

- Utilitarian: sum of agents' costs/utilities
- Degree of Integration: number of agents with at least one other-type neighbor [Agarwal et al. AAAI'20]

Game-Theoretic Schelling Segregation

Game-theoretic version:

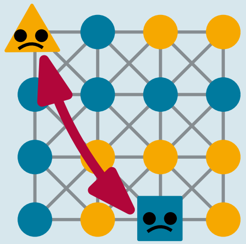
[Chauhan, L., Molitor: SAGT'18]



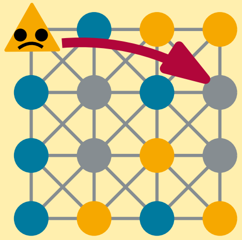
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game



Jump Game



Important concepts:

Jump/Swap Equilibrium (JE/SE):
A strategy profile σ is a JE (SE) if and only if no agent (pair of agents) has an improving jump (swap).

Finite Improvement Property (FIP): A Schelling game has the FIP if starting from any strategy profile, a JE/SE is reached after a finite number of improving jumps/swaps.

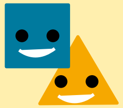
Social Cost/Welfare:

- Utilitarian: sum of agents' costs/utilities
- Degree of Integration: number of agents with at least one other-type neighbor [Agarwal et al. AAAI'20]

Game-Theoretic Schelling Segregation

Game-theoretic version:

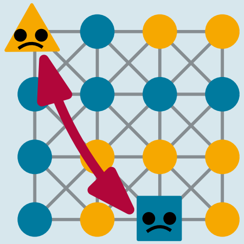
[Chauhan, L., Molitor: SAGT'18]



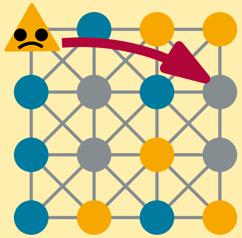
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game



Jump Game



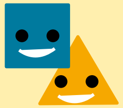
Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

Game-Theoretic Schelling Segregation

Game-theoretic version:

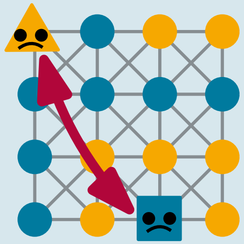
[Chauhan, L., Molitor: SAGT'18]



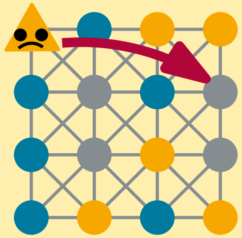
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game



Jump Game



- YES, if $\tau \leq \frac{1}{2}$ or if host graph is regular

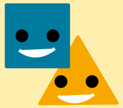
Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

Game-Theoretic Schelling Segregation

Game-theoretic version:

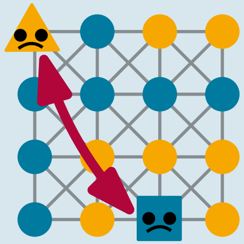
[Chauhan, L., Molitor: SAGT'18]



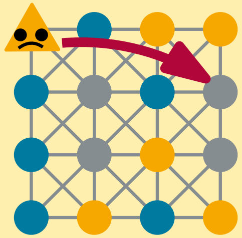
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Swap Game



Jump Game



- YES, if $\tau \leq \frac{1}{2}$ or if host graph is regular
- NO, if $\tau > \frac{1}{2}$ on arbitrary host graph

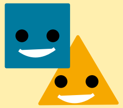
Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

Game-Theoretic Schelling Segregation

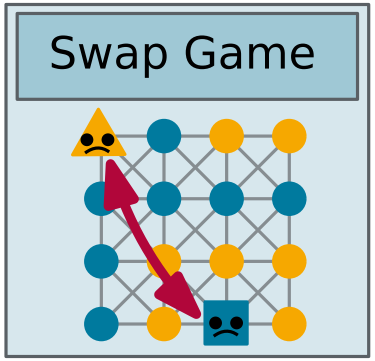
Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

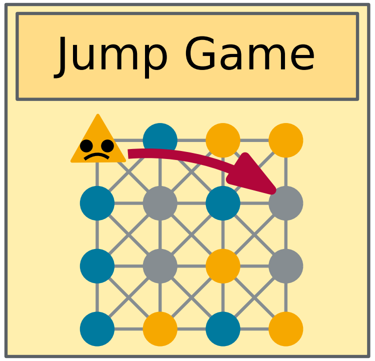
$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$



- YES, if $\tau \leq \frac{1}{2}$ or if host graph is regular
- NO, if $\tau > \frac{1}{2}$ on arbitrary host graph

Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

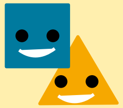


- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph

Game-Theoretic Schelling Segregation

Game-theoretic version:

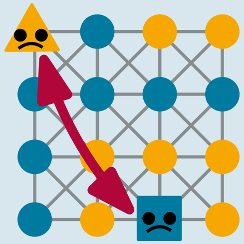
[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Swap Game

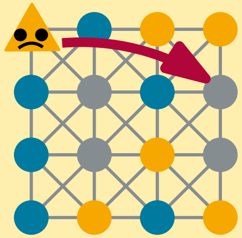


- YES, if $\tau \leq \frac{1}{2}$ or if host graph is regular
- NO, if $\tau > \frac{1}{2}$ on arbitrary host graph

Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

Jump Game

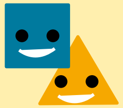


- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph
- NO, if $\tau > \frac{2}{\Delta}$ on Δ-regular host graph

Game-Theoretic Schelling Segregation

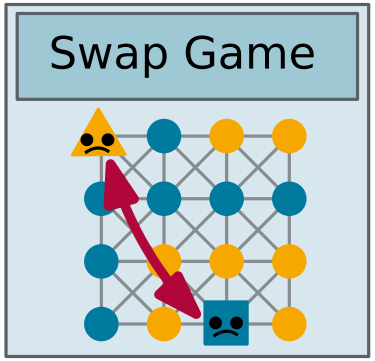
Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

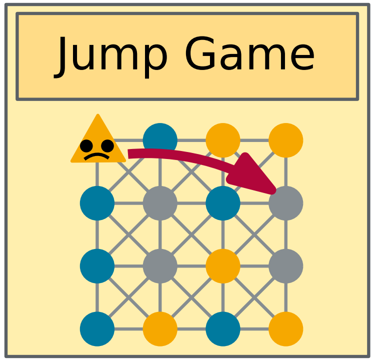
$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$


Swap Game

- YES, if $\tau \leq \frac{1}{2}$ or if host graph is regular
- NO, if $\tau > \frac{1}{2}$ on arbitrary host graph

Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

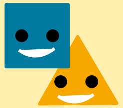

Jump Game

- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph
- NO, if $\tau > \frac{2}{\Delta}$ on Δ-regular host graph
- NO, for any $\tau > 0$ on arbitrary host graph

Game-Theoretic Schelling Segregation

Game-theoretic version:

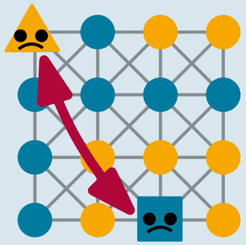
[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Swap Game

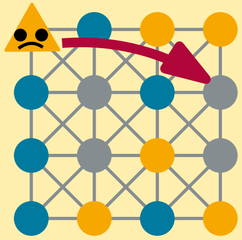


- YES, if $\tau \leq \frac{1}{2}$ or if host graph is regular later!
- NO, if $\tau > \frac{1}{2}$ on arbitrary host graph

Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

Jump Game

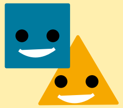


- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph
- NO, if $\tau > \frac{2}{\Delta}$ on Δ-regular host graph
- NO, for any $\tau > 0$ on arbitrary host graph

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

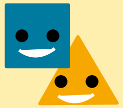
[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- NO, for any $\tau > 0$ on arbitrary host graph

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- NO, for any $\tau > 0$ on arbitrary host graph

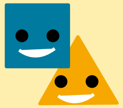
Proofsketch:

- consider the following best response cycle

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

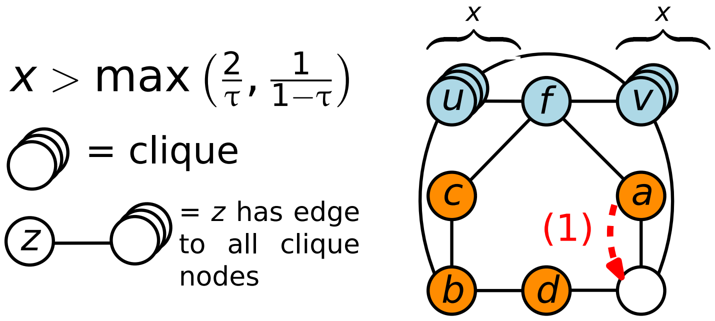
[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- NO, for any $\tau > 0$ on arbitrary host graph

Proofsketch:

- consider the following best response cycle

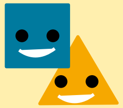
$x > \max\left(\frac{2}{\tau}, \frac{1}{1-\tau}\right)$



Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

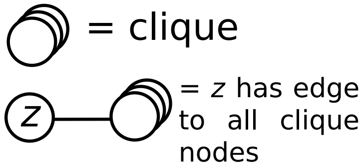
[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

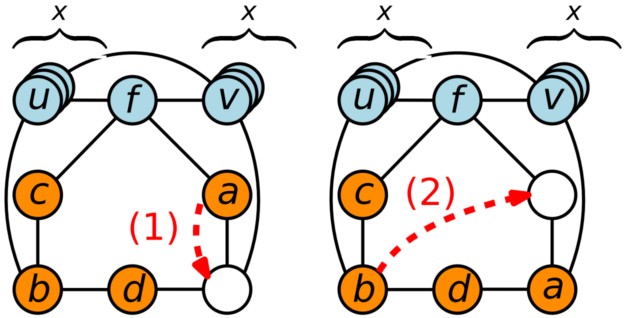
- NO, for any $\tau > 0$ on arbitrary host graph

Proofsketch:

- consider the following best response cycle

$x > \max\left(\frac{2}{\tau}, \frac{1}{1-\tau}\right)$

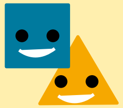




Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

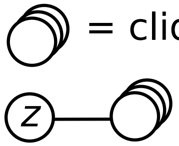
[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- NO, for any $\tau > 0$ on arbitrary host graph

Proofsketch:

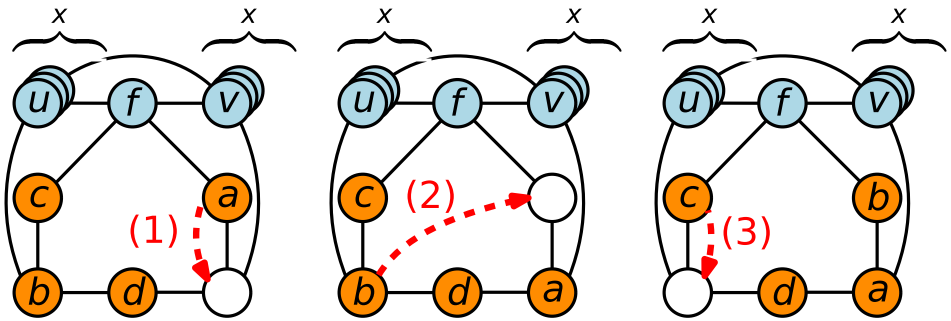
- consider the following best response cycle

$x > \max\left(\frac{2}{\tau}, \frac{1}{1-\tau}\right)$



= clique

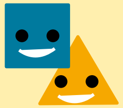
= z has edge to all clique nodes



Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Jump Game

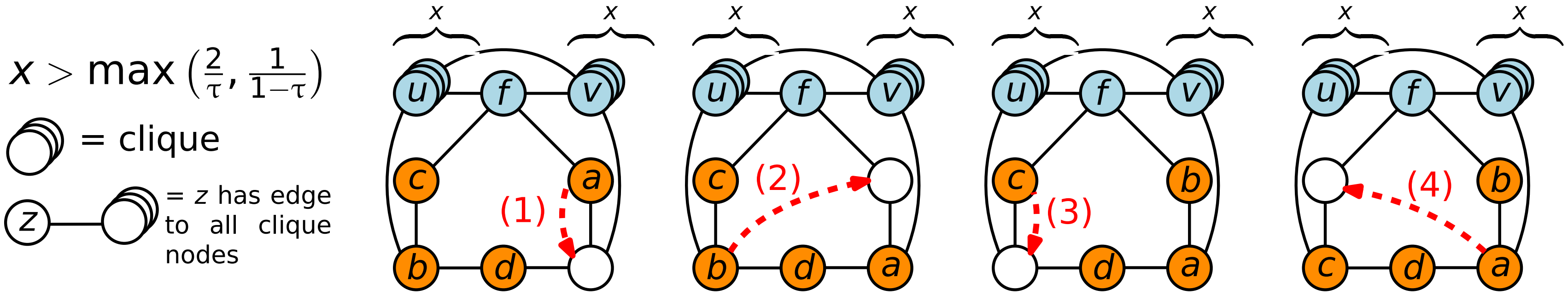
Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- NO, for any $\tau > 0$ on arbitrary host graph

Proofsketch:

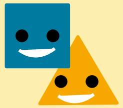
- consider the following best response cycle



Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

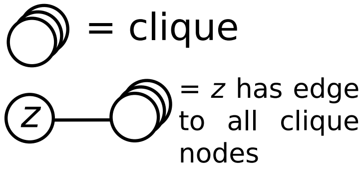
[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

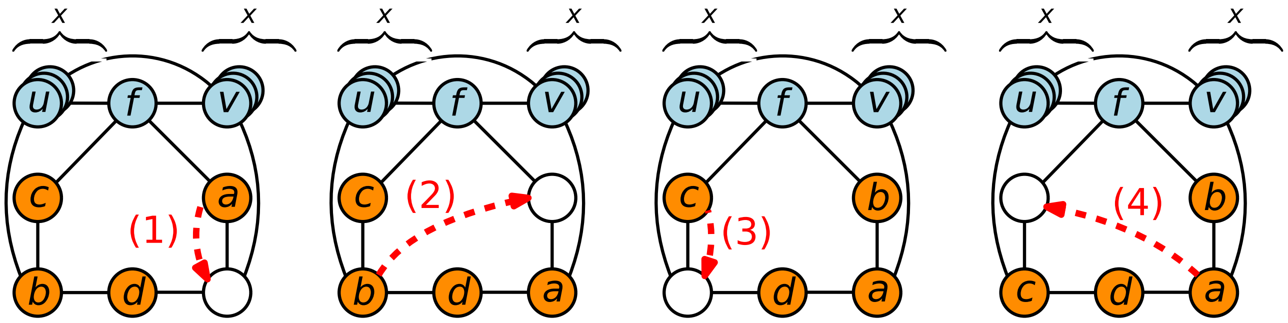
- NO, for any $\tau > 0$ on arbitrary host graph

Proofsketch:

- consider the following best response cycle

$x > \max\left(\frac{2}{\tau}, \frac{1}{1-\tau}\right)$



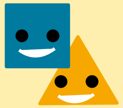


$\Rightarrow$ the Jump Schelling Game on arbitrary host graphs cannot have an ordinal potential function $\Rightarrow$ no FIP ☐

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Jump Game

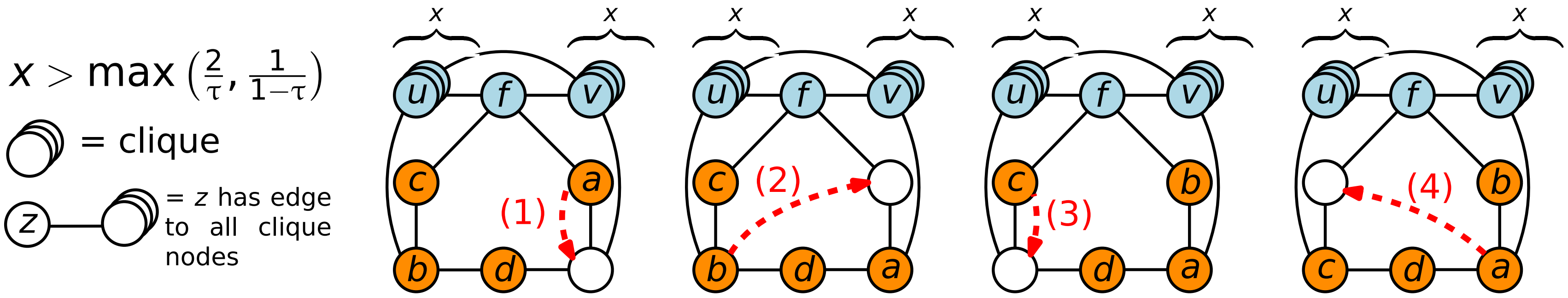
Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- NO, for any $\tau > 0$ on arbitrary host graph

Proofsketch:

- consider the following best response cycle

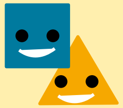


- the above best response cycle has the following remarkable properties:
  - in every step exactly one agent has an improving move
  - all improving moves are unique

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Jump Game

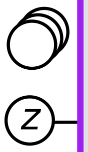
Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- NO, for any $\tau > 0$ on arbitrary host graph

Proo

$x >$



- t

- actually we have proven something even worse:

The Jump Schelling Game on arbitrary host graphs is not **weakly acyclic**.

