Parameterized Complexity of Segment Routing

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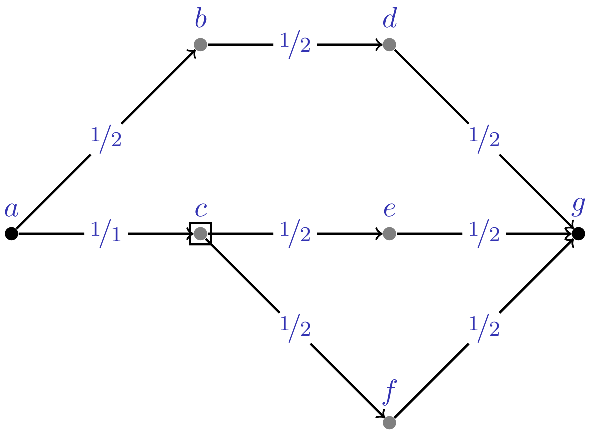
Routing in the Internet



Input:

- the internet (a graph)
- Traffic demand: e. g. $a \to g$

Routing protocol:

- traffic send along shortest path
- multiple shortest paths
  $\rightsquigarrow$ split traffic evenly

Traffic control / Operation:

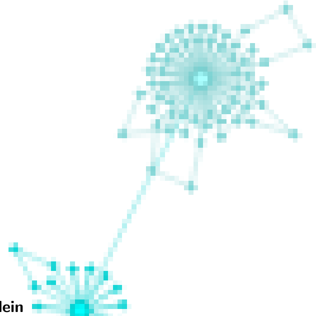
- add waypoints, e. g. c
  $a \to g \rightsquigarrow a \to c \to g$

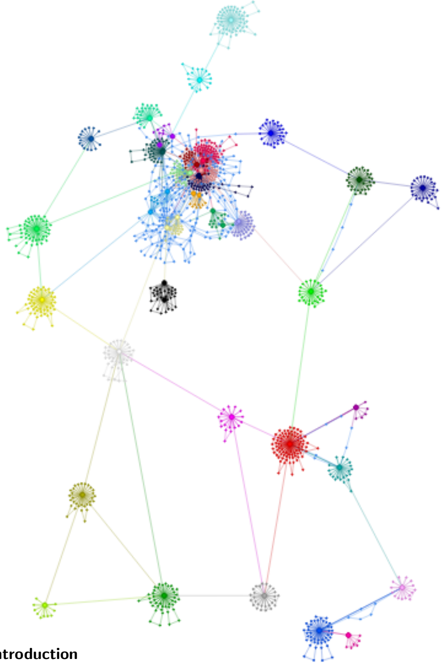
Central question: How to determine waypoints?

Real world networks

France backbone network:

- 1200 routers
- 4000 links
- 100k demands
  (on a typical working day)





Segment Routing

Segment Routing

Input:

- network $(G = (V, E), \omega, c)$,
- budget $k \in \mathbb{N}$,
- d demands $D = \{(s_1, t_1, b_1), \ldots, (s_d, t_d, b_d)\}$

Question:

- $\exists$ feasible routing scheme?

Legend:

- $G = (V, E)$: undirected graph
- ω: edge length (influencing shortest paths)
- c: edge capacities (**do not exceed!!**)
- k: allowed number of waypoints per demand (**small**)
- demand (s_i, t_i, b_i): source $s_i \in V$, destination $t_i \in V$, demand volume $b_i \in \mathbb{N}$

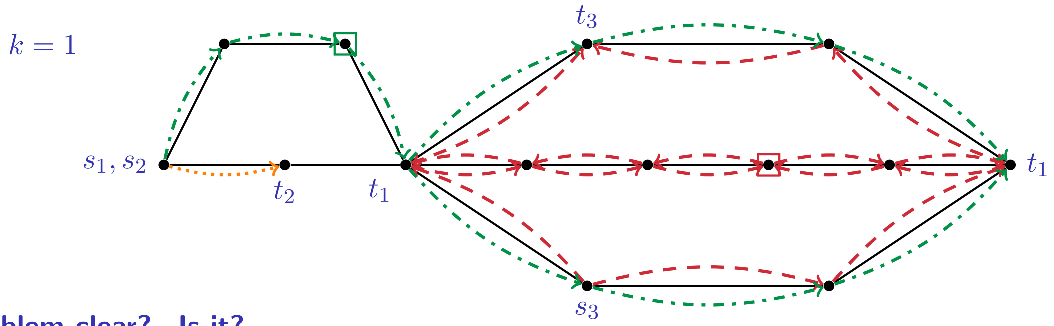
What we (mostly) study:

Unit Segment Routing: $\omega \equiv 1$, $c \equiv 1$, all $b_i = 1$

Unit Segment Routing

Unit Segment Routing

Input: A graph $G = (V, E)$, $k \in \mathbb{N}$, and demands $D = \{(s_1, t_1), \ldots, (s_d, t_d)\}$.
Question: Is there a feasible routing scheme in G using at most k waypoints for each demand?



Problem clear? Is it?

