Reducing Stochastic Games to Semidefinite Programming

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Joint work with Georg Logo and Mateusz Skomra

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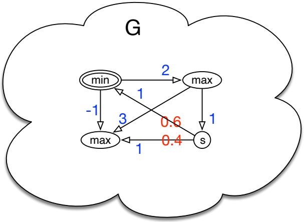
Outline

1. (Stochastic) Mean Payoff Games



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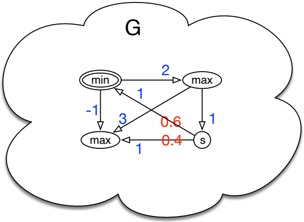
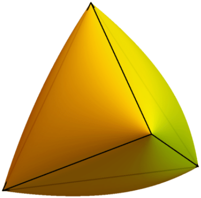
2. Semidefinite Programming (SDP)





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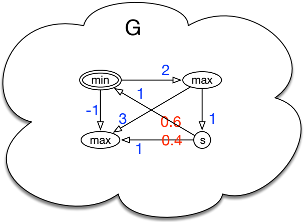

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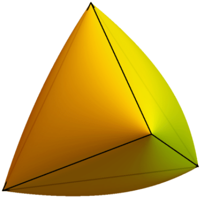

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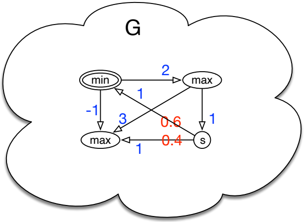

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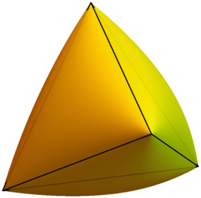

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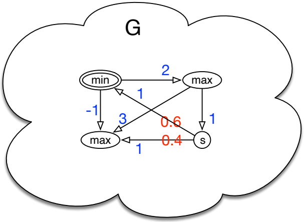

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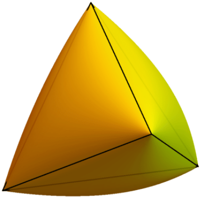

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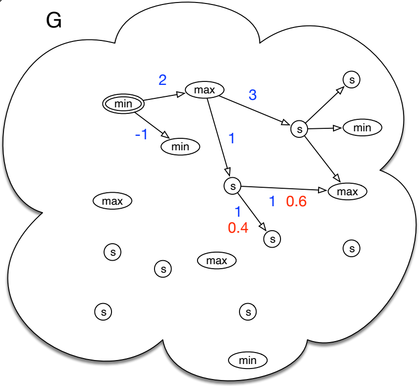

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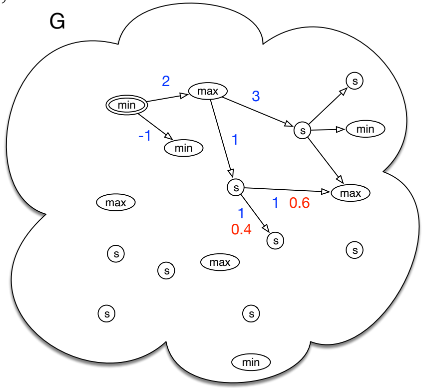

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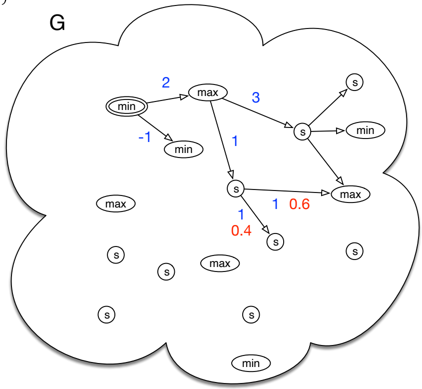

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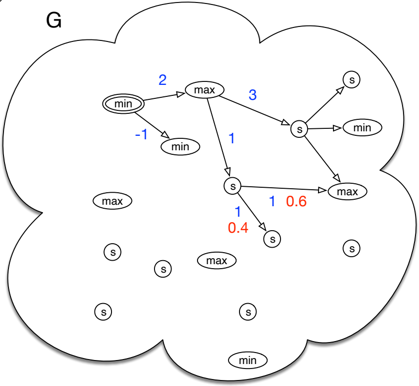