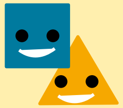
A game is **weakly acyclic** if from every strategy profile there exists a sequence of improving moves that leads to a PNE.

- in every step exactly one agent has an improving move
- all improving moves are unique

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT’18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

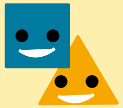
[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE’19]

- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

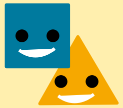
- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph

Proofsketch:
- we define a suitable ordinal potential function

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

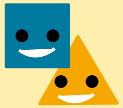
- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph

Proofsketch:
- we define a suitable ordinal potential function
- consider any edge $e = \{u, v\}$; we define its weight depending on $\mathbf{s}$ as $w_{\mathbf{s}}(e)$ as:

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Jump Game

Can agents find JE/SE via iterative improving moves?

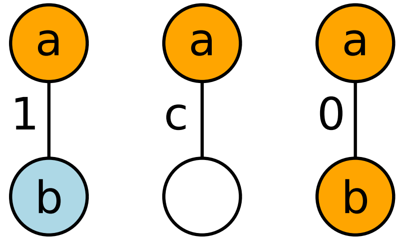
[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph

Proofsketch:

- we define a suitable ordinal potential function
- consider any edge $e = \{u, v\}$; we define its weight depending on $\mathbf{s}$ as $w_{\mathbf{s}}(e)$ as:

$$w_{\mathbf{s}}(e) = \begin{cases} 1, & \text{if } u \text{ and } v \text{ are occupied by agents of different types for } \mathbf{s}, \\ c, & \text{if either } u \text{ or } v\text{, but not both, are empty for } \mathbf{s}, \\ 0, & \text{otherwise,} \end{cases}$$

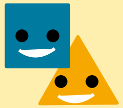


$\frac{1}{2} - \frac{1}{2\Delta} < c < \frac{1}{2}$

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Jump Game

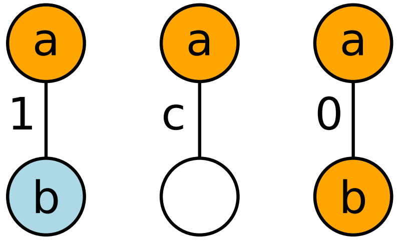
Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph

Proofsketch:
- we define a suitable ordinal potential function
- consider any edge $e = \{u, v\}$; we define its weight depending on $\mathbf{s}$ as $w_{\mathbf{s}}(e)$ as:

$$w_{\mathbf{s}}(e) = \begin{cases} 1, & \text{if } u \text{ and } v \text{ are occupied by agents of different types for } \mathbf{s}, \\ c, & \text{if either } u \text{ or } v\text{, but not both, are empty for } \mathbf{s}, \\ 0, & \text{otherwise,} \end{cases}$$



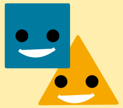
$\frac{1}{2} - \frac{1}{2\Delta} < c < \frac{1}{2}$

- then we define: $\Phi(\mathbf{s}) = \sum_{e \in E} w_{\mathbf{s}}(e)$

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Jump Game

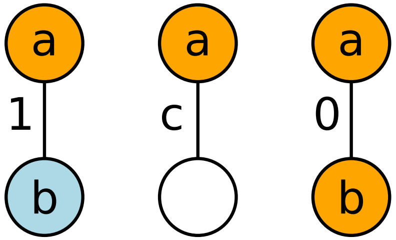
Can agents find JE/SE via iterative improving moves?

[Echzell, Friedrich, L., Molitor, Pappik, Schöne, Sommer, Stangl, WINE'19]

- YES, if $\tau \leq \frac{2}{\Delta}$ on Δ-regular host graph

Proofsketch:
- we define a suitable ordinal potential function
- consider any edge $e = \{u, v\}$; we define its weight depending on $\mathbf{s}$ as $w_{\mathbf{s}}(e)$ as:

$$w_{\mathbf{s}}(e) = \begin{cases} 1, & \text{if } u \text{ and } v \text{ are occupied by agents of different types for } \mathbf{s}, \\ c, & \text{if either } u \text{ or } v\text{, but not both, are empty for } \mathbf{s}, \\ 0, & \text{otherwise,} \end{cases}$$



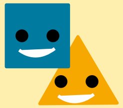
$\frac{1}{2} - \frac{1}{2\Delta} < c < \frac{1}{2}$

- then we define: $\Phi(\mathbf{s}) = \sum_{e \in E} w_{\mathbf{s}}(e)$
- $\Phi(\mathbf{s})$ is an ordinal potential function $\Rightarrow$ the FIP holds □

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



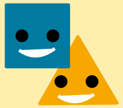
- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

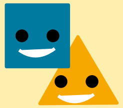
$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

model variants

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Schelling Models with Localized Social Influence: A Game-Theoretic Framework

[Chan, Irfan, Than, AAMAS'20]

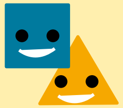
- influence of social network

model variants

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Modified Schelling Games

[Kanellopoulos, Kyropoulou, Voudouris, SAGT'20]

- agent included in neighborhood

Schelling Models with Localized Social Influence: A Game-Theoretic Framework

[Chan, Irfan, Than, AAMAS'20]

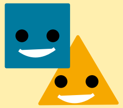
- influence of social network

model variants

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Schelling Models with Localized Social Influence: A Game-Theoretic Framework

[Chan, Irfan, Than, AAMAS'20]

- influence of social network

Modified Schelling Games

[Kanellopoulos, Kyropoulou, Voudouris, SAGT'20]

- agent included in neighborhood

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

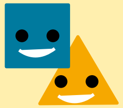
- non-monotone utilities

model variants

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Modified Schelling Games

[Kanellopoulos, Kyropoulou, Voudouris, SAGT'20]

- agent included in neighborhood

Schelling Models with Localized Social Influence: A Game-Theoretic Framework

[Chan, Irfan, Than, AAMAS'20]

- influence of social network

model variants

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

Not All Strangers Are the Same: The Impact of Tolerance in Schelling Games

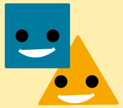
[Kanellopoulos, Kyropoulou, Voudouris, MFCS'22]

- 1-dim ordering of types

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT’18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

model variants

Schelling Models with Localized Social Influence: A Game-Theoretic Framework

[Chan, Irfan, Than, AAMAS’20]

- influence of social network

Modified Schelling Games

[Kanellopoulos, Kyropoulou, Voudouris, SAGT’20]

- agent included in neighborhood

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI’22]

[Friedrich, L., Molitor, Seifert, SAGT’23]

- non-monotone utilities

Not All Strangers Are the Same: The Impact of Tolerance in Schelling Games

[Kanellopoulos, Kyropoulou, Voudouris, MFCS’22]

- 1-dim ordering of types

Schelling Games with Continuous Types

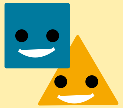
[Bilò, Bilò, Döring, L., Molitor, Schmidt, IJCAI’23]

- type is number in $[0, 1]$

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Modified Schelling Games

[Kanellopoulos, Kyropoulou, Voudouris, SAGT'20]

- agent included in neighborhood

Schelling Models with Localized Social Influence: A Game-Theoretic Framework

[Chan, Irfan, Than, AAMAS'20]

- influence of social network

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

model variants

Diversity-Seeking Jump Games in Networks

[Narayanan, Sabbagh, SAGT'23]

- inverse model

Not All Strangers Are the Same: The Impact of Tolerance in Schelling Games

[Kanellopoulos, Kyropoulou, Voudouris, MFCS'22]

- 1-dim ordering of types

Schelling Games with Continuous Types

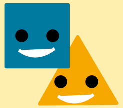
[Bilò, Bilò, Döring, L., Molitor, Schmidt, IJCAI'23]

- type is number in $[0, 1]$

Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Modified Schelling Games
[Kanellopoulos, Kyropoulou, Voudouris, SAGT'20]

- agent included in neighborhood

Schelling Models with Localized Social Influence: A Game-Theoretic Framework
[Chan, Irfan, Than, AAMAS'20]

- influence of social network

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

model variants

Diversity-Seeking Jump Games in Networks
[Narayanan, Sabbagh, SAGT'23]

- inverse model

Schelling Games with Continuous Types
[Bilò, Bilò, Döring, L., Molitor, Schmidt, IJCAI'23]

- type is number in $[0, 1]$

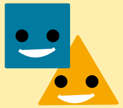
Not All Strangers Are the Same: The Impact of Tolerance in Schelling Games
[Kanellopoulos, Kyropoulou, Voudouris, MFCS'22]

- 1-dim ordering of types

Single-Peaked Schelling Games

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

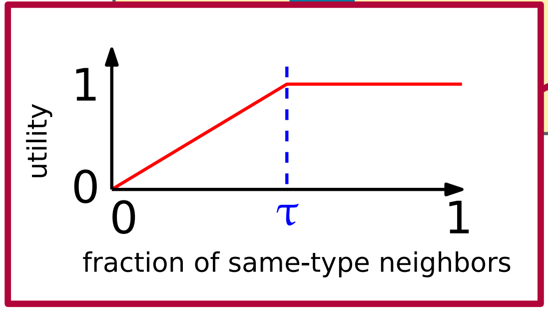
Single-Peaked Schelling Games

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]

- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$



Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

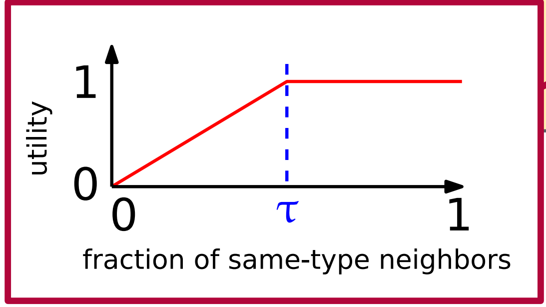
Single-Peaked Schelling Games

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]

- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$



Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

General Social Survey

- since 50 years regulary done in the US

If you could find the housing that you would want and like, would you rather live in a neighborhood that is all black; mostly black; half black, half white; or mostly white?

54% "half black/half white"

Living in a neighborhood where half of your neighbors were blacks?

1988: 57%, 1998: 70%, 2008: 79%, 2018: 82% answered with "neither favor nor oppose" or better
2018: 33% answered with “favor” or “strongly favor”.

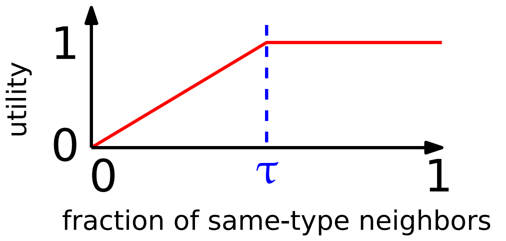
Single-Peaked Schelling Games

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]

- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\text{\#agents of same type in neighborhood}}{\text{\#agents in neighborhood}}\right\}$$



General Social Survey

- since 50 years regulary done in the US

If you could find the housing that you would want and like, would you rather live in a neighborhood that is all black; mostly black; half black, half white; or mostly white?

54% "half black/half white"

Living in a neighborhood where half of your neighbors were blacks?

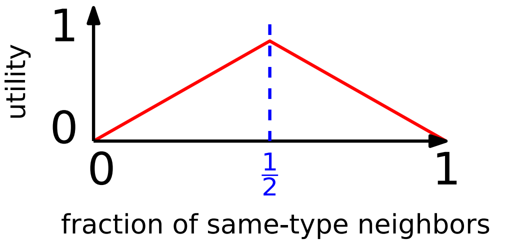
1988: 57%, 1998: 70%, 2008: 79%, 2018: 82% answered with "neither favor nor oppose" or better
2018: 33% answered with “favor” or “strongly favor”.

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities



Single-Peaked Schelling Games

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI’22]

[Friedrich, L., Molitor, Seifert, SAGT’23]

- non-monotone utilities

Single-Peaked Schelling Games

Single-Peaked Function $p(x)$

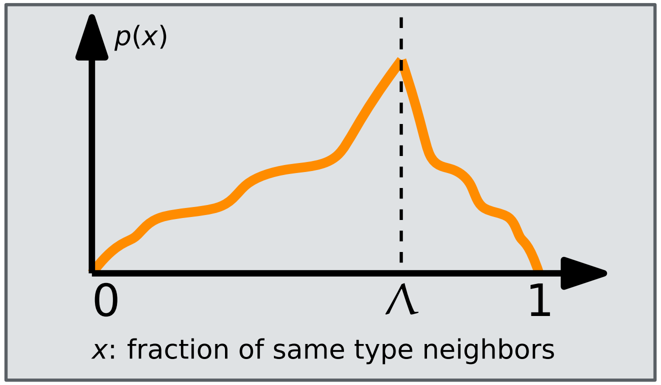
- $p(0) = p(1) = 0$,
- strictly increasing on $[0, \Lambda]$
- stretched symmetry $\forall x \in [\Lambda, 1] : p(x) = p(\frac{\Lambda(1-x)}{1-\Lambda})$

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities



Single-Peaked Schelling Games

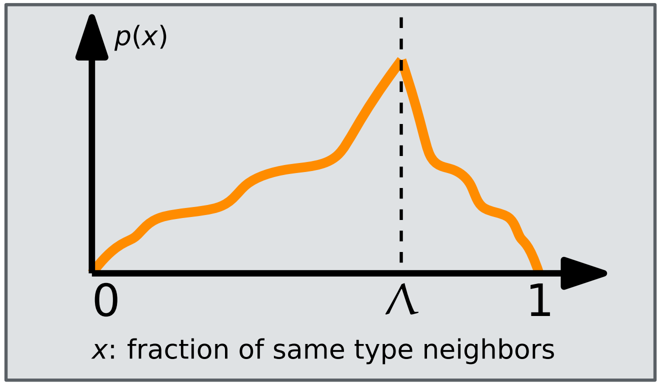
Single-Peaked Function $p(x)$

- $p(0) = p(1) = 0$,
- strictly increasing on $[0, \Lambda]$
- stretched symmetry $\forall x \in [\Lambda, 1] : p(x) = p(\frac{\Lambda(1-x)}{1-\Lambda})$

- focus on $\Lambda \in (0, 1)$
- agents actively strive for integration (instead of only passively accepting it)

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities



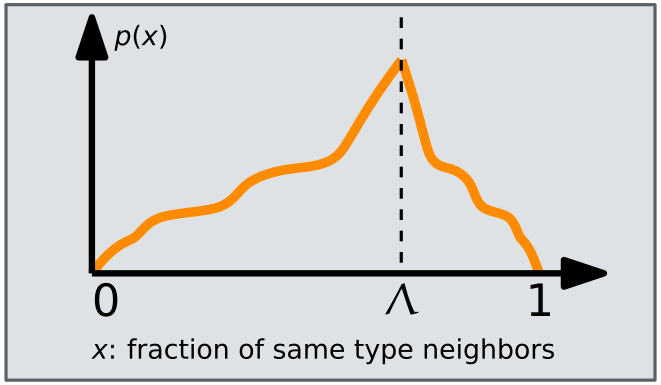
Single-Peaked Schelling Games

Single-Peaked Function $p(x)$

- $p(0) = p(1) = 0$,
- strictly increasing on $[0, \Lambda]$
- stretched symmetry $\forall x \in [\Lambda, 1] : p(x) = p(\frac{\Lambda(1-x)}{1-\Lambda})$