Results for (Unit) Segment Routing

Known:

- brute force: $O(n^{kd}mkd)$ [folklore]
- Solve via Integer Linear Programming or Constraint Programming
  [Bhatia, Hao, Kodialam, Lakshman; INFOCOM 2015], [Callebaut, De Boeck, Fortz; Networks 2023], [Hartert, Schaus, Vissicchio, and Bonaventure; CP 2015], [Jadin, Aubry, Schaus, Bonaventur; INFOCOM 2019]
- **Segment Routing**: weakly NP-hard [Hartert, Schaus, Vissicchio, and Bonaventure; CP 2015]

Our results:

	param.	undirected	directed
unit	k	NP-hard if $k=1$	NP-hard if $k=1$
	$k+\tau$	NP-hard if $k=1,\tau=4$	?
	d	NP-hard if $d=4$	NP-hard if $d=3$
	$d+k$	W[1]-hard	W[1]-hard
unary	n	W[1]-hard even if $k=1$	W[1]-hard even if $k=1$
	$k+\tau$	NP-hard if $k=1,\tau=2$	NP-hard if $k=1,\tau=2$

Legend:

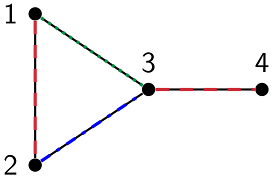
- n: # vertices
- m: # edges
- k: # waypoints
- d: # demands
- τ: vertex cover number (delete τ vertices to remove all edges)

NP-hardness

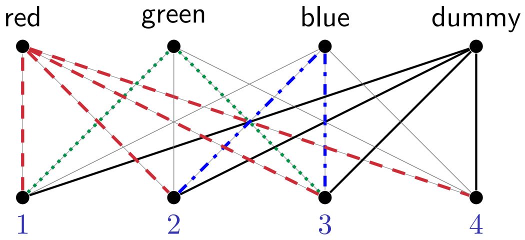
Reduce **3-Edge Coloring** to **Unit Segment Routing**

3-Edge Coloring

Input: An undirected graph $G = (V, E)$.
Question: Is there a proper edge coloring of G, i. e. an edge coloring such that no two adjacent edges have the same color, using at most 3 colors?



G



Demands $\widehat{=}$ edges in G: $(1,2),(1,3),(2,3),(3,4)$

dummy demands: $(1, \text{dummy}), (2, \text{dummy}), (3, \text{dummy}), (4, \text{dummy})$

Cactus graphs

Definition

A graph is a cactus if every edge is part of at most one cycle.

$\geq 1/3$ of networks in Internet Topology Zoo dataset are cactus graphs

[Amiri, Foerster, Jacob, Schmid; ACM SIGCOMM Computer Communication Review; 2018]

Results:

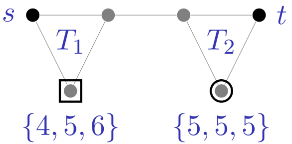
- **Segment Routing** (weakly) NP-hard on cycles

  [Hartert, Schaus, Vissicchio, and Bonaventure; CP 2015]

- **Segment Routing** (strongly) NP-hard on cactus graphs ($k = 1$)

  Proof by example:

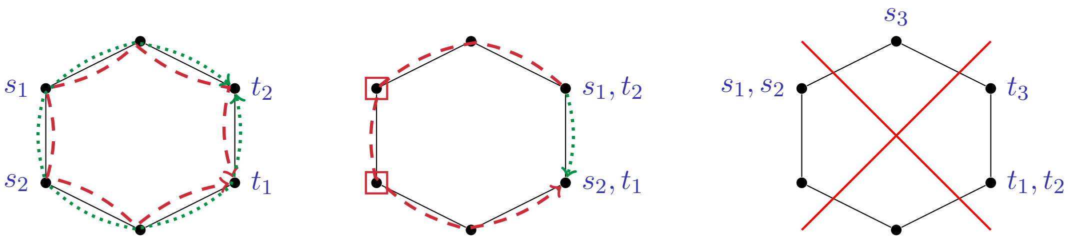
  - 6 items of size $4, 5, 5, 5, 5, 6$; bin size 15
  - solution: $\{4, 5, 6\}, \{5, 5, 5\}$



- Polynomial-time algorithm on cactus graphs for **Unit Segment Routing**

Cycles, Cacti & Hope

Unit Segment Routing on cycles $\rightsquigarrow$ case distinctions:



- Extension to cactus graphs: dynamic programming (non-trivial)
- **Wishful thinking:** Extend to non-unit weights with (fpt-)approximation?! (using approximation for **Bin Packing**)

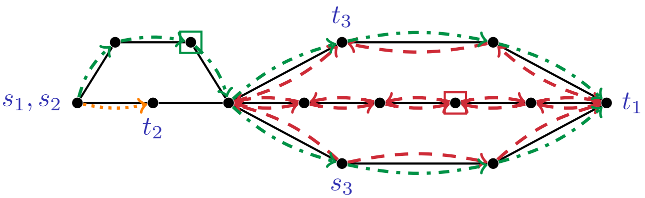
Conclusion

Summary:

- **Segment Routing** very hard
- special structure: very sparse networks, few waypoints, many demands

Open questions:

- structure of demands exploitable?
- approximation



Thank you!