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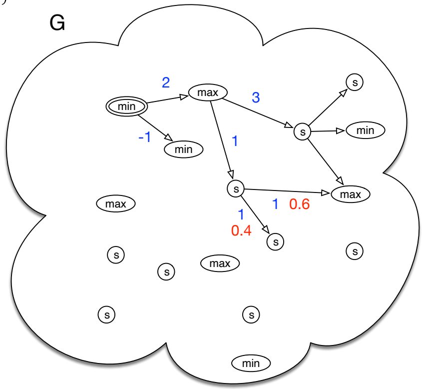

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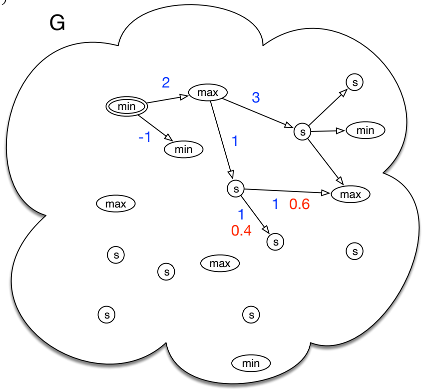

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- Quasi-polynomial algorithms for parity games (Calude+Jain+Khoussainov+Li+Stephan'2022) don't work for MPGs

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Example. $S = \left\{(x, y) \mid \begin{pmatrix} 1 & x \\ x & y \end{pmatrix} \succeq 0\right\} = \left\{(x, y) \mid y - x^2 \geq 0\right\}$

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Theorem (B.+Loho+Skomra'2025).

There is a polynomial-time reduction from simple stopping stochastic games to the feasibility problem for SDP.

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An **instance** of the Max-average constraint satisfaction problem:
consists of conjunction of constraints of the form

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See, e.g., Bertrand+Bouyer-Decitre+Fijalkov+Skomra'2023.

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$\mathtt{x}_{i_0} \leq \mathtt{x}_{i_1} + \cdots + \mathtt{x}_{i_k}$	for every constraint $x_{i_0} \leq \max(x_{i_1}, \dots, x_{i_k})$ in ϕ,
$\mathtt{x}_{i_0}^2 \leq \mathtt{x}_{i_1} \mathtt{x}_{i_2}$	for every constraint $x_{i_0} \leq \frac{x_{i_1} + x_{i_2}}{2}$ in ϕ,
$\mathtt{x}_{i_0} = t^c$	for every constraint $x_{i_0} = c$ in ϕ,
$\mathtt{x}_{i_0} \geq 0$	for every $i_0 \in \{1, \dots, n\}$.

Non-archimedian SDPs

K: set of Puiseux series over $\mathbb{R}$

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Max-average instance satisfiable if and only if this SDP has solution in K.

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- This reduction also works if t is replaced by a large number.

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Max-average instance satisfiable if and only if this SDP has solution in K.

Idea:

- This reduction also works if t is replaced by a large number.
- Using quantifier-elimination: double exponential is large enough.

Large Numbers

Problem: our reduction is not polynomial-time!

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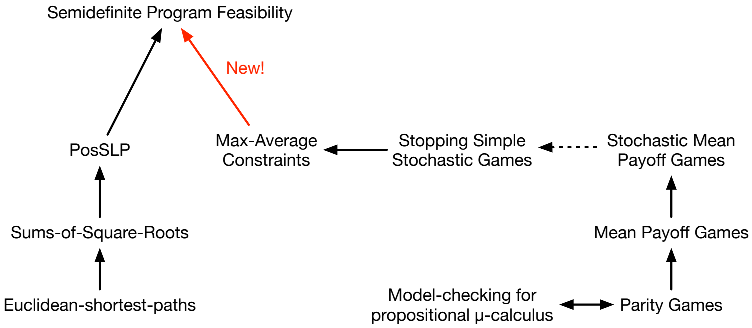
Problem: need to express $x = 2^{2^n}$ for our reduction.

Solution: Use SDP duality theory to find small expression for $x = 2^{2^n}$.

Summary



Semidefinite Program Feasibility
New!
PosSLP
Max-Average Constraints
Stopping Simple Stochastic Games
Stochastic Mean Payoff Games
Sums-of-Square-Roots
Mean Payoff Games
Euclidean-shortest-paths
Model-checking for propositional μ-calculus
Parity Games