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

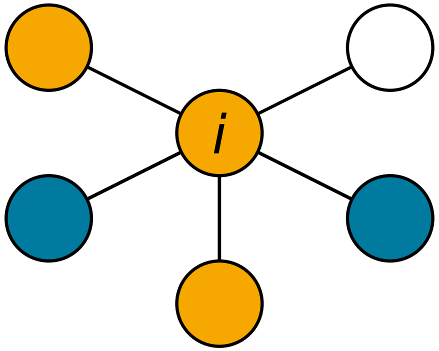
- non-monotone utilities



x: fraction of same type neighbors

- focus on $\Lambda \in (0, 1)$
- agents actively strive for integration (instead of only passively accepting it)

Consider agent placement σ:



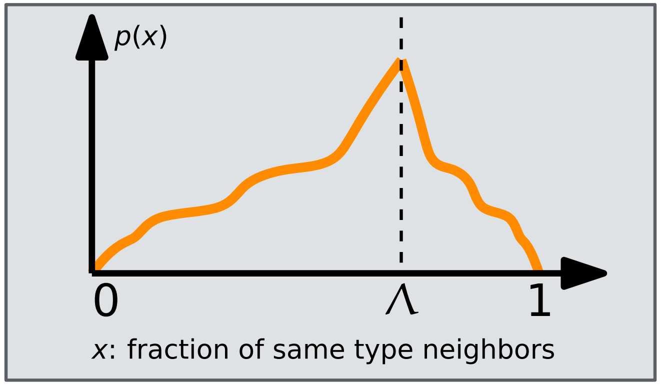
Single-Peaked Schelling Games

Single-Peaked Function $p(x)$

- $p(0) = p(1) = 0$,
- strictly increasing on $[0, \Lambda]$
- stretched symmetry $\forall x \in [\Lambda, 1] : p(x) = p(\frac{\Lambda(1-x)}{1-\Lambda})$

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

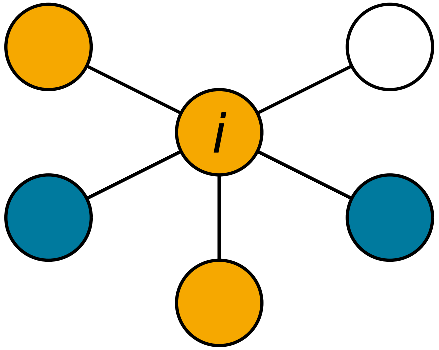


- focus on $\Lambda \in (0, 1)$
- agents actively strive for integration (instead of only passively accepting it)

Consider agent placement σ:

- fraction of same-type agents in neighborhood of agent i:

$$f_i(\sigma) = \frac{|N[i, \sigma] \cap C(i)|}{|N[i, \sigma]|}$$



Single-Peaked Schelling Games

Single-Peaked Function $p(x)$

- $p(0) = p(1) = 0$,
- strictly increasing on $[0, \Lambda]$
- stretched symmetry $\forall x \in [\Lambda, 1] : p(x) = p(\frac{\Lambda(1-x)}{1-\Lambda})$

- focus on $\Lambda \in (0, 1)$
- agents actively strive for integration (instead of only passively accepting it)

Consider agent placement σ:

- fraction of same-type agents in neighborhood of agent i:

$$f_i(\sigma) = \frac{|\boxed{N[i,\sigma]} \cap C(i)|}{|N[i,\sigma]|}$$

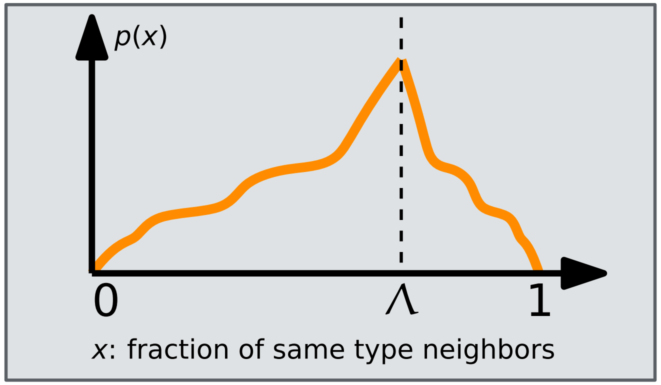
closed neighborhood of agent *i*

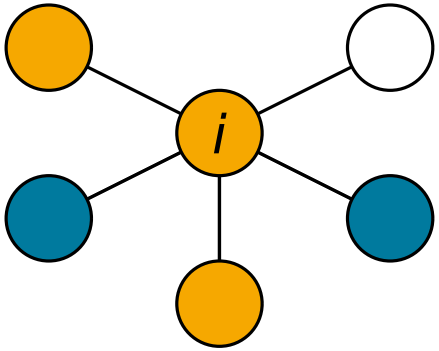
Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

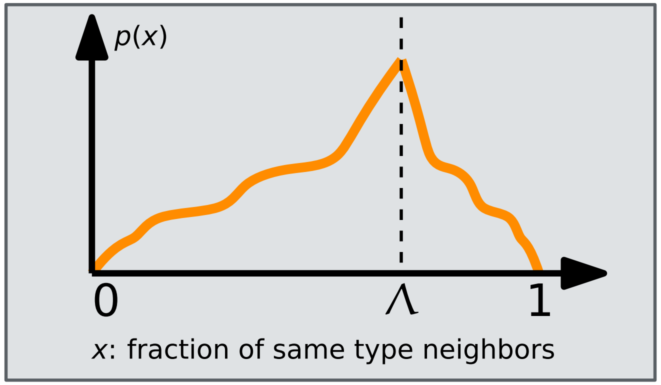




Single-Peaked Schelling Games

Single-Peaked Function $p(x)$
- $p(0) = p(1) = 0$,
- strictly increasing on $[0, \Lambda]$
- stretched symmetry $\forall x \in [\Lambda, 1] : p(x) = p(\frac{\Lambda(1-x)}{1-\Lambda})$

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI’22]
[Friedrich, L., Molitor, Seifert, SAGT’23]
- non-monotone utilities



x: fraction of same type neighbors

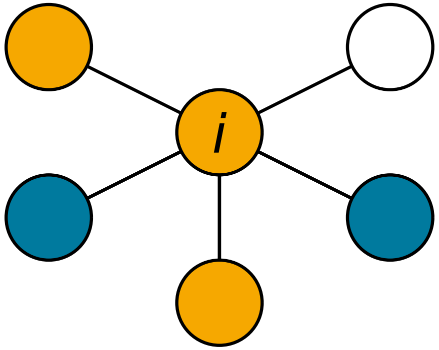
- focus on $\Lambda \in (0, 1)$
- agents actively strive for integration (instead of only passively accepting it)

Consider agent placement σ:
- fraction of same-type agents in neighborhood of agent i:

$$f_i(\sigma) = \frac{|\boxed{N[i,\sigma]} \cap \boxed{C(i)}|}{|N[i,\sigma]|}$$

closed neighborhood of agent i

set of nodes having i’s type



Single-Peaked Schelling Games

Single-Peaked Function $p(x)$

- $p(0) = p(1) = 0$,
- strictly increasing on $[0, \Lambda]$
- stretched symmetry $\forall x \in [\Lambda, 1] : p(x) = p(\frac{\Lambda(1-x)}{1-\Lambda})$

- focus on $\Lambda \in (0, 1)$
- agents actively strive for integration (instead of only passively accepting it)

Consider agent placement σ:

- fraction of same-type agents in neighborhood of agent i:

$$f_i(\sigma) = \frac{|\boxed{N[i, \sigma]} \cap \boxed{C(i)}|}{|N[i, \sigma]|}$$

closed neighborhood of agent i

set of nodes having i's type

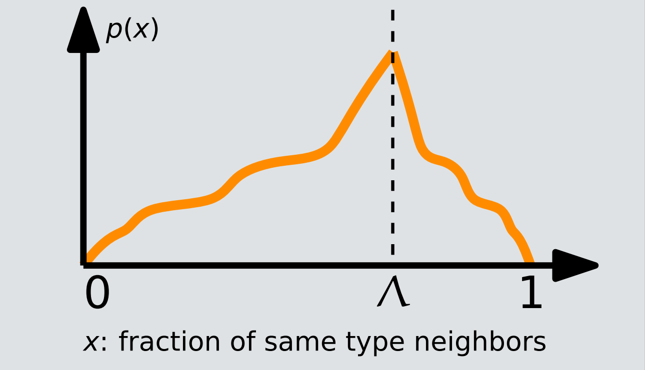
- utility of agent i: $\boxed{U_i(\sigma) = p\left(f_i(\sigma)\right)}$

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games

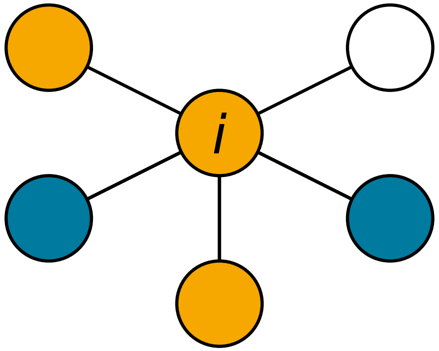
[Bilò, Bilò, L., Molitor, IJCAI'22]

[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities



x: fraction of same type neighbors



Single-Peaked Schelling Games

Single-Peaked Function $p(x)$

- $p(0) = p(1) = 0$,
- strictly increasing on $[0, \Lambda]$
- stretched symmetry $\forall x \in [\Lambda, 1] : p(x) = p(\frac{\Lambda(1-x)}{1-\Lambda})$

- focus on $\Lambda \in (0, 1)$
- agents actively strive for integration (instead of only passively accepting it)

Consider agent placement σ:

- fraction of same-type agents in neighborhood of agent i:

$$f_i(\sigma) = \frac{|\boxed{N[i, \sigma]} \cap \boxed{C(i)}|}{|N[i, \sigma]|}$$

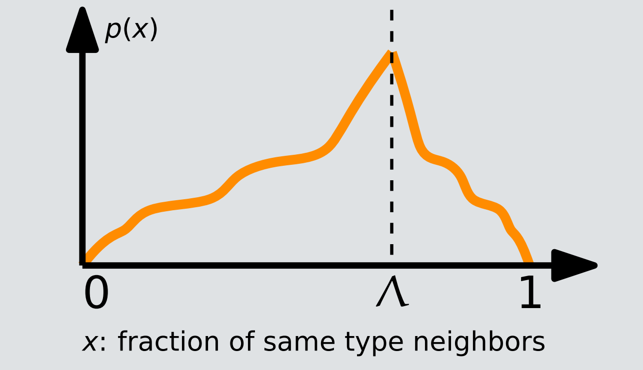
closed neighbor-hood of agent *i*

set of nodes having *i*'s type

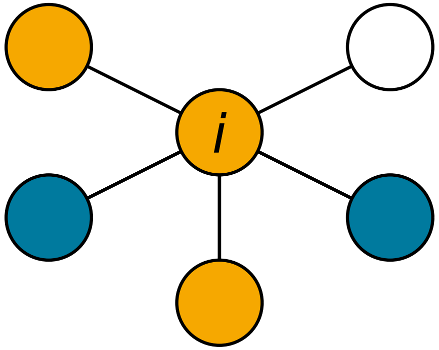
- utility of agent i: $\boxed{U_i(\sigma) = p\left(f_i(\sigma)\right)}$

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities



$U_i(\sigma) = p\left(\frac{3}{5}\right)$



Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

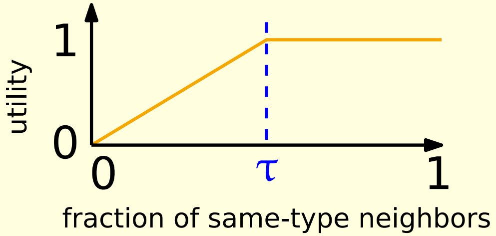
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



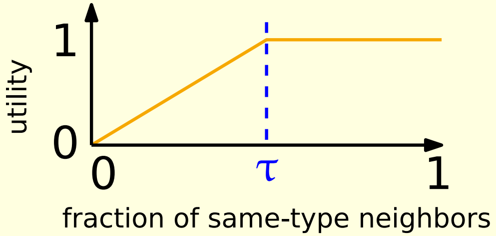
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



Example Result:

Convergence guarantee on Δ-regular graph.

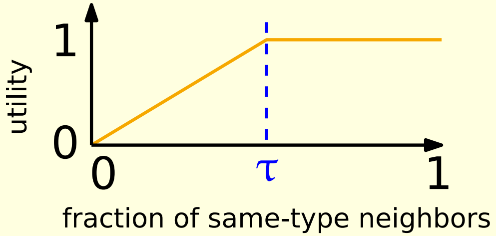
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow$ Φ increases

generalized ordinal potential function

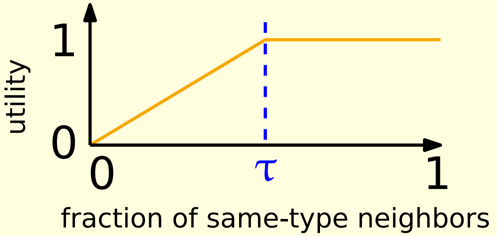
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



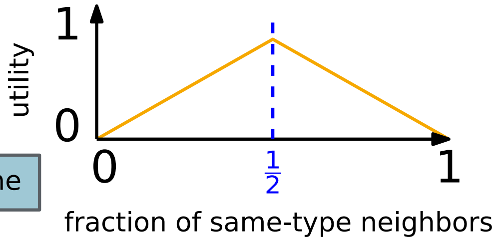
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow$ Φ increases

generalized ordinal potential function

single-peaked utilities:



Swap Game

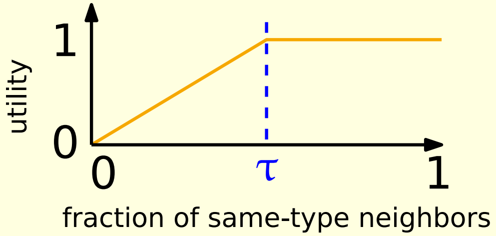
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



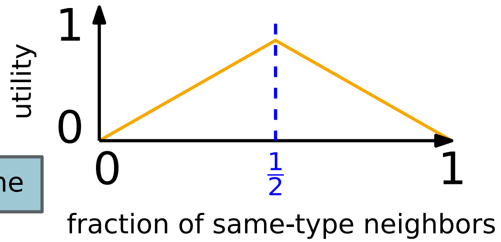
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow$ Φ increases

generalized ordinal potential function

single-peaked utilities:



Swap Game

Example Results:

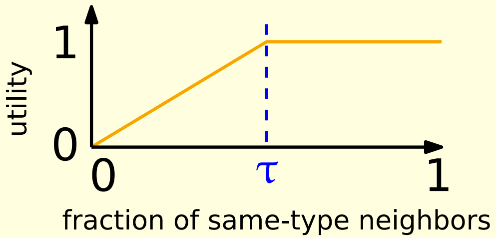
- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ

Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]
- non-monotone utilities

already seen: monotone utilities

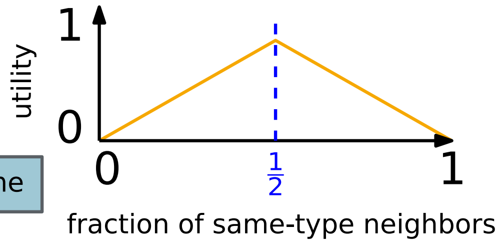


Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges
- improving swap $\Rightarrow$ Φ increases

generalized ordinal potential function

single-peaked utilities:



Swap Game

Example Results:

peak $\leq \frac{1}{2}$:	convergence guarantee on almost-regular graphs via Φ
peak $> \frac{1}{2}$:	no convergence, no equilibria even on regular graphs

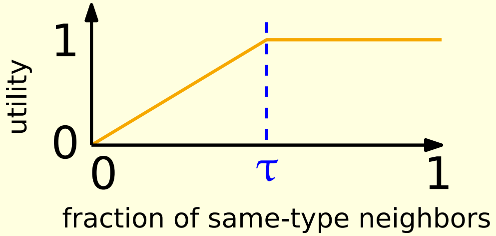
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



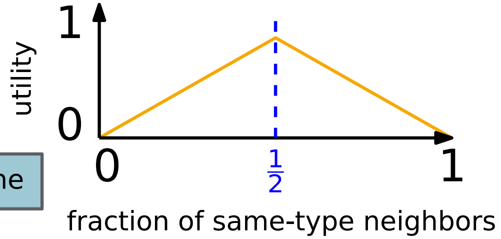
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow$ Φ increases

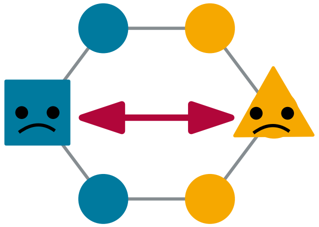
generalized ordinal potential function

single-peaked utilities:



Swap Game

Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

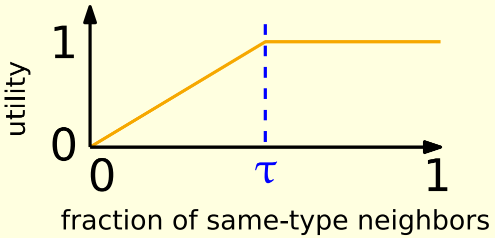
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



Example Result: Convergence guarantee on Δ-regular graph.

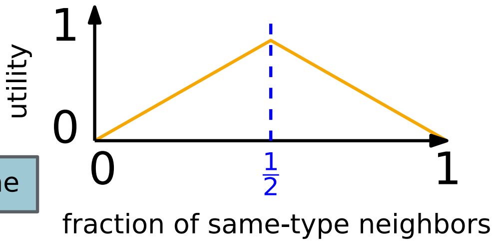
Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow$ Φ increases

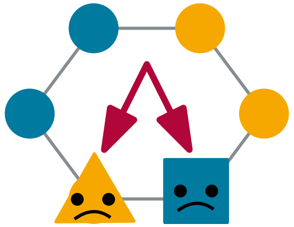
generalized ordinal potential function

single-peaked utilities:

Swap Game



Example Results:



- peak $\le \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

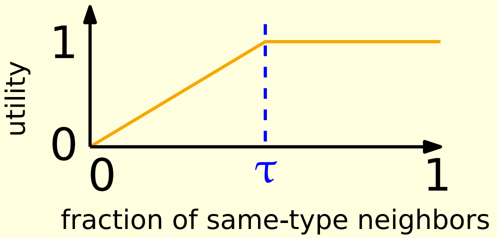
no convergence, no equilibria even on rings or paths

Single-Peaked Schelling Games

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]
- non-monotone utilities

Can agents find JE/SE via iterative improving moves?

already seen: monotone utilities

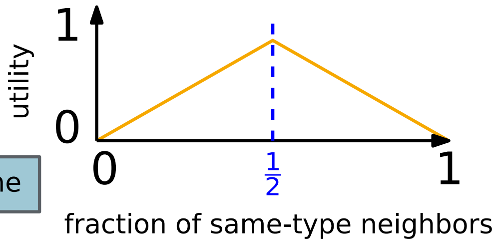


Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges
- improving swap $\Rightarrow$ Φ increases

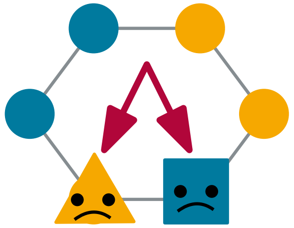
generalized ordinal potential function

single-peaked utilities:



Swap Game

Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths

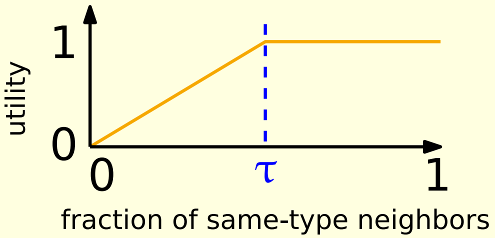
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



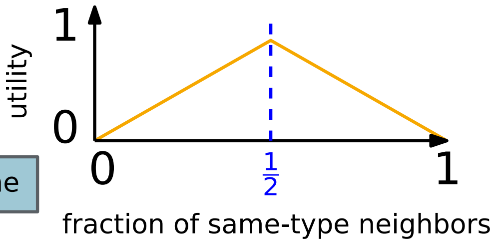
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow$ Φ increases

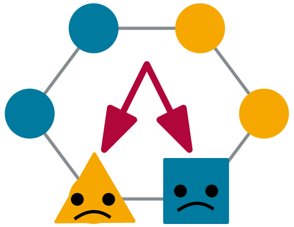
generalized ordinal potential function

single-peaked utilities:



Swap Game

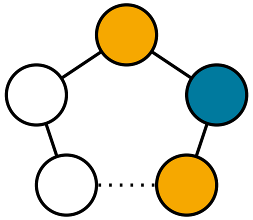
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths



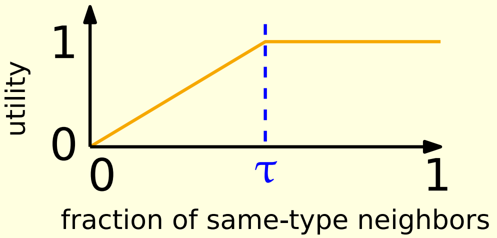
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



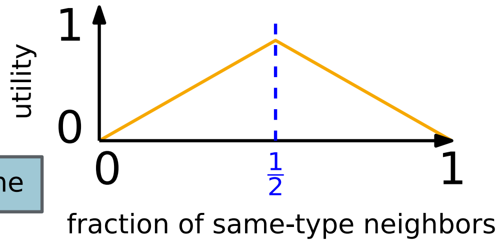
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap ⇒ Φ increases

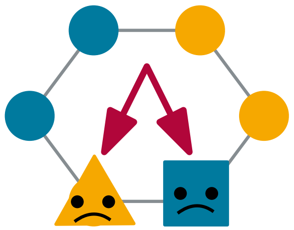
generalized ordinal potential function

single-peaked utilities:



Swap Game

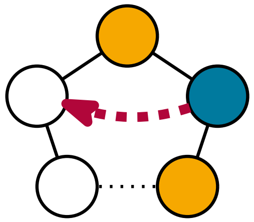
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths



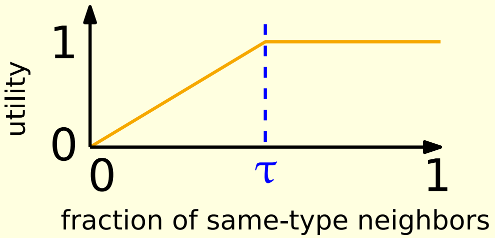
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



Example Result: Convergence guarantee on Δ-regular graph.

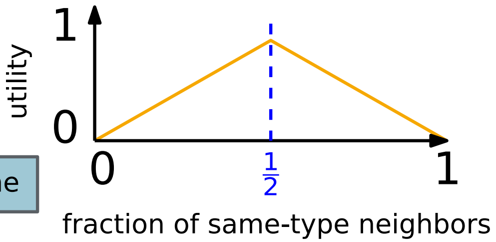
Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow$ Φ increases

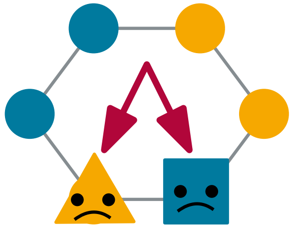
generalized ordinal potential function

single-peaked utilities:

Swap Game



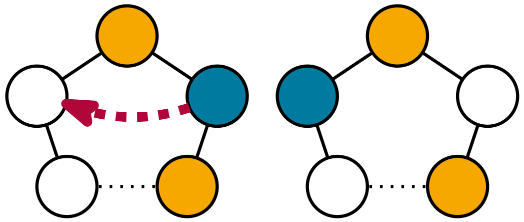
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths

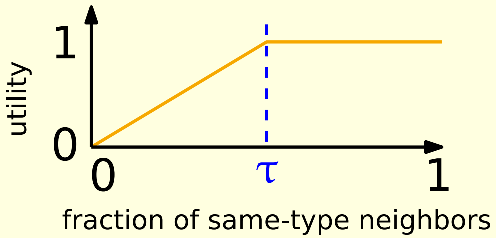


Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI’22]
[Friedrich, L., Molitor, Seifert, SAGT’23]
- non-monotone utilities

already seen: monotone utilities



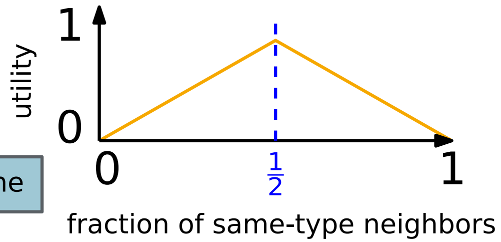
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges
- improving swap $\Rightarrow$ Φ increases

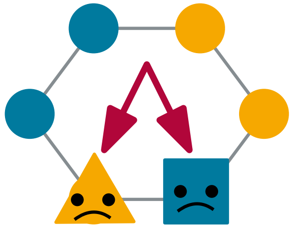
generalized ordinal potential function

single-peaked utilities:

Swap Game



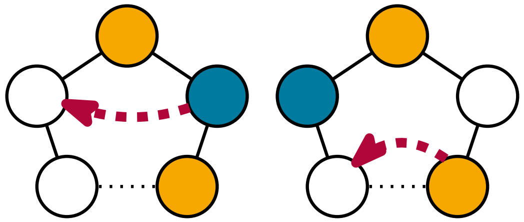
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths



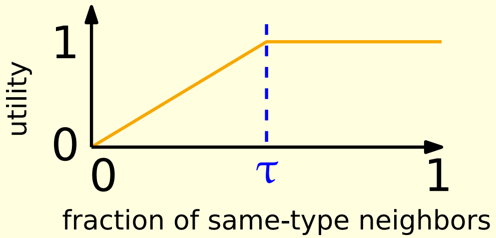
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI’22]
[Friedrich, L., Molitor, Seifert, SAGT’23]

- non-monotone utilities

already seen: monotone utilities



Example Result: Convergence guarantee on Δ-regular graph.

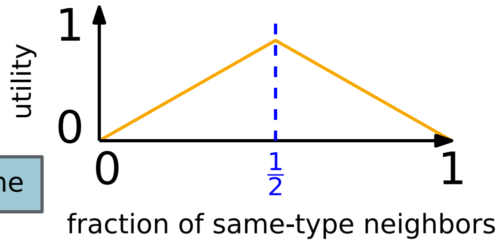
Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow$ Φ increases

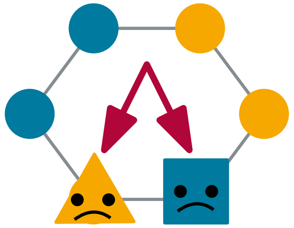
generalized ordinal potential function

single-peaked utilities:

Swap Game



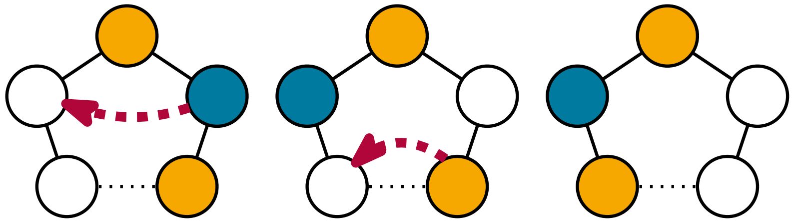
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths

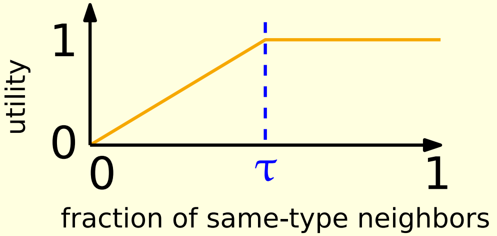


Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]
- non-monotone utilities

already seen: monotone utilities



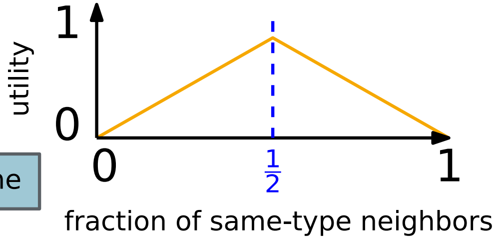
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges
- improving swap $\Rightarrow$ Φ increases

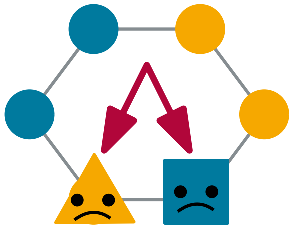
generalized ordinal potential function

single-peaked utilities:

Swap Game



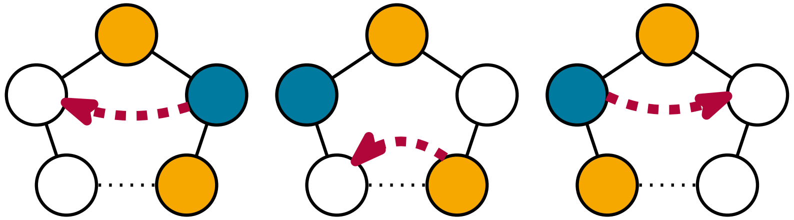
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths

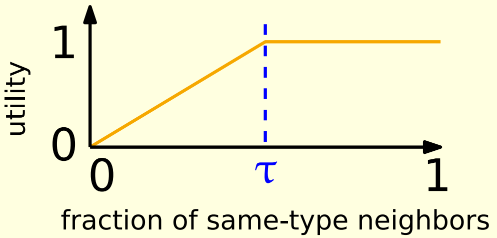


Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI’22]
[Friedrich, L., Molitor, Seifert, SAGT’23]
- non-monotone utilities

already seen: monotone utilities

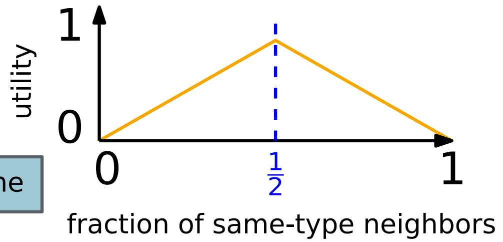


Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges
- improving swap $\Rightarrow$ Φ increases

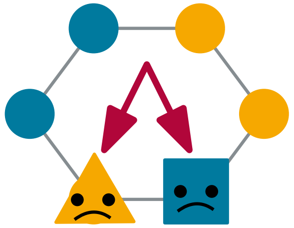
generalized ordinal potential function

single-peaked utilities:



Swap Game

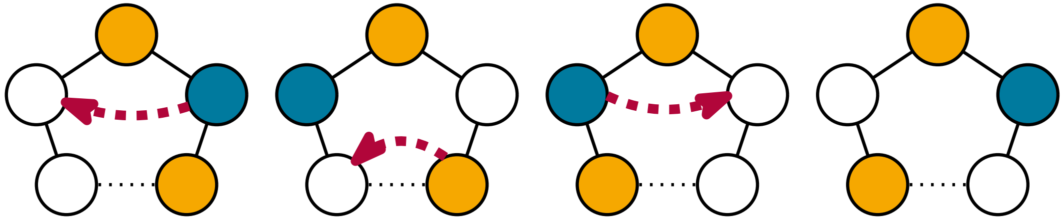
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths



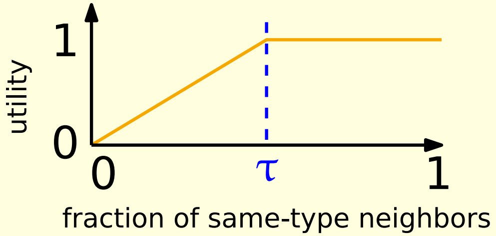
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



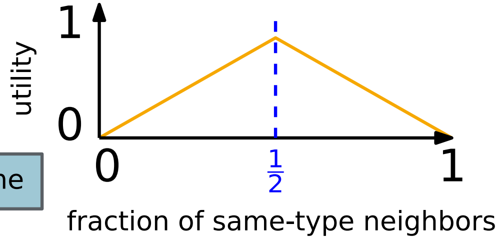
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow \Phi$ increases

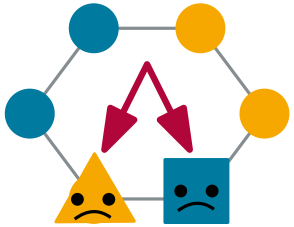
generalized ordinal potential function

single-peaked utilities:



Swap Game

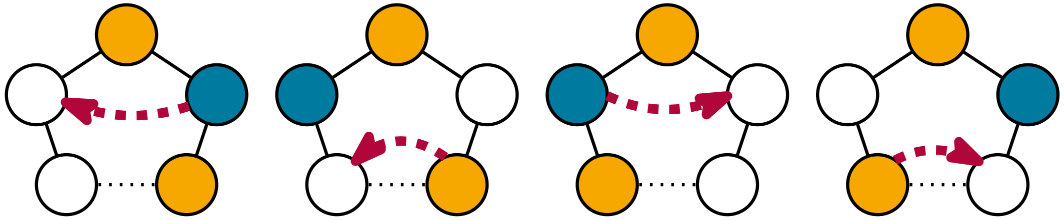
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths



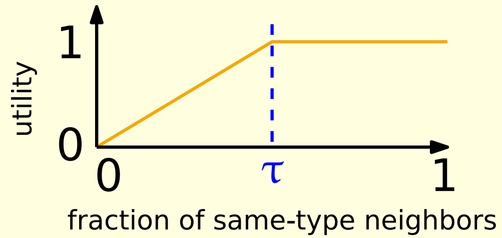
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



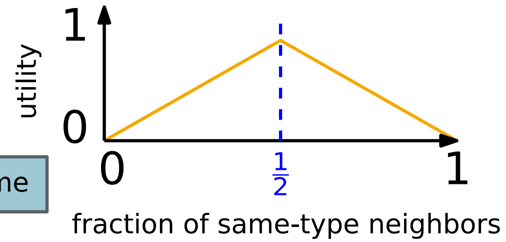
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap $\Rightarrow \Phi$ increases

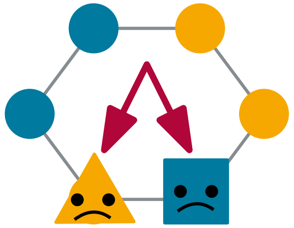
generalized ordinal potential function

single-peaked utilities:



Swap Game

Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

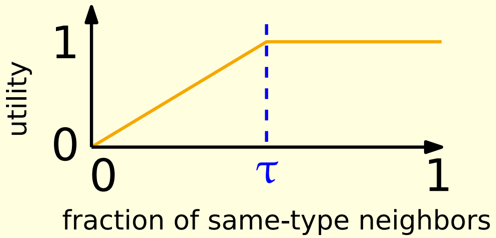
- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths
- peak $\leq \frac{1}{2}$: no convergence guarantee

Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]
- non-monotone utilities

already seen: monotone utilities

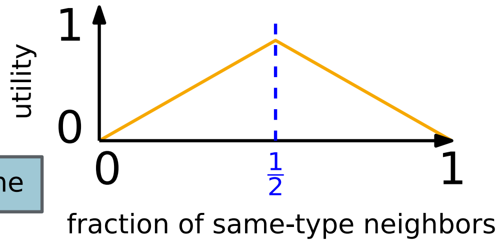


Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges
- improving swap $\Rightarrow \Phi$ increases

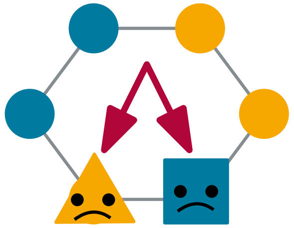
generalized ordinal potential function

single-peaked utilities:



Swap Game

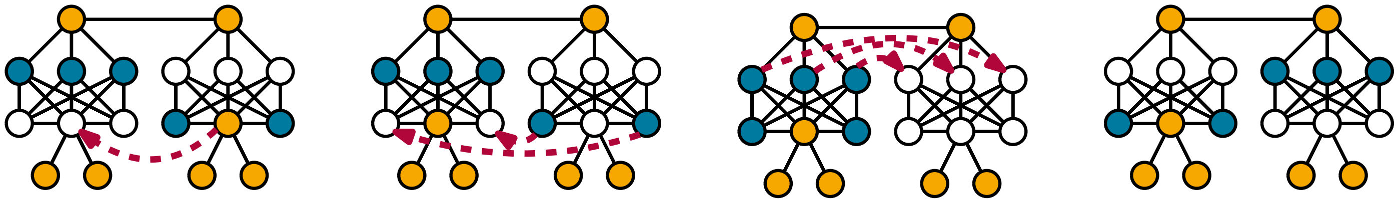
Example Results:



- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

Jump Game

- peak $\geq \frac{1}{2}$: no convergence, no equilibria even on rings or paths
- peak $\leq \frac{1}{2}$: no convergence guarantee



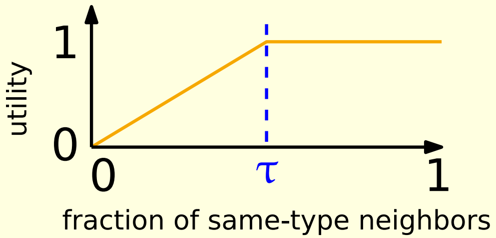
Single-Peaked Schelling Games

Can agents find JE/SE via iterative improving moves?

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

already seen: monotone utilities



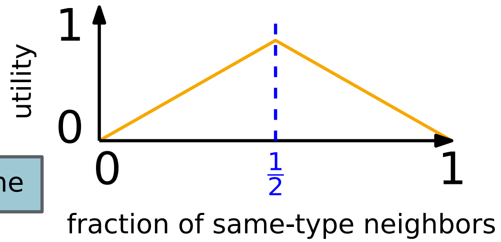
Example Result: Convergence guarantee on Δ-regular graph.

Proof: Φ = # of monochromatic edges

- improving swap ⇒ Φ increases

generalized ordinal potential function

single-peaked utilities:



Swap Game

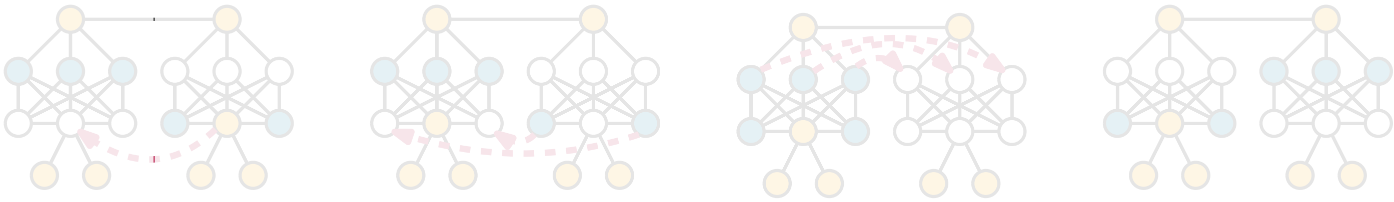
Jump Game

Example Results:

- peak $\leq \frac{1}{2}$: convergence guarantee on almost-regular graphs via Φ
- peak $> \frac{1}{2}$: no convergence, no equilibria even on regular graphs

- peak $\geq \frac{1}{2}$: no convergen
- peak $\leq \frac{1}{2}$: no convergence guarantee

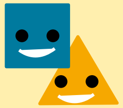
Observation: More realistic model has drastically different properties compared to Schelling's model.



Game-Theoretic Schelling Segregation

Game-theoretic version:

[Chauhan, L., Molitor: SAGT'18]



- model with strategic agents on an arbitrary host graph
- agents only care about their neighborhood; agent is content if $\frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}} \geq \tau$

$$cost_i(\mathbf{s}) = \max\left\{0, \tau - \frac{\#\text{agents of same type in neighborhood}}{\#\text{agents in neighborhood}}\right\}$$

Modified Schelling Games
[Kanellopoulos, Kyropoulou, Voudouris, SAGT'20]

- agent included in neighborhood

Schelling Models with Localized Social Influence: A Game-Theoretic Framework
[Chan, Irfan, Than, AAMAS'20]

- influence of social network

Tolerance is Necessary for Stability: Single-Peaked Swap Schelling Games
[Bilò, Bilò, L., Molitor, IJCAI'22]
[Friedrich, L., Molitor, Seifert, SAGT'23]

- non-monotone utilities

model variants

Diversity-Seeking Jump Games in Networks
[Narayanan, Sabbagh, SAGT'23]

- inverse model

Schelling Games with Continuous Types
[Bilò, Bilò, Döring, L., Molitor, Schmidt, IJCAI'23]

- type is number in $[0, 1]$

Not All Strangers Are the Same: The Impact of Tolerance in Schelling Games
[Kanellopoulos, Kyropoulou, Voudouris, MFCS'22]

- 1-dim ordering of types

Schelling Games with Continuous Types

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI’23]

- agents’ type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

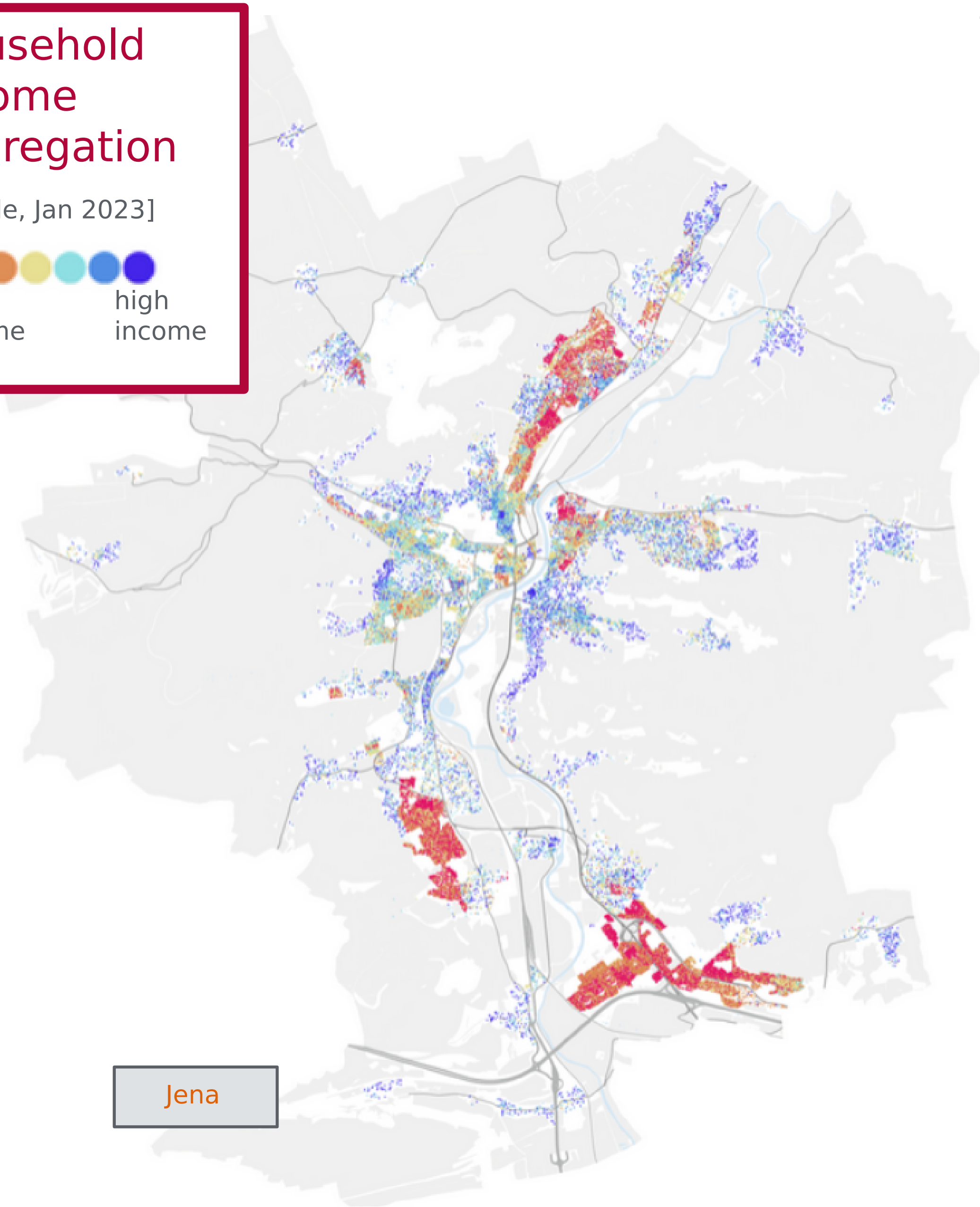
Schelling Games with Continuous Types

Household Income Segregation

[zeit.de, Jan 2023]

● ● ● ● ● ●

low income — high income



Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI’23]

- agents’ type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

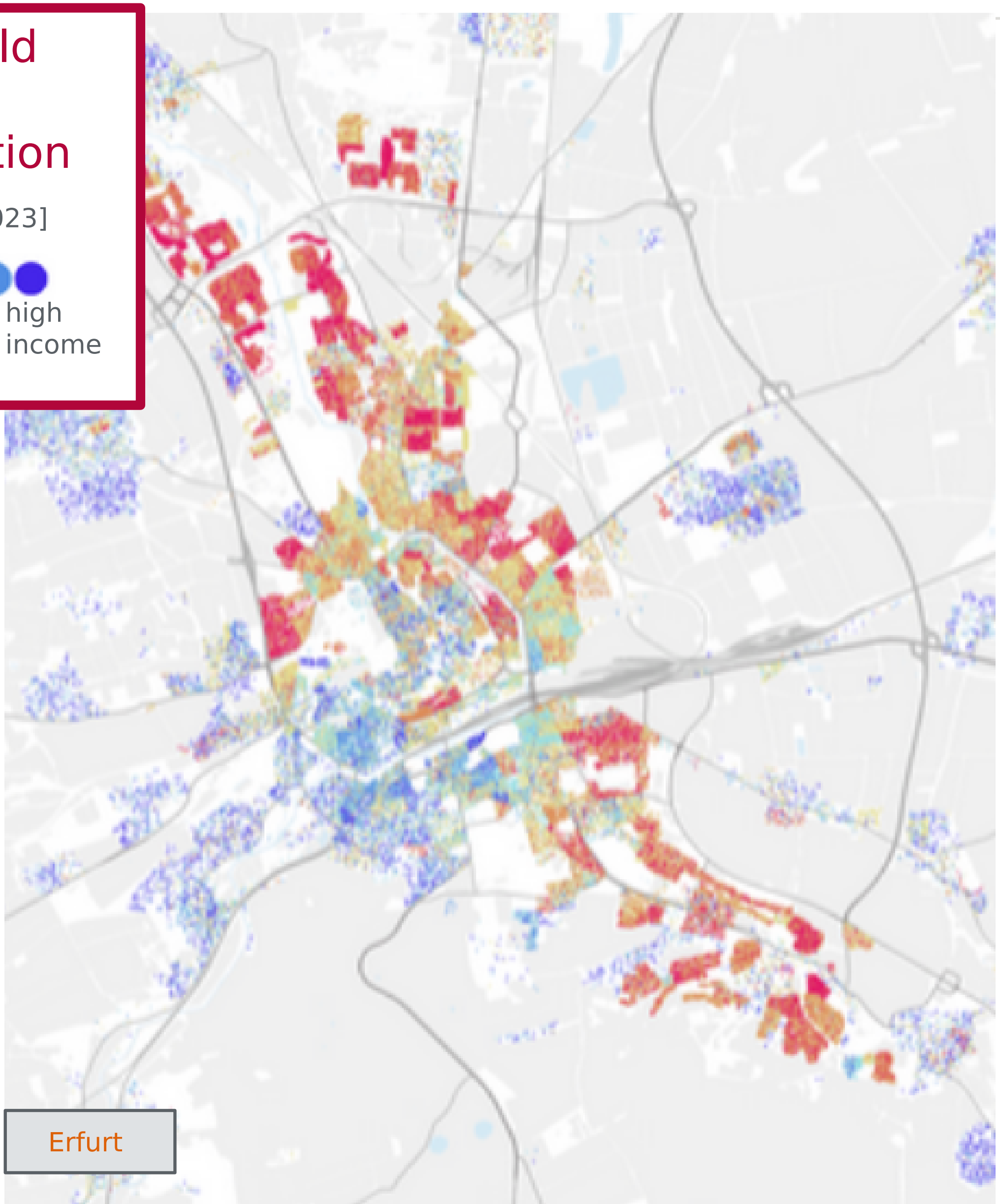
Schelling Games with Continuous Types

Household Income Segregation

[zeit.de, Jan 2023]

●●●●●●

low income — high income



Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

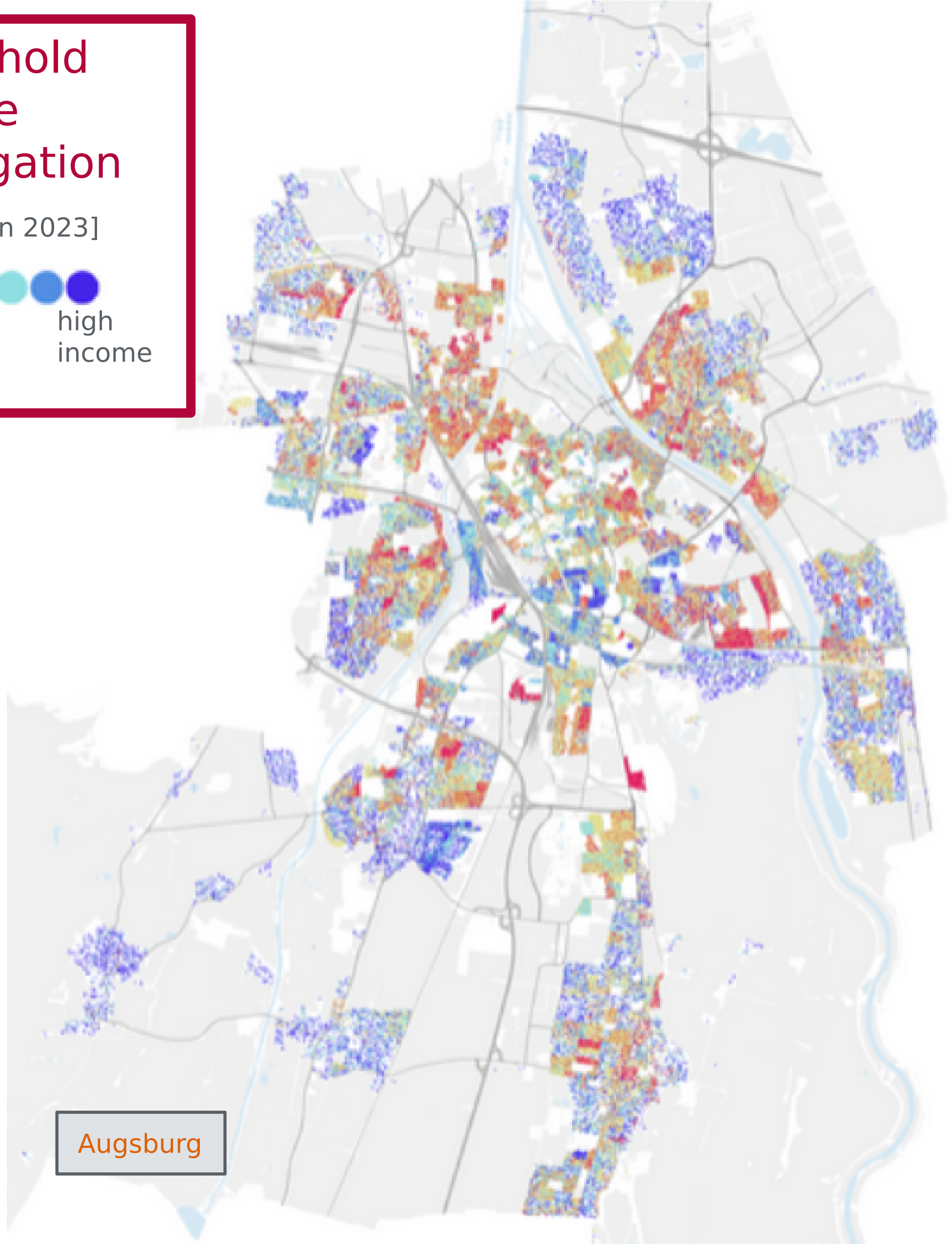
Schelling Games with Continuous Types

Household Income Segregation

[zeit.de, Jan 2023]

● ● ● ● ● ●

low income — high income



Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI’23]

- agents’ type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

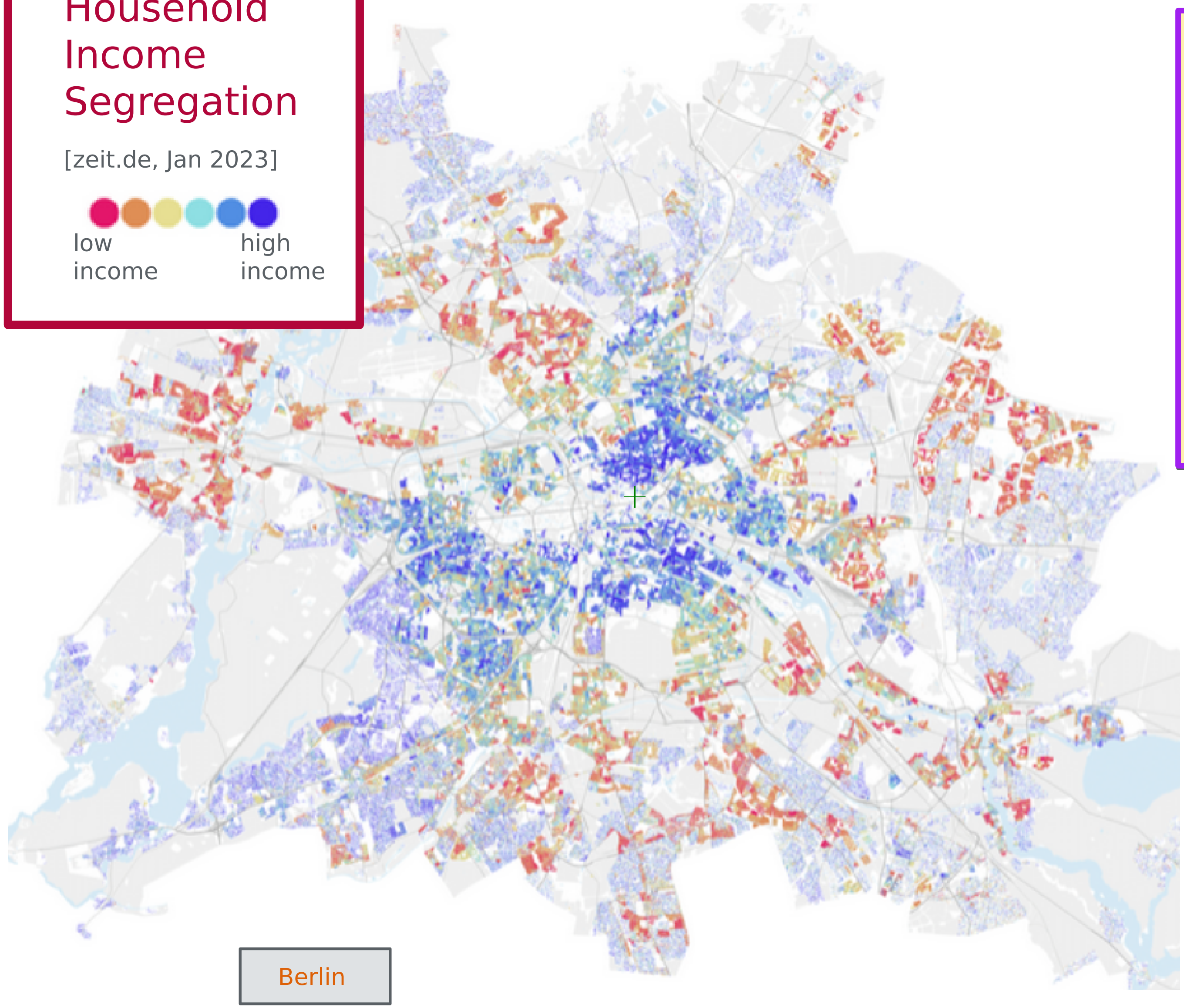
Schelling Games with Continuous Types

Household Income Segregation

[zeit.de, Jan 2023]

● ● ● ● ● ●

low income — high income



Schelling Games with Continuous Types

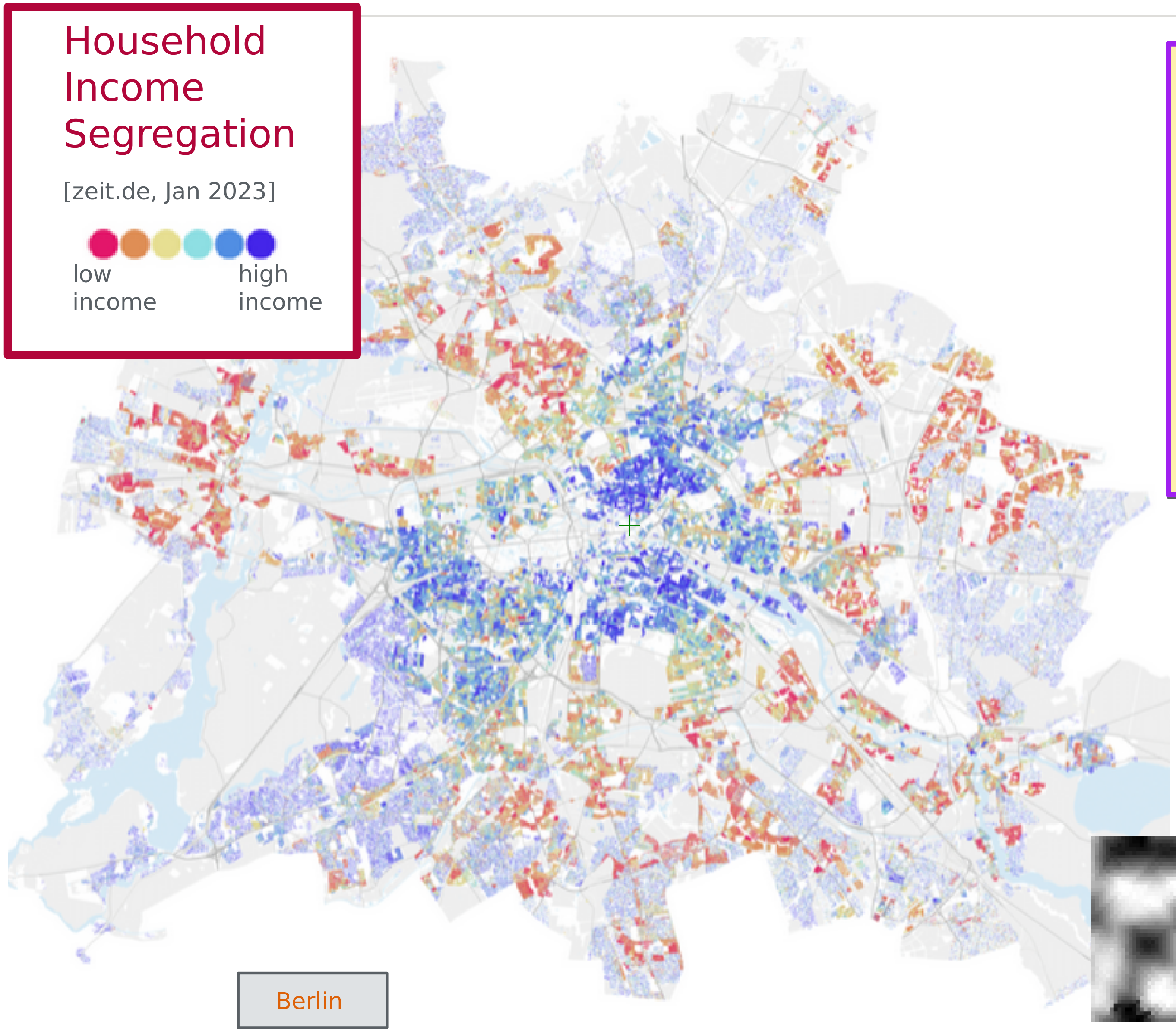
[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI’23]

- agents’ type is continuous in $[0, 1]$, e.g., age or income

0 1

- three types of cost functions: dist to max, dist to avg, cutoff

Schelling Games with Continuous Types



Household Income Segregation

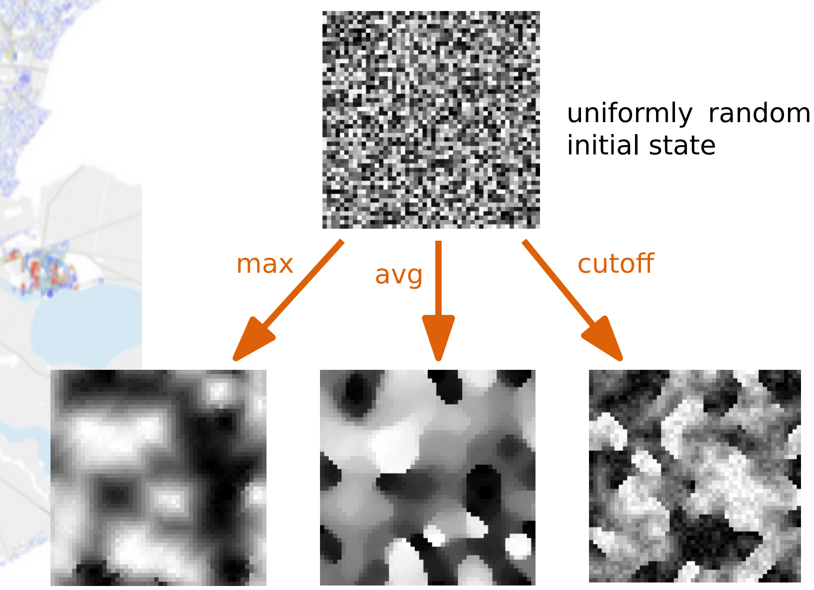
[zeit.de, Jan 2023]

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff



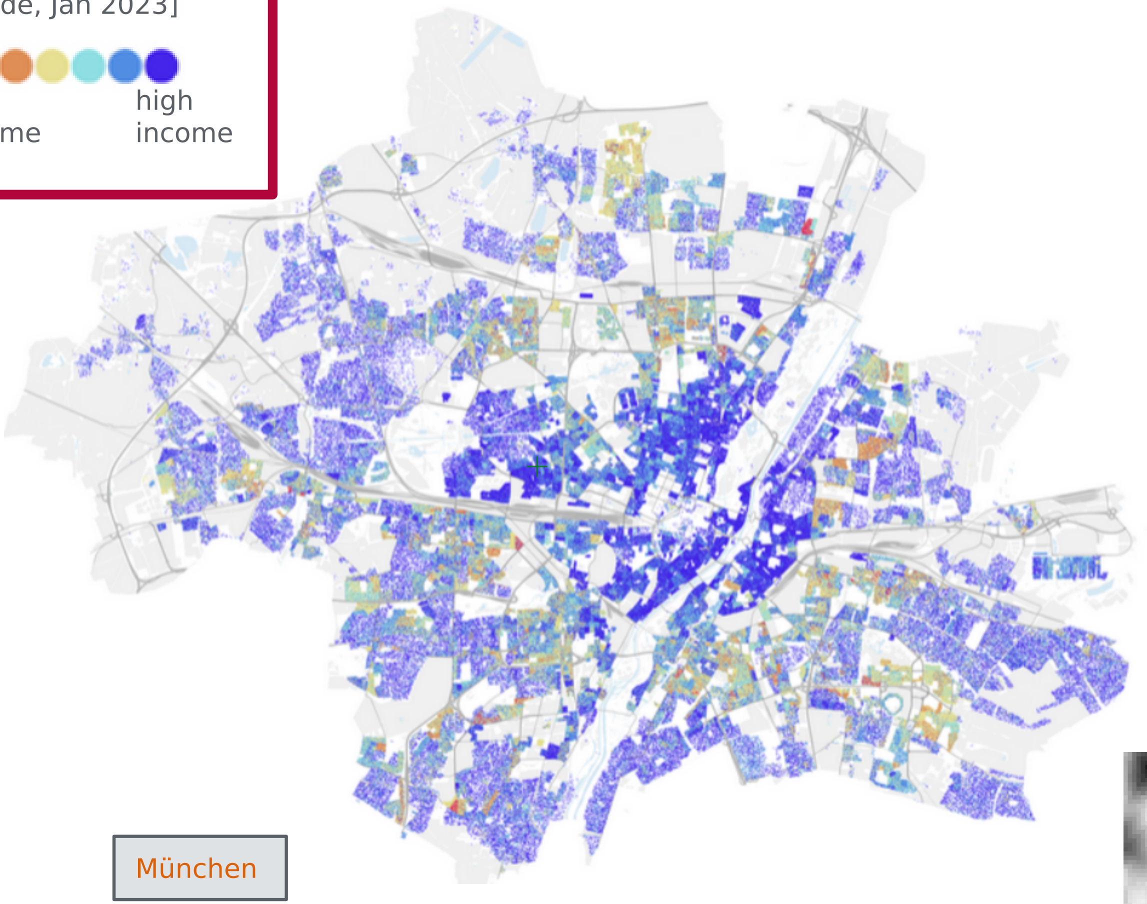
Schelling Games with Continuous Types

Household Income Segregation

[zeit.de, Jan 2023]

● ● ● ● ● ●
low income — high income

Fun fact:

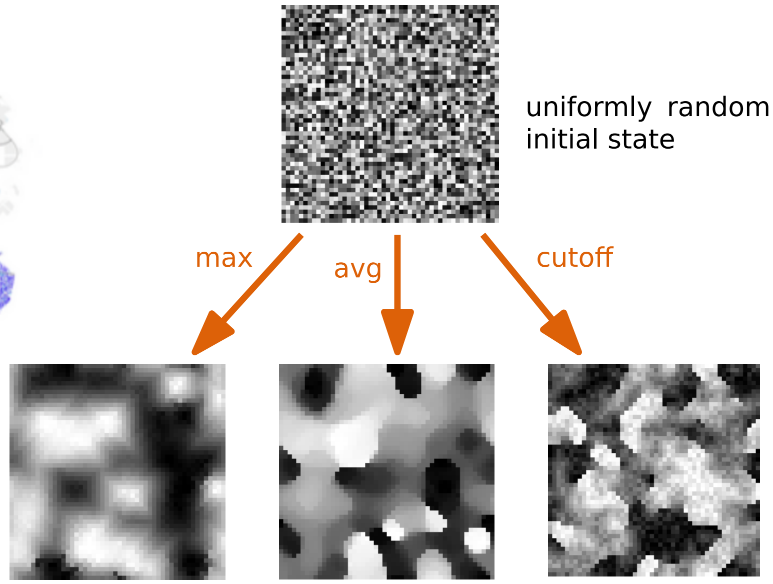


Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in [0, 1], e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff



Schelling Games with Continuous Types

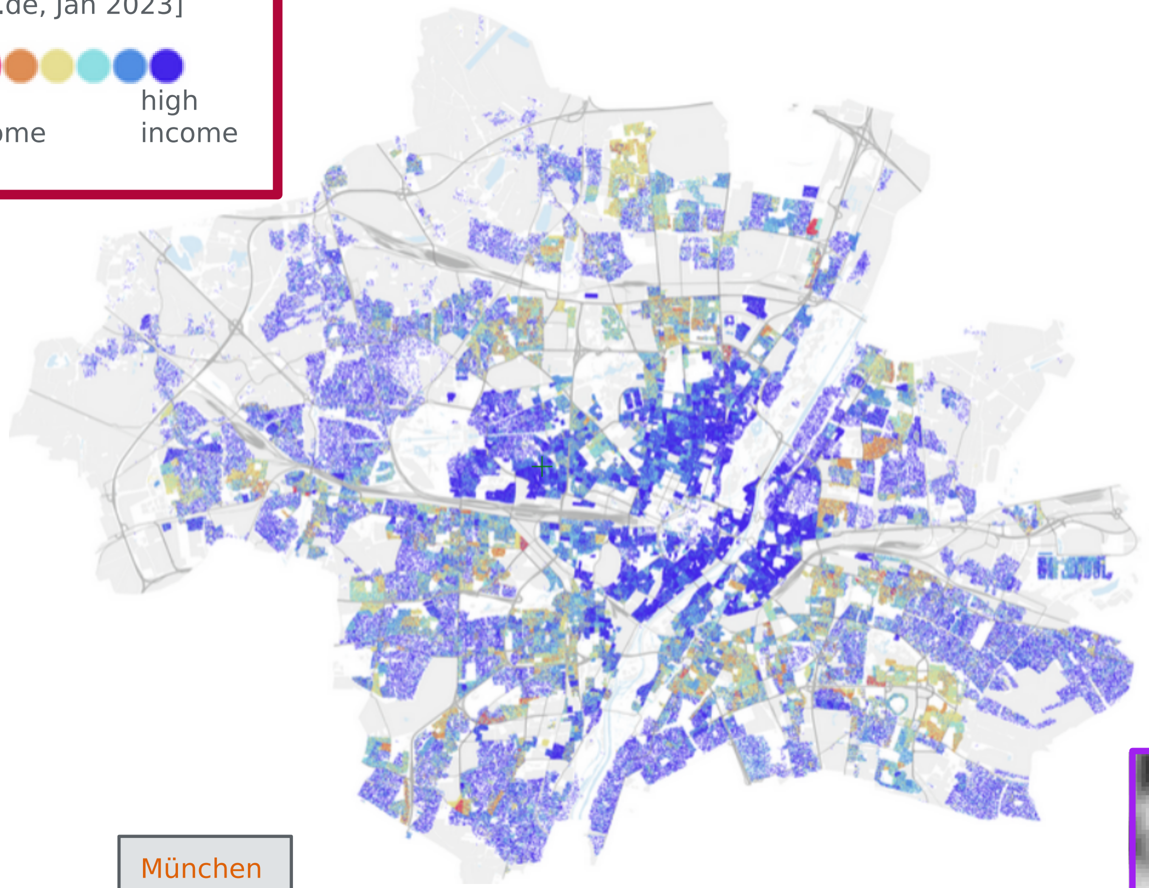
Household Income Segregation

[zeit.de, Jan 2023]

● ● ● ● ● ●

low income — high income

Fun fact:



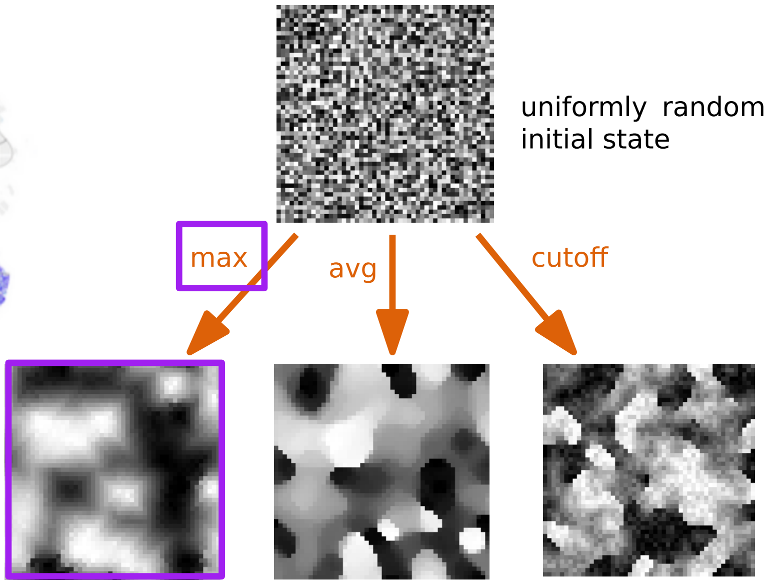
München

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI’23]

- agents’ type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff



uniformly random initial state

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \to [0, 1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \rightarrow [0, 1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \to [0, 1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Surprising answer: YES, on all graphs!

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \rightarrow [0,1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Surprising answer: YES, on all graphs!

Proofsketch:

Swap Game

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0,1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \to [0,1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Surprising answer: YES, on all graphs!

Proofsketch:

Swap Game

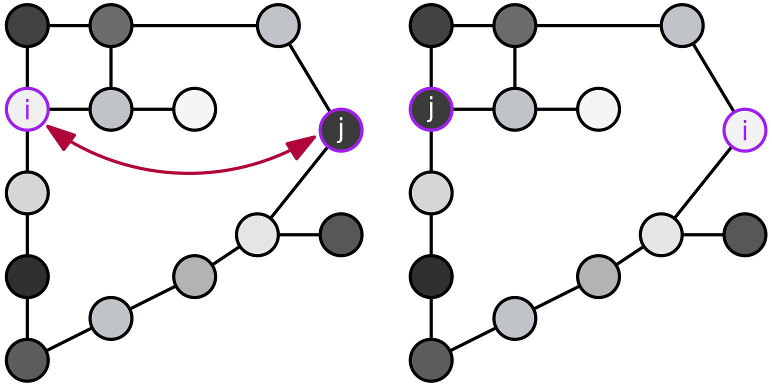
- consider placement σ and a swap of agents i and j that yields placement σ_{ij}

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0,1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff



placement σ

placement σ_{ij}

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \to [0,1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Surprising answer: YES, on all graphs!

Proofsketch:

Swap Game

- consider placement σ and a swap of agents i and j that yields placement σ_{ij}
- consider agent k with

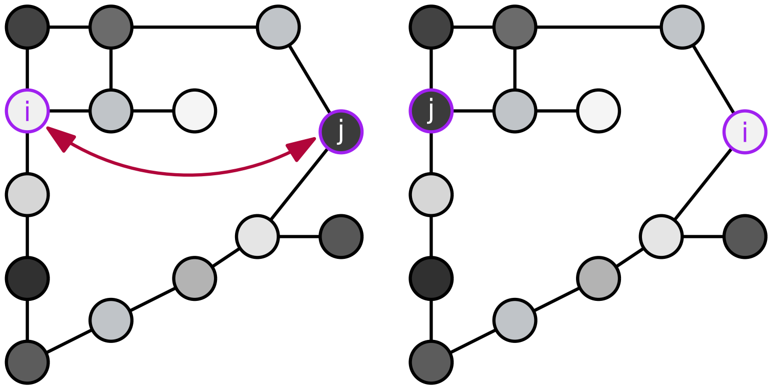
$$cost_k(\sigma_{ij}) \geq \max\{cost_i(\sigma), cost_j(\sigma)\}$$

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0,1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff



placement σ placement σ_{ij}

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \rightarrow [0, 1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Surprising answer: YES, on all graphs!

Proofsketch:

Swap Game

- consider placement σ and a swap of agents i and j that yields placement σ_{ij}
- consider agent k with

$$cost_k(\sigma_{ij}) \geq \max\{cost_i(\sigma), cost_j(\sigma)\}$$

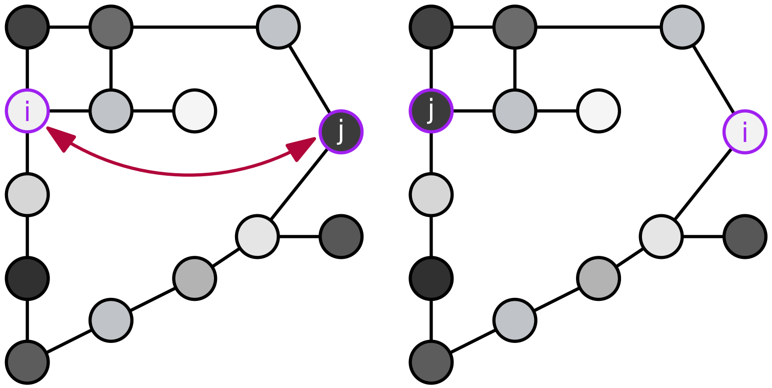
- then it holds that $cost_k(\sigma) \geq cost_k(\sigma_{ij})$

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI’23]

- agents’ type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff



placement σ placement σ_{ij}

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \rightarrow [0, 1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Surprising answer: YES, on all graphs!

Proofsketch:

Swap Game

- consider placement σ and a swap of agents i and j that yields placement σ_{ij}
- consider agent k with

$$cost_k(\sigma_{ij}) \geq \max\{cost_i(\sigma), cost_j(\sigma)\}$$

- then it holds that $cost_k(\sigma) \geq cost_k(\sigma_{ij})$
- consider non-increasingly sorted cost vector Φ

$$\Phi(\sigma) = (\bullet, \bullet, \mathrm{cost}_k(\sigma), \bullet, \bullet, \mathrm{cost}_j(\sigma), \bullet, \bullet, \bullet, \mathrm{cost}_j(\sigma), \bullet, \bullet, \bullet)$$

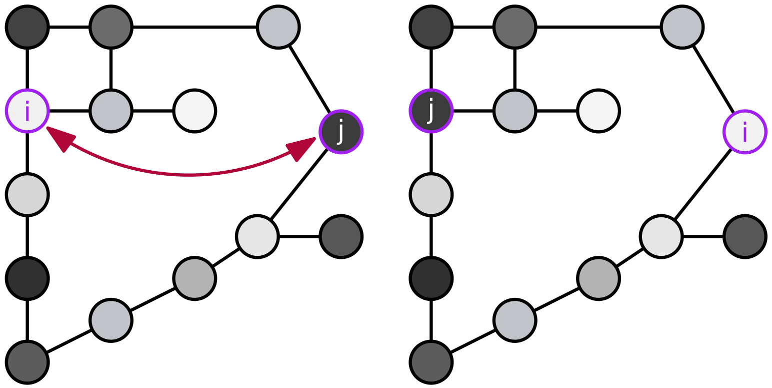
$$\Phi(\sigma_{ij}) = (\bullet, \bullet, \bullet, \mathrm{cost}_k(\sigma_{ij}), \bullet, \bullet, \bullet, \mathrm{cost}_j(\sigma_{ij}), \bullet, \bullet, \bullet, \mathrm{cost}_j(\sigma_{ij}), \bullet)$$

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff



placement σ placement σ_{ij}

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \to [0, 1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Surprising answer: YES, on all graphs!

Proofsketch:

Swap Game

- consider placement σ and a swap of agents i and j that yields placement σ_{ij}
- consider agent k with

$$cost_k(\sigma_{ij}) \geq \max\{cost_i(\sigma), cost_j(\sigma)\}$$

- then it holds that $cost_k(\sigma) \geq cost_k(\sigma_{ij})$
- consider non-increasingly sorted cost vector Φ

$$\Phi(\sigma) = (\bullet, \bullet, \text{cost}_k(\sigma), \bullet, \bullet, \text{cost}_j(\sigma), \bullet, \bullet, \bullet, \text{cost}_j(\sigma), \bullet, \bullet, \bullet)$$

$$\Phi(\sigma_{ij}) = (\bullet, \bullet, \bullet, \text{cost}_k(\sigma_{ij}), \bullet, \bullet, \bullet, \text{cost}_j(\sigma_{ij}), \bullet, \bullet, \bullet, \text{cost}_j(\sigma_{ij}), \bullet)$$

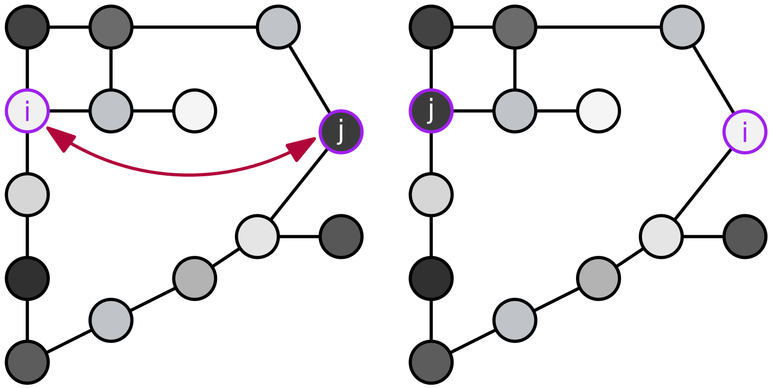
- improving swap by agents i and j $\Rightarrow$ vector Φ decreases lexicographically

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff



placement σ placement σ_{ij}

Schelling Games with Continuous Types

Maximum Type-Distance Game

- type function $t : [n] \to [0, 1]$
- utility of agent i: $cost_i(\sigma) = \max_{j \in N(i)} |t(i) - t(j)|$

Can agents find JE/SE via iterative improving moves?

Surprising answer: YES, on all graphs!

Schelling Games with Continuous Types

[Bilò, Bilò, Döring, L., Molitor, Schmidt. IJCAI'23]

- agents' type is continuous in $[0, 1]$, e.g., age or income

- three types of cost functions: dist to max, dist to avg, cutoff

Proofsketch:

Swap Game

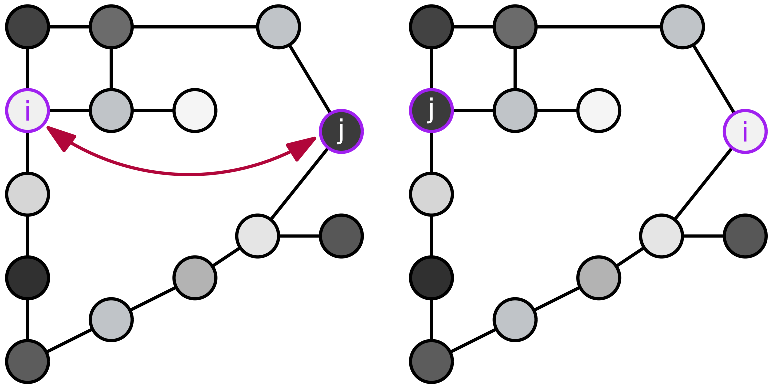
- consider placement σ and a swap of agents i and j that yields placement σ_{ij}
- consider agent k with

$$cost_k(\sigma_{ij}) \geq \max\{cost_i(\sigma), cost_j(\sigma)\}$$

- then it holds that $cost_k(\sigma) \geq cost_k(\sigma_{ij})$
- consider non-increasingly sorted cost vector Φ

$$\Phi(\sigma) = (\bullet, \bullet, \mathrm{cost}_k(\sigma), \bullet, \bullet, \mathrm{cost}_i(\sigma), \bullet, \bullet, \bullet, \mathrm{cost}_j(\sigma), \bullet, \bullet, \bullet)$$

$$\Phi(\sigma_{ij}) = (\bullet, \bullet, \bullet, \mathrm{cost}_k(\sigma_{ij}), \bullet, \bullet, \bullet, \mathrm{cost}_j(\sigma_{ij}), \bullet, \bullet, \bullet, \mathrm{cost}_i(\sigma_{ij}), \bullet)$$



placement σ placement σ_{ij}

- improving swap by agents i and j $\Rightarrow$ vector Φ decreases lexicographically

Jump Game analogous proof (have to deal with isolated agents) □

Schelling Games: Open Problems

Jump/Swap Equilibrium (JE/SE):
A strategy profile σ is a JE (SE) if and only if no agent (pair of agents) has an improving jump (swap).

Finite Improvement Property (FIP): A Schelling game has the FIP if starting from any strategy profile, a JE/SE is reached after a finite number of improving jumps/swaps.

Social Cost/Welfare:

- Utilitarian: sum of agents’ costs/utilities
- Degree of Integration: number of agents with at least one other-type neighbor

Schelling Games: Open Problems

Jump/Swap Equilibrium (JE/SE):
A strategy profile σ is a JE (SE) if and only if no agent (pair of agents) has an improving jump (swap).

- for many models JE/SE existence on many graph classes still open!
- characterization missing (only few examples of non-existence known)

Finite Improvement Property (FIP): A Schelling game has the FIP if starting from any strategy profile, a JE/SE is reached after a finite number of improving jumps/swaps.

Social Cost/Welfare:

- Utilitarian: sum of agents' costs/utilities
- Degree of Integration: number of agents with at least one other-type neighbor

Schelling Games: Open Problems

Jump/Swap Equilibrium (JE/SE):
A strategy profile σ is a JE (SE) if and only if no agent (pair of agents) has an improving jump (swap).

- for many models JE/SE existence on many graph classes still open!
- characterization missing (only few examples of non-existence known)

Finite Improvement Property (FIP): A Schelling game has the FIP if starting from any strategy profile, a JE/SE is reached after a finite number of improving jumps/swaps.

- for some models tight characterization of FIP still open
- convergence speed; influence of activation order
- which equilibria are reachable?

Social Cost/Welfare:

- Utilitarian: sum of agents’ costs/utilities
- Degree of Integration: number of agents with at least one other-type neighbor

Schelling Games: Open Problems

Jump/Swap Equilibrium (JE/SE):
A strategy profile σ is a JE (SE) if and only if no agent (pair of agents) has an improving jump (swap).

- for many models JE/SE existence on many graph classes still open!
- characterization missing (only few examples of non-existence known)

Finite Improvement Property (FIP): A Schelling game has the FIP if starting from any strategy profile, a JE/SE is reached after a finite number of improving jumps/swaps.

- for some models tight characterization of FIP still open
- convergence speed; influence of activation order
- which equilibria are reachable?

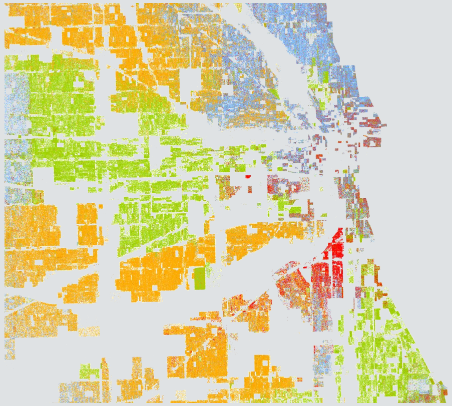
Social Cost/Welfare:
- Utilitarian: sum of agents' costs/utilities
- Degree of Integration: number of agents with at least one other-type neighbor

- better measures of obtained segregation strength missing
- how to find socially good equilibria?

Conclusion

Conclusion

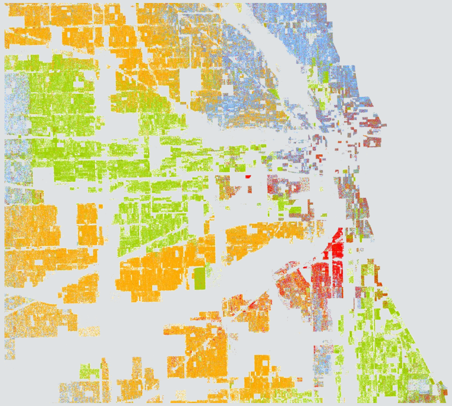
Real-World Segregation:



Residential Segregation in Chicago

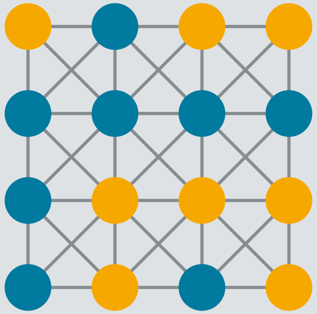
Conclusion

Real-World Segregation:



Residential Segregation in Chicago

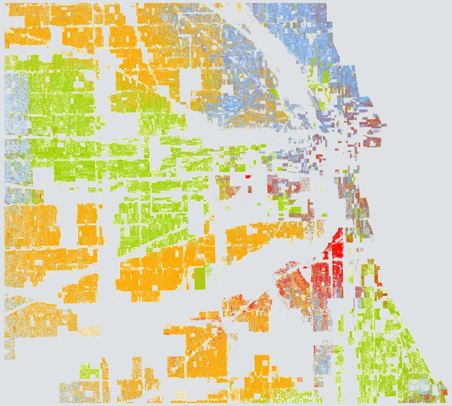
Schelling's Model:



- agent content if at least τ-fraction of neighbors is of own type

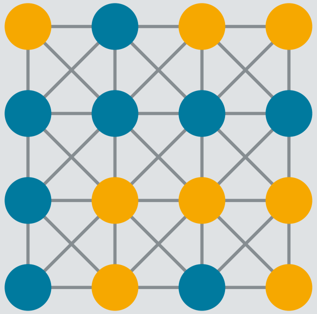
Conclusion

Real-World Segregation:



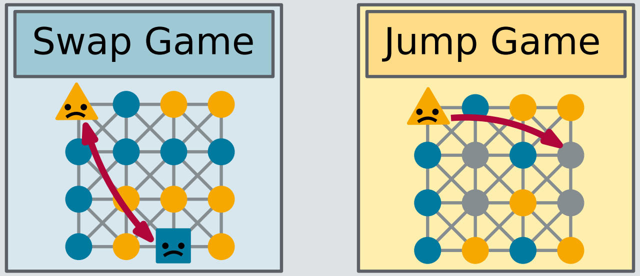
Residential Segregation in Chicago

Schelling’s Model:



- agent content if at least τ-fraction of neighbors is of own type

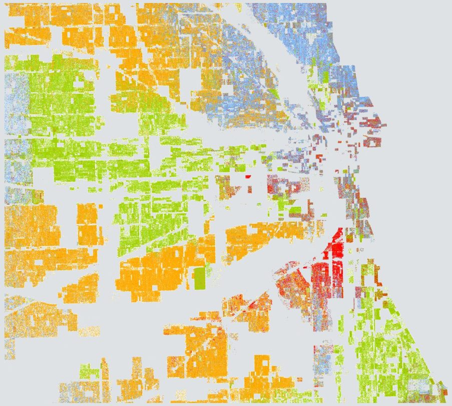
Game-Theoretic Version:



- agent utility depends on neighborhood type distribution
- equilibrium if no profitable swap/jump is possible

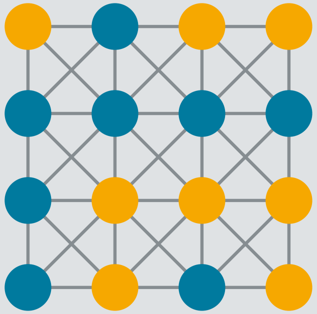
Conclusion

Real-World Segregation:



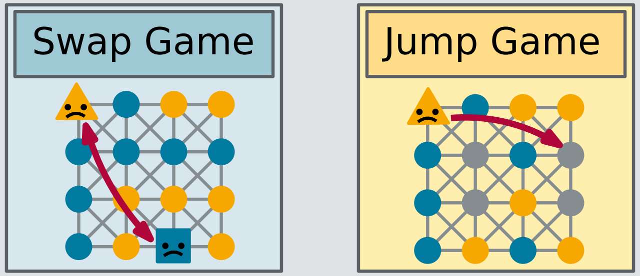
Residential Segregation in Chicago

Schelling’s Model:



- agent content if at least τ-fraction of neighbors is of own type

Game-Theoretic Version:

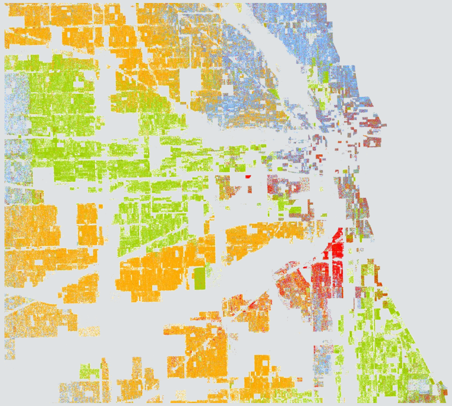


- agent utility depends on neighborhood type distribution
- equilibrium if no profitable swap/jump is possible

- we focussed on game dynamics and equilibrium existence

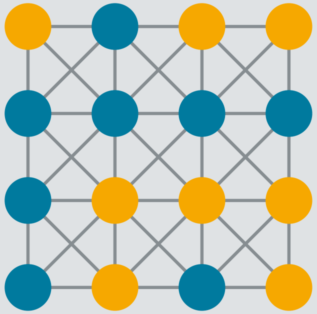
Conclusion

Real-World Segregation:



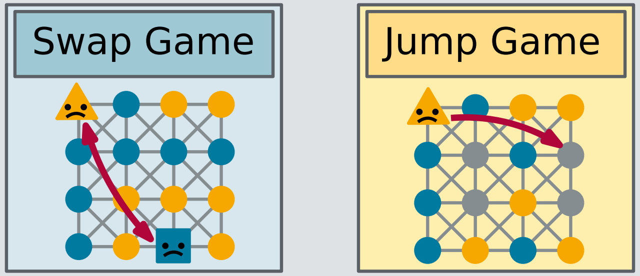
Residential Segregation in Chicago

Schelling’s Model:



- agent content if at least τ-fraction of neighbors is of own type

Game-Theoretic Version:



- agent utility depends on neighborhood type distribution
- equilibrium if no profitable swap/jump is possible

- we focussed on game dynamics and equilibrium existence

Many more questions to explore!

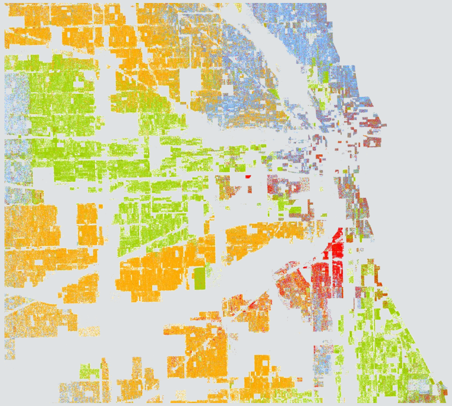
Interested?

We are looking for PhD-students and PostDocs!

mail: pascal.lenzner@uni-a.de

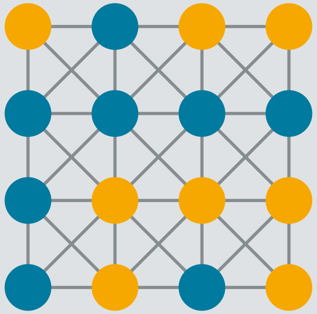
Conclusion

Real-World Segregation:



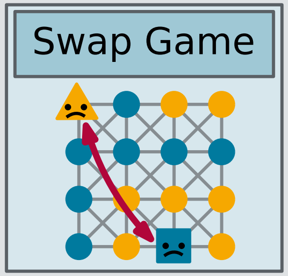
Residential Segregation in Chicago

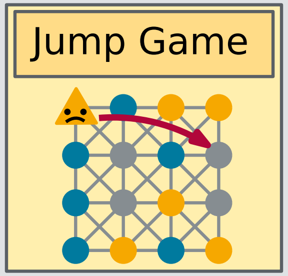
Schelling's Model:



- agent content if at least τ-fraction of neighbors is of own type

Game-Theoretic Version:





- agent utility depends on neighborhood type distribution
- equilibrium if no profitable swap/jump is possible

- we focussed on game dynamics and equilibrium existence

Play with Schelling's model?

https://ncase.me/polygons/

Many more questions to explore!

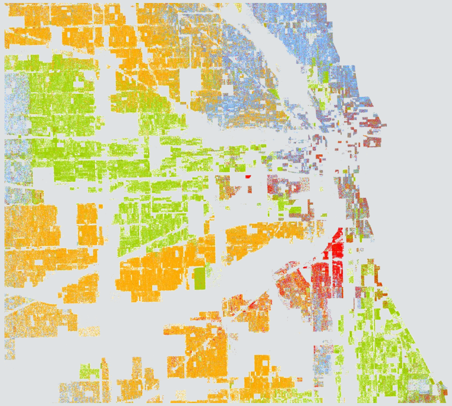
Interested?

We are looking for PhD-students and PostDocs!

mail: pascal.lenzner@uni-a.de

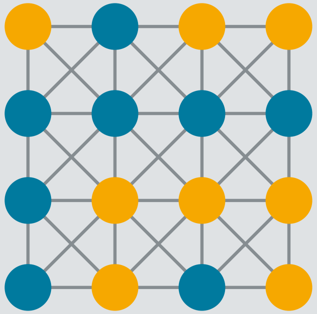
Conclusion

Real-World Segregation:



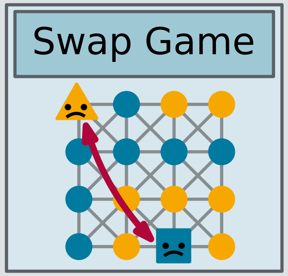
Residential Segregation in Chicago

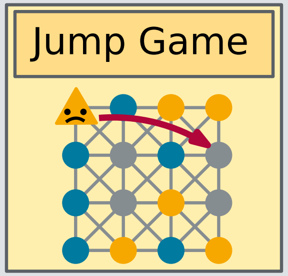
Schelling's Model:



- agent content if at least τ-fraction of neighbors is of own type

Game-Theoretic Version:



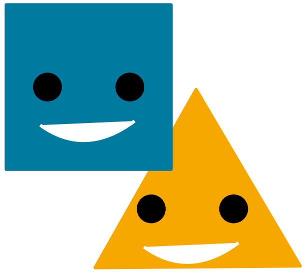


- agent utility depends on neighborhood type distribution
- equilibrium if no profitable swap/jump is possible

- we focussed on game dynamics and equilibrium existence

Play with Schelling's model?

https://ncase.me/polygons/



Questions?

Many more questions to explore!

Interested?

We are looking for PhD-students and PostDocs!

mail: pascal.lenzner@uni-a.de